Strategies for Optimizing GEO Debris Search

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ABSTRACT

An effective optical debris search strategy at GEO requires a compromise between high search rates (greater coverage, leak proof fence) and sufficient dwell times within a sensor's individual pixel (greater sensitivity). In this study we simulate various search strategies to evaluate their effectiveness in meeting these two objectives. We evaluate three basic search strategies using a specialized step and stare scan used by the ESA Space-Debris Telescope on Tenerife, Canary Islands [1]: (1) one-dimensional scans centered on the GEO belt, (2) multiple one-dimensional, constant declination scans and (3) a square wave scan pattern centered on the GEO belt. We evaluate each of these search scans using various encounter statistics to determine which provides the greatest coverage. We also investigate the possibility of minimizing solar phase angles during the search to achieve increased target brightness. Based on these results we focus on the one-dimensional scans centered on the GEO belt. We also examine the likelihood that targets with high inclinations and/or drift rates (both typical characteristics of debris objects) will *not* stay in a sensor's single pixel field of view for the full integration time. We developed analytical equations to (1) calculate the expected dwell time within a single pixel and (2) calculate the likelihood that a streak will dwell within a single pixel for the full integration time, given the relative angular rate of the satellite and the sensor detector size and integration time. An expected pixel dwell time that is less than the full integration time could be justification for reducing the integration time, which would result in increased scan rates without sacrificing sensitivity.

1. INTRODUCTION

An accurate knowledge of the debris population at geosynchronous earth orbit (GEO) is of importance to any owner/operator of satellite missions operating in this orbit regime. GEO debris surveys are conducted periodically by various organizations to find new (previously uncataloged) or lost debris objects [1,2,3,4]. Oftentimes the surveys are not designed to search the full volume of space visible from a given site, but rather to focus on specific regions of interest. Our study examines optimal strategies for large area searches for high-inclination GEO objects. The purpose of this study is to evaluate various search routines that target all objects within a sensor's field of regard while maximizing sensitivity.

An effective debris search strategy at GEO requires high search rates to achieve greater coverage and to create a leak proof fence and sufficient dwell times within a sensor's individual pixel to achieve greater sensitivity. As these requirements are competing, an objective of this study was to find the optimal solution that maximizes both search rate and sensitivity.

In this study we simulated various search strategies to evaluate their effectiveness in meeting these two objectives. The effectiveness of each scan pattern was evaluated using the metrics outlined in the table below (where an encounter is defined as the passage of an object through the sensor field of view).

Table 1. Overview of metrics used to evaluate the simulated search strategies

Metric	Objective
Number of encounters for individual objects on a single night	Obtain multiple observations of individual objects over the course of a single night to improve the quality of the resulting orbit determination
Number of days between encounters for individual objects	Reduce the maximum number of days between encounters to improve the likelihood of associating observations of the same object made on different nights
Non-leakiness of search	Encounter all GEO objects within the sensor's field of regard during each scan
Sensitivity of individual encounters	Maximize dwell time within a sensor's individual pixels to increase the likelihood of detecting debris

Optimizing all of the factors above with a single search routine is difficult because of the conflict between scanning faster for greater area coverage and scanning slower (longer integration time) for greater sensitivity. For example, higher search rates will increase the number of encounters but at the same time will reduce the pixel dwell time for each encounter (and vice versa). We evaluate various search strategies, identify the conflicts in meeting the various objectives, and develop methods that will provide the optimal solution against all objectives.

The primary focus of this analysis was on the evaluation of search techniques designed to target high inclination objects. Encounter statistics are the primary method of comparison, though optical sensitivity also plays a large role in the success of a GEO debris search. Optical sensitivity is affected by both the integration time and the pixel size. Although the signal increases with integration time, the pixel size determines the amount of background noise received, thus potentially limiting sensitivity. Two external factors that affect optical sensitivity are relative target motion and solar geometry. The motion of the target will impact the dwell time within individual pixels and the solar geometry will impact the amount of illumination of the target.

For this study we used an internally developed suite of software modeling tools to develop the GEO search simulations. The search routines were then run against a set of pseudo-randomly generated satellites in GEO.

