PHASE ANGLE: WHAT IS IT GOOD FOR?

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ABSTRACT

The concept of phase angle as a variable of interest when reporting resident space object (RSO) brightness information is widespread throughout the community. Phase angle is the angle between the direction to the Sun and the direction to the observer, as seen at the object being observed. If the object were the Moon, a near-zero degree phase angle would be full moon. Phase angle is widely used in the astronomical community, and has been adopted by the satellite community as well. Phase angle is very useful for some objects (e.g., spherical objects with Lambertian reflectance properties), but rapidly loses its utility when applied to objects which have different properties.

It takes four angles to describe the illumination and observing conditions for any RSO observation, two angles describing the direction to the illumination source, and two angles describing the direction to the observer. Replacing those four angles with a single angle, the phase angle, yields an inherent loss of information. For RSOs with complex structures, or whose orientation with respect to the Sun and the observer changes with time, the correlation of brightness with phase angle is poor.

This presentation will discuss phase angle, how correlations with phase angle break down for most RSOs, and discuss under what conditions phase angle is a useful parameter, and under what conditions it is not.

1. BACKGROUND

There seems to be a wide-held assumption within the space surveillance community that if one observes an Earth-orbiting RSO, the maximum irradiance of that RSO will occur at minimum phase angle. While this is usually, but not always, true for an ensemble of thousands of measurements of the same RSO at varying phase angles, it is often not the case when dealing with a set of observations over a single RSO pass. It is not at all uncommon to be come across a set of data for which the brightness is not greatest at minimum phase angles. This often results in a question or concern that the data is anomalous, either because the data is faulty, or the RSO has some very strange reflection properties. We will show that one need not employ either of these situations to explain situations where the irradiance is not greatest at minimum phase angle.

The use of phase angle is attractive, because it is very easy to calculate. One need not know the shape or attitude profile of the RSO. All that is needed is the position of the RSO, the Sun, and the observer for each observation. Combine that with its success in it use by the astronomical community for the analysis of solar system objects, measuring the reflected solar illumination from these natural bodies, and the argument for its use for Earth-orbiting RSOs may seem compelling.

As with any tool, it is important to understand the advantages and limitations of the tool, to determine when is it most properly employed. There are many examples in the literature that demonstrate that phase angle is a valuable tool for use in analyzing stable geosynchronous orbits [3][6][7]. There is very little evidence that phase angle is valuable in orbital regimes outside of geosynchronous orbit [1].

2. DEFINITIONS

We will use the term illumination geometry to describe the geometrical relationship between the positions of the Sun, the RSO, and the observer, as well as the orientation of the RSO. Unless a surface of the RSO is both illuminated by the Sun, and visible to the observer, that surface will not contribute to the observed irradiance. Note that we will ignore multiple reflections, and transparent surfaces, in this analysis. Except for pathological cases, e.g. a retroreflector, these phenomena will weaken any argument for the utility of phase angle. We will also only consider direct solar illumination.

One can choose a number of coordinate systems to describe the illumination geometry. A very convenient coordinate system to use is a body reference system, attached to the RSO, that both translates and rotates with the translation and rotation of the RSO. In this coordinate system, the position of the Sun and the position of the observer are varying in time, as the RSO moves in its orbit. It is sufficient for this portion of the analysis to consider only the direction to the Sun and observer. While rigorous interpretation of radiometric signatures requires normalization of distances, both to the Sun and to the observer, in this case we are only concerned about directions, so unit vectors will suffice.

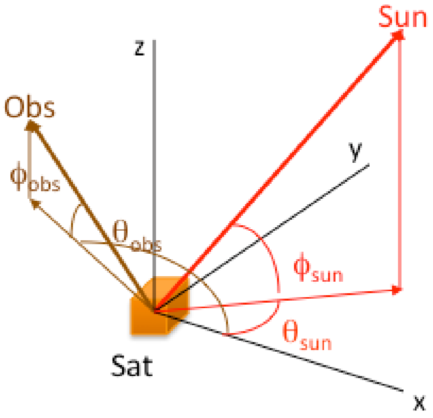


Figure 1: Body reference coordinate system

Fig. 1 shows the 4 angles that are required to keep track of these positions. Two angles are required to describe the position of the Sun, and two angles to describe the position of the observer. It is important to remember that this coordinate system is in the RSO body coordinate frame, so that the coordinate system rotates and translates with the RSO.