2. GEO SEARCH STRATEGIES

The primary purpose of this study was to evaluate general search strategies for finding previously uncataloged GEO debris objects. Therefore we did not consider strategies where smaller search fields were selected based on higher density regions. All of the search strategies considered for this study involve complete sweeps of the GEO belt region visible from the ground optical observing site.

2.1 Sensor Scan Method

For this study we adopted the step and stare method developed by the ESA Space-Debris Telescope on Tenerife, Canary Islands [1]. This scan technique combines the benefits of a single frame step and stare (reduced target motion across the focal plane) with the benefits of a sidereal scan, thus optimizing both sensitivity and metric accuracy. In this method, three minor-frames are taken using a fixed mount exposure; between sub-frames the telescope slews to a position so the star background is identical in each minor-frame (Fig. 1). The three sidereal minor-frames are followed by a major frame where the step is equal to the field of view size. The use of multiple frames (versus the more common single frame step and stare) verifies the detection of an object in the field of view and also provides enough data to perform a preliminary orbit determination. Additionally, the use of a fixed stare (versus a sidereal stare) allows for longer dwell times within individual pixels which improves detection sensitivity. As a result of the multiple benefits, we used this Tenerife scan technique for each of the step and stare search strategies that we modeled.

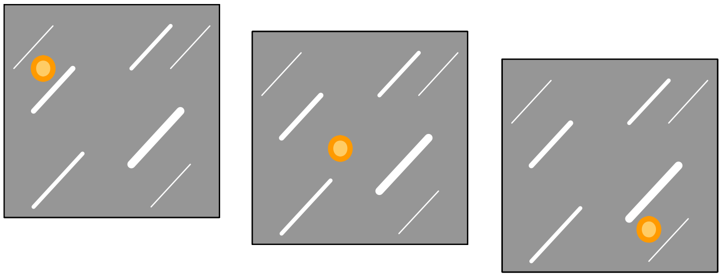

Fig. 1. Illustration of the three frame step and stare. The telescope tracks the objects (orange point) during the exposure and is then repositioned between exposures to observe the same field in the sky (the stars form streaks on each image) [1]

2.2. Search Strategies

Three search strategies were considered: (1) one-dimensional scans along the GEO belt, (2) multiple constant declination scans offset from the GEO belt, and (3) a square wave (or saw tooth) scan pattern centered on the GEO belt. The second two methods are generally considered beneficial when it comes to the detection of high inclination objects, which is important because most GEO debris objects have non-zero inclinations. Analysis of the U.S. space catalog shows the inclination of GEO debris to be distributed between one and fifteen degrees [3].

2.2.1. One-Dimensional Scans Along the GEO Belt

One-dimensional search scans along the GEO belt were modeled using one-way (east to west) scans. The time required to slew back to the east side of the scan is insignificant compared to the time required to perform a complete sweep of the GEO belt and thus a one-way scan does not pose a disadvantage compared to a scan that sweeps back and forth across the GEO belt (east to west and then back to east).

2.2.2. Multiple One-Dimensional Scans Offset from the GEO Belt

One-dimensional scans at constant declination offset from the GEO belt were modeled using declinations either above or below the GEO belt. Our software can handle any combination of declination offsets. Each scan pattern was repeated over the simulation duration and multiple combinations of different constant declination scans were performed to evaluate various off-belt strategies. However, the disadvantage of this method is that the sensor is guaranteed *not* to encounter satellites during an individual sweep if the inclination of the satellite is below the declination used for that particular sweep.