$$\hat{n}_{Sun}(t) = \left[\cos\theta_{Sun}(t)\cos\phi_{Sun}(t)\right]\hat{x} + \left[\sin\theta_{Sun}(t)\cos\phi_{Sun}(t)\right]\hat{y} + \left[\sin\phi_{Sun}(t)\right]\hat{z}$$

$$\hat{n}_{Obs}(t) = \left[\cos\theta_{Obs}(t)\cos\phi_{Obs}(t)\right]\hat{x} + \left[\sin\theta_{Obs}(t)\cos\phi_{Obs}(t)\right]\hat{y} + \left[\sin\phi_{Obs}(t)\right]\hat{z}$$

$$Phase\ angle\,(t) = \arccos\left[\hat{n}_{Sun}(t) \bullet \hat{n}_{Obs}(t)\right]$$

Figure 2: Phase angle defined

There is an inherent loss of information when you take 4 angles and combine them into a single angle. There is an infinite number of values for those 4 angles that will yield a given phase angle. That is, there is a wide range of illumination geometries that are characterized by a single phase angle, all of which can result in very different light curves. We define a light curve as the irradiance as a function of time.

In general, if you want to simulate a light curve, or if you wish to schedule an observation to catch some phenomenon, e.g. an Iridium flash, or if you wish to configure a camera system to make optimum measurements, you need to keep track of all of four or those angles. If you wish to analyze a light curve, to understand why the light curve looks unusual, or to infer the characteristics of the RSO from a measured light curve, you need to use all four angles.

Let's remind ourselves why phase angle is so often used. In many situations the RSO shape or attitude profile are not known. If they are, they should be used. If they are not, that means the four angles are not known. One may be tempted to simply use phase angle. To quote one of my colleagues, it often comes down to simply, "it's what you can measure."

Let's establish a qualitative score card that will tell us when it is appropriate to use phase angle. The score will depend on two situations. We will base that score on how useful phase angle is to predict the signature of a RSO. We will also base the score on how useful phase angle is to help understand the behavior of a measured signature to infer characteristics of the RSO, which is often very useful in anomaly resolution.

We will evaluate several examples of different orbit classes, and look at RSO stability. The orbit classes we will consider will include an example of a geosynchronous orbit (geo), low-Earth orbit (LEO), medium Earth orbit (MEO), and a Molniya orbit (Moly). We will consider vector-vector stabilized RSOs, as well as a tumbling RSO. A spin-stabilized RSO has similar characteristics in this analysis as a tumbling RSO, and so won't be treated as its own case.

A vector-vector stable attitude profile is one where one vector associated with the RSO is pointed in a specific direction (e.g., nadir, Sun, inertial direction), and a second, orthogonal vector associated with the RSO is restricted to the plane formed by the first direction and a second direction. An example would be a RSO in a nadir-velocity attitude profile, where the z axis of the spacecraft would be aligned to the nadir direction, while the x axis would be aligned in plane defined by the nadir and velocity vectors. In this example, if the orbit were circular, the x axis would be aligned in the velocity direction.

3. ANALYSIS

Since we are only interested in directions, it doesn't make any difference what the shape of the RSO is, we are only keeping track of the position of the observer and the Sun in the RSO coordinate system. We will use two tools to understand how the positions of the Sun and the observer are changing as a function of time. The first will be a plot of the four angles as a function of time, along with the plot of the phase angle as a function of time. The second will be a plot on a unit sphere, centered on the RSO, that show the path of the directions to the Sun and the observer.

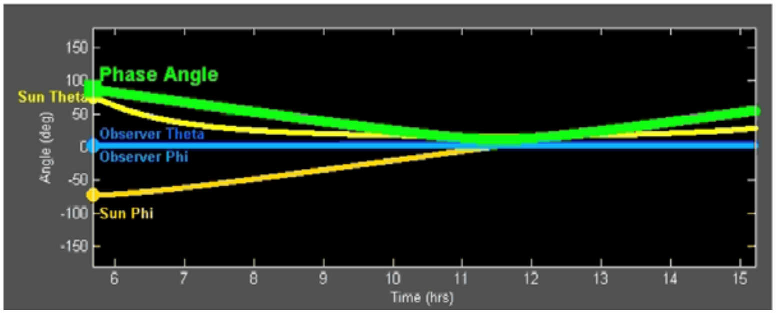


Figure 3: Angles as a function of time

The first example should help demonstrate these two tools. For our first case let's look at a vector-vector stable RSO flown in a geo orbit. The first tool is shown in Fig. 3. The two angles describing the position of the Sun are shown as the yellow lines, while the two angles describing the position of the observer are shown as blue lines. The phase angle is shown as the green line.