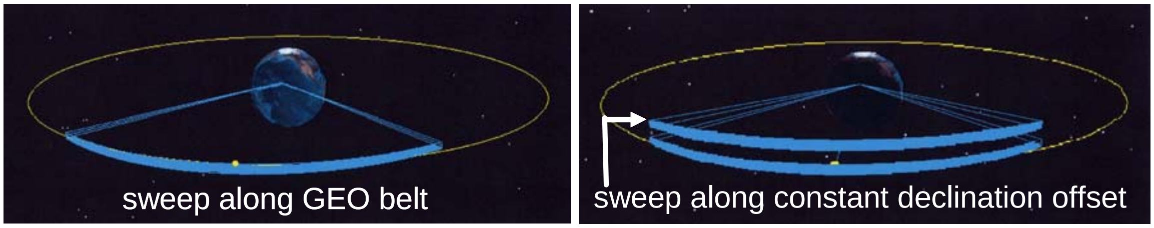

Fig. 2. Illustration of the one-dimensional search pattern

2.2.3. Square Wave Scan

A square wave scan searches above and below the GEO belt in a square wave (saw-tooth) pattern. The sweep begins at a user-specified angular offset below the belt and scans in a direction perpendicular to the GEO belt. Once the sensor reaches the equivalent angular offset above the GEO belt it steps in a direction parallel to the GEO belt (with a step size equal to the width of the FOV) and sweeps back down to the angular offset below the GEO belt. This pattern repeats along the entire GEO belt visible from the site. The benefit of this method is that it provides more complete coverage of the volume of space within a specified declination band. However, the time required to complete a single sweep of the GEO belt with this method is significantly longer than with the one-dimensional search methods.

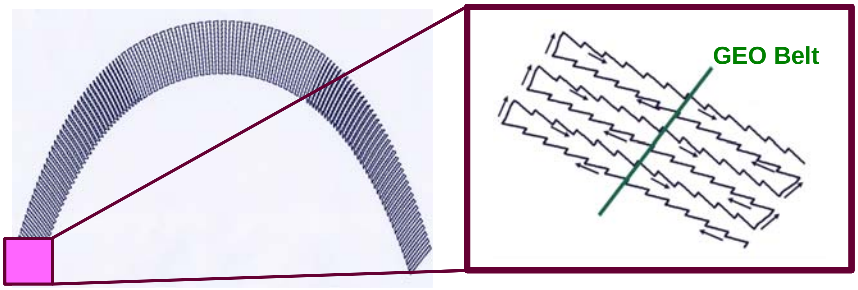

Fig. 3. Illustration of the square-wave search pattern (The jagged edges are a result of the three sidereal sub-frames - sensor motion is parallel to the GEO belt between the sub-frames and perpendicular to the GEO belt between the major frames)

3. SIMULATION RESULTS

Since we want to maximize the likelihood of encountering every GEO object in a region of space, the primary metrics used for comparison were the total number of encounters for an individual object over the simulation period and the maximum gap time between encounters of an individual object. We focused exclusively on encounter statistics and did not incorporate a model for the probability of detecting a target within the optical system field of view.

We assumed going into this study that the optimal solution would be to perform continuous sweeps along the GEO belt. Every object at GEO must cross the GEO belt twice per revolution, therefore no object would be impossible to encounter with this strategy. In addition, the scan could be performed quickly enough to increase the likelihood of multiple tracks on individual objects within a few days of each other. As a consequence of this bias, we performed two pair-wise comparisons: (1) continual sweeps along the GEO belt vs. continual sweeps offset from the GEO belt by a variety of declination offsets, and (2) continual sweeps along the GEO belt vs. the square wave scan.

3.1. Sample Case Set A

For the first case set we selected the sensor configuration as described in Table 2. [We also assumed no overlap between adjacent major frames.]

Table 2. Optical Site/Sensor Description

Parameter	Value
Location	33. 8° N Latitude
Field of View	0.5° x 0.5°
Integration Time	1 sec
Readout Time	1 sec
Minimum Elevation	10°

Each simulation run included 60 satellites with different inclinations (inclination randomly selected from a uniform distribution between 1 and 15 degrees). The baseline simulation used a single one-dimensional scan centered on the GEO belt. The comparison simulations used multiple constant declination one-dimensional scans with various repeating declination offset patterns. The simulated scan patterns are summarized in Table 3.

Table 3. Description of the one-dimensional scan patterns simulated

Case	Pattern (repeating pattern of boresight declination offset angle from the GEO belt)	Time to complete single scan (hrs)
1	0° (no offset from GEO belt)	1.30
2	1°... 15° (0.5° increments)	37.66
3	1°… 15° (1° increments)	19.48
4	1°... 5° (1° increments)	6.52
5	2°... 10° (2° increments)	6.51
6	1°, 5°, 1°, 10°, 1°, 15°	7.79
7	5°	1.30

We simulated the number of encounters for each case in Table 3 over one month (January). As Table 3 indicates, the off-belt strategies require many hours to complete a full pattern and in some cases multiple nights would be required to perform one full sweep.

In four of the seven cases all 60 satellites were encountered at least once during the simulation (Cases 1, 2, 3, and 6). Cases 5 and 7 did not encounter any of the satellites with inclination lower than the declination offset used for the scan (which was expected). Case 4 did not encounter two of the 60 satellites (both had an inclination near 12°). The average number of encounters for satellites in each of fourteen one degree inclination bins is provided in Fig. 4.

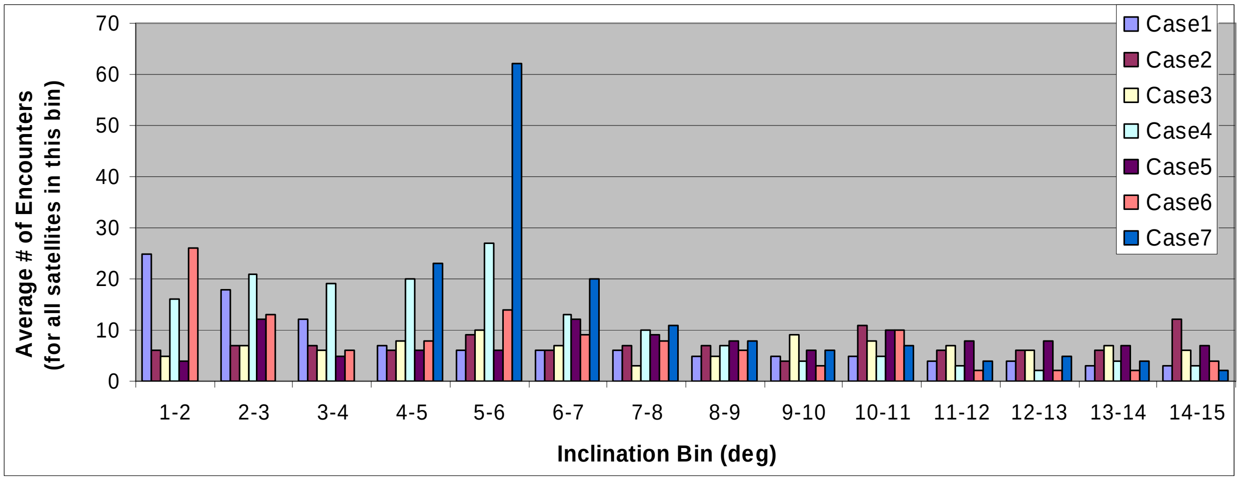

Fig. 4. Encounter results for one month simulations

Fig. 4 shows that each of the search strategies simulated outperformed the other strategies for at least one of the inclination bins, but none exhibited an overall advantage. The results show that the performance of Case 1 is comparable to some of the offset cases, though it is not the best strategy for any of the high inclination targets. Case 7 significantly outperforms the other cases for targets with inclinations within a degree of the declination offset used.

3.2. Sample Case Set B

For this case set we selected a subset of Case Set A, with the focus being the long term success of the various patterns. We selected three patterns (case #1, #3 and #4 from Case Set A) and ran them each for one year against five of the satellites from Case Set A[1] and the same sensor configuration described in Table 2. The objective of this analysis was to determine which pattern provides better overall encounter statistics in the long run. The three basic metrics used to evaluate the search patterns were: (1) total number of encounters over the simulation duration (one year), (2) maximum gap in time between encounters on the same objects, and (3) average gap in time between encounters on the same object. Figs. 5, 6, and 7 below show the results.