The second tool we will use is shown in Fig. 4. The yellow line shows how the position of the Sun has changed over the observation period, while the blue line shows the position of the observer has changed.

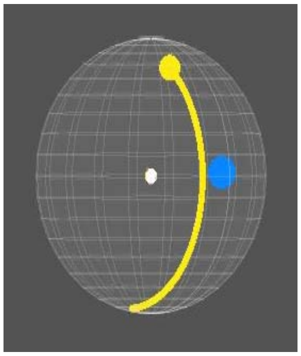


Figure 4: Path of Sun and observer in body coordinate system

For this particular case, a vector-vector stable RSO in geo, one would expect that the vector to the observer would be unchanged over time. If the RSO were in a true geostationary orbit, the vector to the observer would be unchanged. In this case the RSO was not in a true geostationary orbit, so the vector to the observer will change slightly during a pass. Similarly, at the beginning of the pass the Sun will be in one position, but over the course of the 9-hour pass, the position of the Sun in this coordinate system will change as a function of time.

Fig. 5 shows the two tools on the same plot. This is the format we will use for the remainder of the analysis.

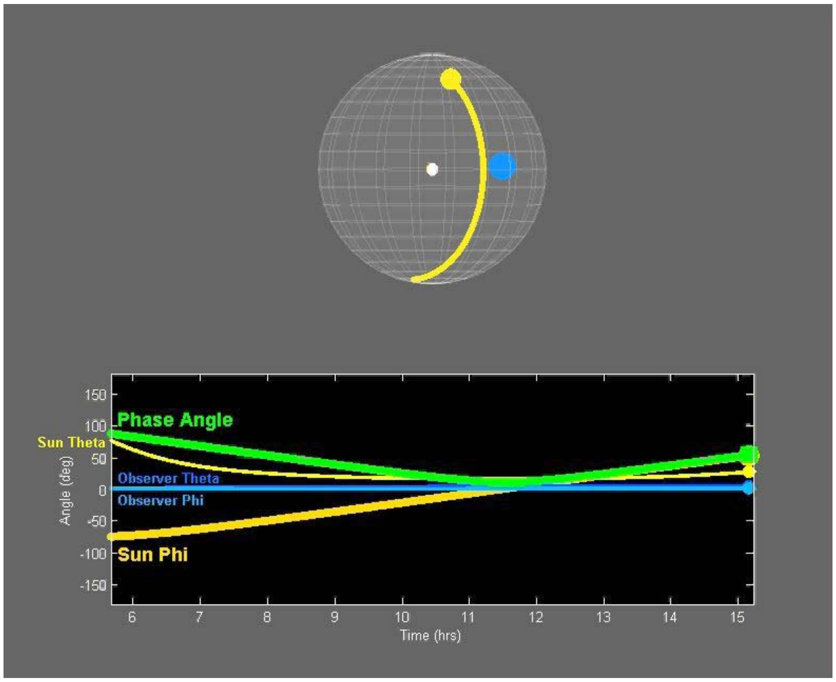


Figure 5: Vector-vector stabilized RSO in geo orbit

A RSO in a geostationary orbit stays in a constant position with respect to any point on the ground, so you would expect the position of the observer to remain constant, as we see here. The fact that we see some minor change in the ground position over 9 hours indicates that the orbit we've chosen for this simulation is not truly geostationary, but has a slight inclination, or eccentricity, or both.

The position of the Sun in this coordinate system is changing in phi in a fairly linear sense, while in theta it is fairly constant over much of the evening. The Sun begins the pass in the general direction of the anti-velocity vector, and ends the pass in the general direction of the velocity vector. Note that for relatively small phase angles (less than 40 degrees), 3 of the 4 direction angles are relatively constant. If we were to restrict the 4th direction angle, Sun Phi, to positive values, as we do for the traditional definition of phase angle, Sun Phi and phase angle track nicely with each other. For this reason we can say that the phase angle is a good measure of the direction to the Sun. For those reasons, we rate using the phase angle in the analysis of this particular case with a score of B.