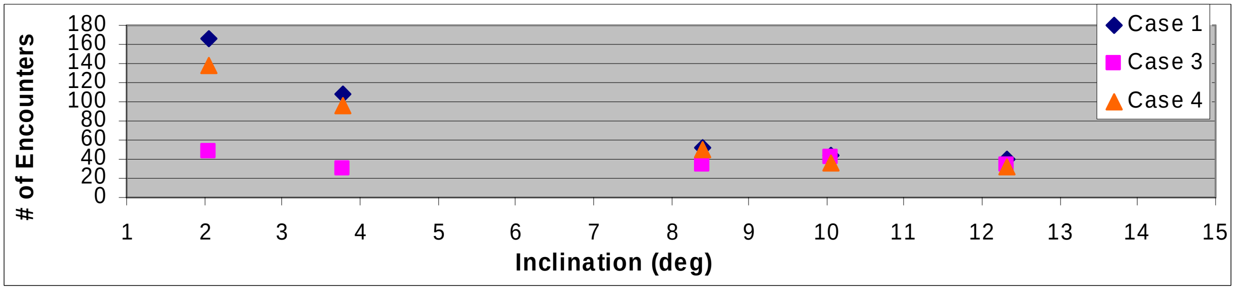

Fig. 5. Total number of encounters over a one year simulation for five targets with inclination between one and fifteen degrees

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[1] Satellite inclinations of 2.05°, 3.79°, 8.41°, 10.06°, and 12.31°

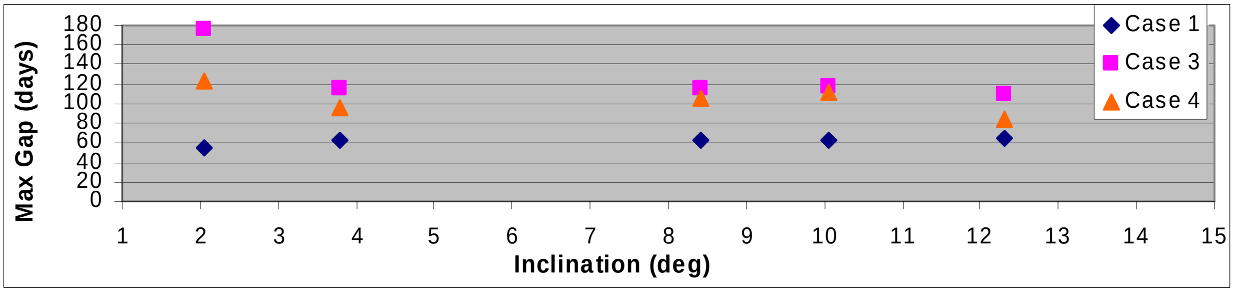

Fig. 6. Maximum separation in days between encounters on individual targets (with inclination between one and fifteen degrees) over a one year simulation

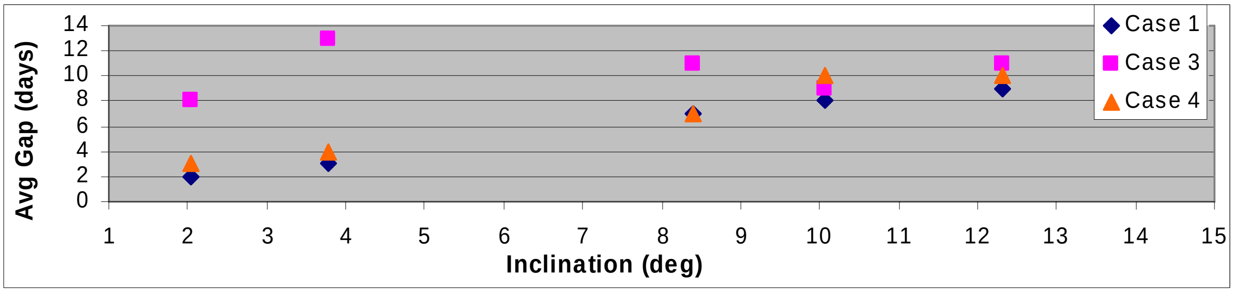

Fig. 7. Average separation in days between encounters on individual targets (with inclination between one and fifteen degrees) over a one year simulation

The results in Fig. 5, 6, and 7 show that the one-dimensional search strategy centered on the GEO belt (Case 1) performed as well or better than the off-belt strategies against the various encounter metrics. Fig. 6, in particular, shows a clear advantage in using on-belt scans in terms of decreasing the maximum gap time in days between subsequent encounters on the same objects.

Currently we are investigating whether one-dimensional scans offset from the GEO belt might be beneficial for reducing the relative north/south motion of the target (in order to improve effective integration time).

3.3. Square Wave Pattern

A single sweep of the GEO belt using a square wave pattern takes a significant amount of time (usually longer than the length of one night). The actual sweep duration depends on a number of factors such as integration time, readout time, and the maximum scan declination off of the GEO belt.

Table 4. Sample scan durations for a specific site and sensor configuration (see Table 2)

Declination Offset (above and below)	Duration (hours)
1°	6.81
2°	14.13
5°	32.93
10°	64.83
15°	98.00

As Table 4 shows, a single sweep takes the better part of a night to complete, and if an offset of more than a couple degrees is used, a single sweep will require multiple nights to complete. Based on these results we concluded that the square wave method would not perform well against any of the metrics used to define a successful search, and we did not perform any further analysis using this scan type.

4. OPTICAL SENSITIVITY

The ability of the optical system to detect debris is an important metric in the evaluation of a search routine. Two factors that affect the achieved sensitivity are (1) the time the target spends in an individual pixel, and (2) the solar phase angle of the encounter. As mentioned earlier, search scans that improve the effective sensitivity of a given optical system can have a negative impact on the encounter statistics (specifically the total number of encounters on individual objects and the gap time between encounters). We analyzed the pixel dwell times using the search scans above to determine whether the effective integration time, as a result of target motion across the focal plane, is less than the actual integration time. We also examined the solar phase angle that resulted from the search scans to determine whether certain scans could minimize phase angle and increase target brightness.

4.1. Target Motion Analysis

We calculate optical sensor limiting sensitivities assuming that the target remains within the field of view of a single detector element for the full integration period. A longer integration period would necessarily result in greater sensitivity. Longer integration times are possible for stationary (station-kept) GEO satellites (only to the point where background illumination begins to have a significant impact), but it is not the case for non-stationary GEO debris. The motion of the target image on the focal plane determines the actual pixel dwell time.

Most debris at GEO (especially older debris) have non-synchronous periods and non-zero inclinations. Unlike a station-kept GEO, this causes them to move with respect to the ground, sometimes with considerable speed. The faster a target moves, the less time it will spend in a single pixel, reducing the sensitivity of the encounter. Using the search scans discussed above, we analyzed the effect (if any) of the relative motion of inclined, drifting targets on the sensitivity of each encounter.

We developed analytical and empirical methods to examine the effect of target motion on optical sensitivity. We also developed closed-form equations to calculate both the mean time a streak spends within a single detector element and the fraction of streaks that dwell in the pixel for the full integration time (as a function of satellite angular rate). Additionally, Monte Carlo simulation was used to validate the results of the analytical approach.

The average time a streak spends within a single detector element (t) is a function of the sensor's integration time (τ) and the width of the detector element (L) as well as the target's angular rate (v) at the time of the encounter. This will not be a fixed value since the relative angular rate of the target varies over each orbit revolution as well as with the longitudinal offset of the target from the observing site. The analytically derived formula calculates the average dwell time for all streaks that enter an arbitrary pixel. This method does not exclude cases where a longer portion of the streak resides in an adjacent pixel. Additionally, these equations assume the streak is a straight line with infinitesimal width and that the scan rate is constant during the streak. We are currently in the process of developing more detailed closed-form equations to evaluate the expected pixel dwell time conditioned on the pixel containing the majority of the streak, as well as a method to incorporate streak width.

The expected dwell time, t, is calculated as a function of the dimensionless parameter β, which is the ratio of the integration time to the pixel transit time (or, equivalently, the ratio of the streak length to the pixel width). The following equations assume a uniform distribution in the direction of the streaks; however the explicit angular dependence was examined and found to be negligible.

$$\beta = \frac{v \cdot \tau}{L} \tag{1}$$

$$t = \frac{\tau}{1 + \frac{4}{\pi}\beta} \tag{2}$$

where:
τ = integration time
v = angular rate
L = pixel width

We also developed a relation to calculate the fraction of streaks (*f*) that will dwell within a single pixel for the full integration time. This formula can be used to help optimize the search routine by identifying an upper bound on the integration time. It would not be advantageous to increase the integration time if the expected relative motion between the site and the target indicates that there is a low (or possibly no) likelihood that the target would dwell within a single pixel for the full integration time.

$$f = \frac{\pi + \beta^2 - 4\beta}{\pi + 4\beta} \qquad \beta \le 1 \tag{3}$$

$$f = \frac{\pi - 2 - \beta^2 - 4\cos^{-1}\left(\frac{1}{\beta}\right) + 4\sqrt{\beta^2 - 1}}{\pi + 4\beta} \qquad 1 \le \beta \le \sqrt{2} \tag{4}$$

$$f = 0 \qquad \beta \ge \sqrt{2} \tag{5}$$

The fraction of streaks that will dwell within a single pixel for the full integration time is a function of the sensor's integration time and pixel size as well as the relative rate of the target as viewed from the site. Fig. 8 is a plot of the above equations as a function of relative rate for a range of optical system configurations (integration time over pixel size).