However, many professional observers keep track not just of phase angle, but of signed longitudinal and latitudinal phase angles [5]. This means they effectively differentiate between the left and right side of minimum phase angle, as well as the latitude differences between different observers, or the seasonal position of the Sun. They are effectively keeping track of two angles, which is sufficient to measure the direction of the Sun, while the direction to a ground observer is constant. For those observers we would rate the use of phase angle for vector-vector stabilized geo RSOs as A.

It should be clear that these grades are very subjective, and one could argue that the grades should be raised or lowered slightly. The grades should, however, be fairly consistent relative to each other. A higher grade effectively means that the use of phase angle for that case is more meaningful and valuable, given the subjective criteria we established at the beginning of this paper, than for a case with a lower grade.

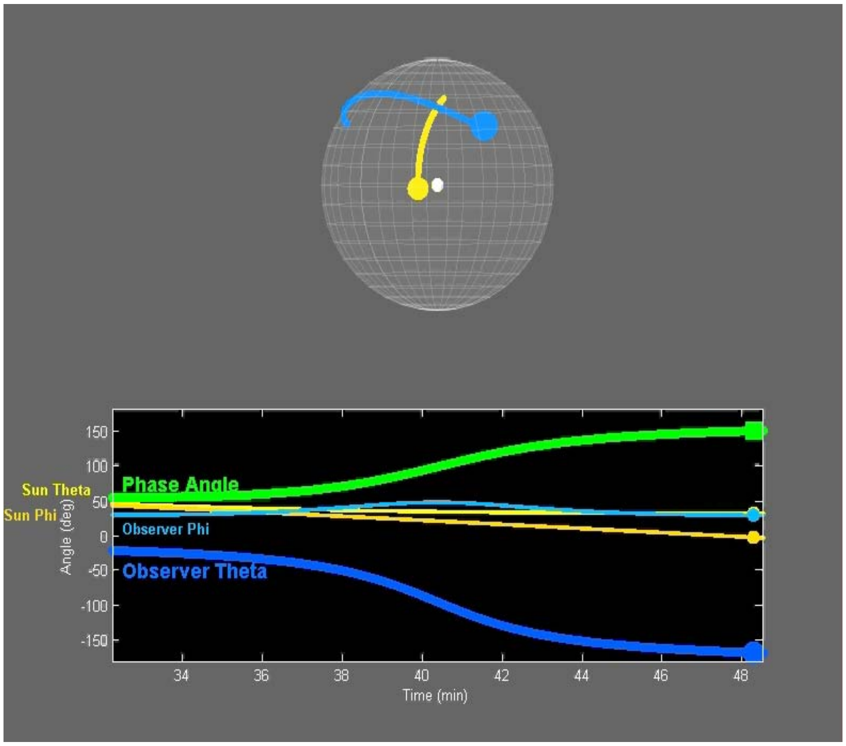


Figure 6: Vector-vector stabilized RSO in LEO

For our next case consider a vector-vector stabilized RSO in LEO orbit, Fig 6. Note that the duration for this case is much shorter. The duration is what one would expect for a normal RSO pass for each type of orbit. Because the duration is so short, the vector to the Sun does not change as much as for the geo case. Unlike the geo case, however, the vector to the observer changes over the length of the pass. The position of the observer begins in the general direction of the velocity vector, varies throughout the pass, and ends in the general direction of the anti-velocity vector. It is clear that the phase angle is a fairly good measure of observer position in the Observer Theta angle. Phase angle in this case does a better job of tracking the observer position than it does the Sun position, but all of the angles are changing and not well correlated to the phase angle. Because this case is not as compelling as the previous geo case, we give this case a grade of C.