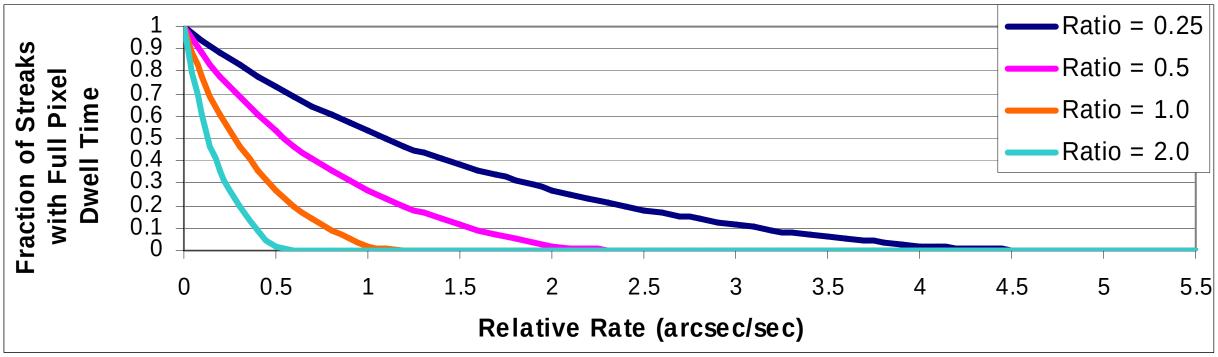

Fig. 8. Plot of the fraction of streaks that dwell within a single pixel for the full integration time, as a function of integration time, pixel width, and relative angular rate of the target as viewed from the site (the ratio in the figure is integration time over pixel width or, equivalently, β/ν)

As Fig. 8 shows, when the ratio of integration time to pixel size is large, the fraction of streaks that dwell within a single pixel for the full integration time is significantly reduced. Therefore, increasing the integration time for a sensor with a relatively small pixel size will not provide the intended increase in sensitivity since most observed debris will not dwell within a single pixel for the full integration time. Though it is intuitively obvious that maximum sensitivity will not be achieved for fast moving debris, these results provide an analytical method for determining how much benefit can be gained from increasing the integration time given a fixed pixel size.

GEO debris are typically in non-synchronous, inclined orbits with non-zero eccentricity. Thus relative angular motion will be observed in both azimuth and elevation at the optical site. The magnitude of the observed angular motion will vary depending on the site latitude, the location of the target relative to the site (in azimuth), and the target orbit. Typically, the relative angular rate for debris will be greater than 0.5 arcsec/sec and rates upwards of 2+ arcsec/sec would not be unexpected for highly inclined targets.[2]

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[2] Relative angular rates are larger for targets due south/north of the site (as compared to targets at the east/west end of the field of regard), and are also larger for lower latitude sites (as compared to higher latitude sites).

At this point we have only considered streak length versus integration time for a fixed pixel size. We are in the process of modeling a two-dimensional streak which will allow us to analyze the fraction of the pixel area that is covered by the streak. Arbitrarily increasing the pixel size by binning and integrating longer will cost sensitivity at some point because the streak covers smaller and smaller fractions of the pixel (and background noise increases for larger pixels). Our new model will allow us to add noise to the system and further investigate optimal strategies for speed, sensitivity and encounter statistics.

4.2. Solar Phase Angle Analysis

In addition to integration time, the solar phase angle impacts target brightness and determines the extent to which the optical system can detect debris for a given encounter. The solar phase angle, or the angle between the ground site and the sun, as viewed from the target, for a fixed point in space varies over a night and the year and is not the same for points above, below, or on the GEO belt. Small phase angles correspond to larger percentages of the target illuminated by the sun. Therefore, it would be desirable to develop search scans that provide more encounters with low solar phase angles, allowing smaller debris to be tracked.

Unfortunately, the point on the GEO belt of lowest solar phase angle moves approximately sidereally across the sky throughout the night. In order to execute a search that follows the point in space (on or near the GEO belt) with the minimum solar phase angle, the sensor would have to search at the sidereal rate. This would significantly reduce the search rate because only one sweep of the GEO belt could be performed each night.

In addition, during much of the year scans above or below the GEO belt result in lower solar phase angles than scans along the GEO belt. The maximum benefit (in terms of the reduction of the solar phase angle) is equivalent to the offset used. Thus a ten degree offset below the GEO belt during northern hemisphere summer months would, at most, result in a reduction of the solar phase angle of ten degrees. In general, significant gains in target brightness are only realized in cases where the declination offset is considerable and the solar phase angle along the GEO belt is particularly large for the night(s) of the scan. This indicates that off-belt scans to minimize the solar phase angle would be beneficial on occasion, but the decline in encounter statistics due to both the lower search rate and not encountering any objects with an inclination below the offset prevents the optimization of a search scan on solar phase angle alone.

5. SUMMARY/CONCLUSIONS

We evaluated various GEO debris search methods for this study using both encounter statistics and sensitivity as metrics. Unfortunately, factors that improve object encounter rate adversely affect sensitivity and, similarly, factors that improve sensitivity adversely affect encounter rate. We look for integration times such that both sufficient sensitivity and search rates were achieved. It was discovered that a target's relative motion limits the amount of time spent in an individual pixel, so it was possible to select a maximum integration time above which sensitivity would not increase.

Two search patterns were compared against a one-dimensional scan centered on the GEO belt. The first, a square wave search scan pattern, was eliminated from consideration because of the significant amount of time required to perform a single scan. The second, linear one-dimensional scans at constant declinations offset from the GEO belt, constituted a more effective solution than the square wave pattern for targeting high inclination objects, although still did not appear to provide a clear overall advantage over the one-dimensional scans centered on the GEO belt against targets with a range of inclinations. However, a single one-dimensional scan at a declination offset from the GEO belt equivalent to the inclination of the target (within ± the width of one field of view) is optimal for that case. There are cases in which an offset might be desirable for meeting objectives other than those identified in this study (such as minimizing the number of active - low inclination - targets encountered during the search). In certain circumstances a scan incorporating a declination offset from the GEO belt can provide a small advantage in solar phase angle, thus increasing the relative brightness of debris. However, unless the solar phase angle is particularly large along the GEO belt for the night(s) of the scan, the advantage will be minimal.

Our expectation had been that the one-dimensional search strategy with a scan centered on the GEO belt would outperform the off-belt strategies. However, the results of our simulations show there is no obvious "best" strategy for implementing a one-dimensional search routine. The simulation results indicate that there is no clear advantage, in terms of general encounter statistics, in performing on or off-belt GEO debris searches.

The results of this study show GEO debris searches with a scan centered on the GEO belt perform well against all metrics considered. For general debris surveys where the entire field of regard is considered, there is no distinct advantage in performing a search designed to target high inclination objects.

6. REFERENCES

1. Flury, W., A. Massart, T. Schildknecht, U. Hugentobler, J. Kuusela, Z. Sodnik, Searching for Small Debris in the Geostationary Ring, ESA Bulletin 104 - November 2000
2. Schillknecht, T., R. Musci, M. Ploner, J. de Leon Cruz, L. de Fatima Dominguez Palmero, An Optical Survey for Space Debris in the Geostationary Ring, Proceedings of the 2003 AMOS Technical Conference, Maui, Hawaii, September 8-12, 2003
3. Matney, M., et al., Recent Results from NASA's GEO Debris Optical Surveys, Proceedings of the 2003 AMOS Technical Conference, Maui, Hawaii, September 8-12, 2003
4. Schillknecht, T., R. Musci, M. Ploner, J. de Leon Cruz, L. de Fatima Dominguez Palmero, Properties of the High Area-to-Mass Ratio Space Debris Population in GEO, 2005 AMOS Technical Conference, Maui, Hawaii, September 5-9, 2005