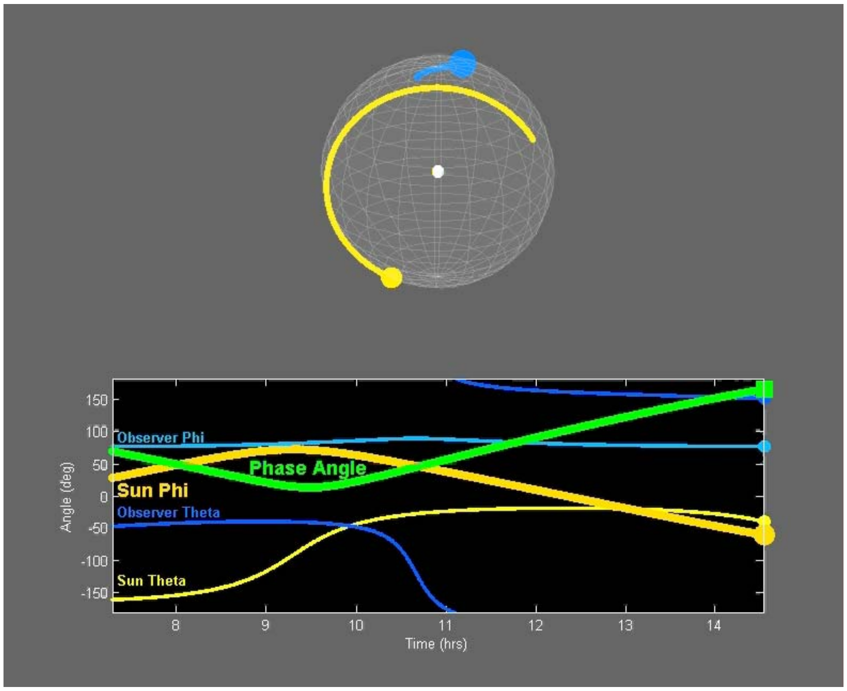


Figure 7: Vector-vector stabilized RSO in MEO

For our next case, look at a vector-vector stable RSO in MEO, Fig. 7. Although the phase angle has a fairly nice correlation with Sun Phi, it has a very poor correlation with the other angles. The other angles are changing too much, even at low phase angles, for this case to be very appropriate for the use of phase angle to predict or interpret light curves. For this reason we choose to give this a grade of D.

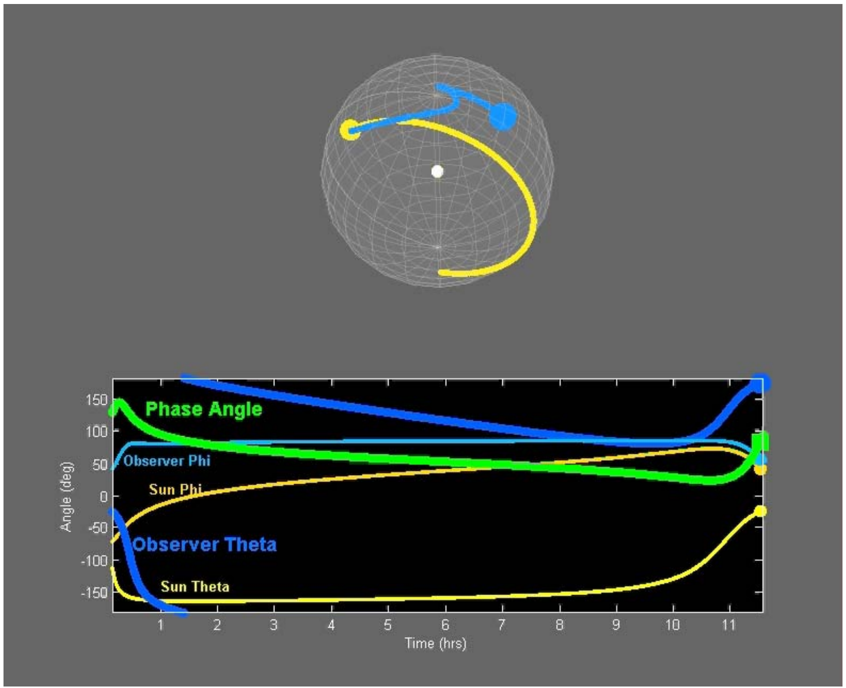


Figure 8: Vector-vector stabilized RSO in Moly orbit

For our last case of a vector-vector stabilized RSO, we will consider such a RSO in a Molniya orbit, Fig. 8. While there are portions of this plot that show a fairly good correlation with Observer Theta angle, the other angles show poor correlation. This case is very similar to what was seen in the case of MEO, so we will grade this case as a D also.

Table 1: Scores for vector-vector stabilization

Orbit type	Stability	Score
Geostationary	V-V Stable	B
Geostationary	V-V Stable signed long/lat phase angle	A
LEO	V-V Stable	C
MEO	V-V Stable	D
Molniya	V-V Stable	D

Let's summarize the grades we have given to the various scenarios, shown in Table 1. The only good grades are for stable RSOs in geo orbit. For LEO, the grade is acceptable, while for MEO and Moly orbits, the utility is marginal at best.

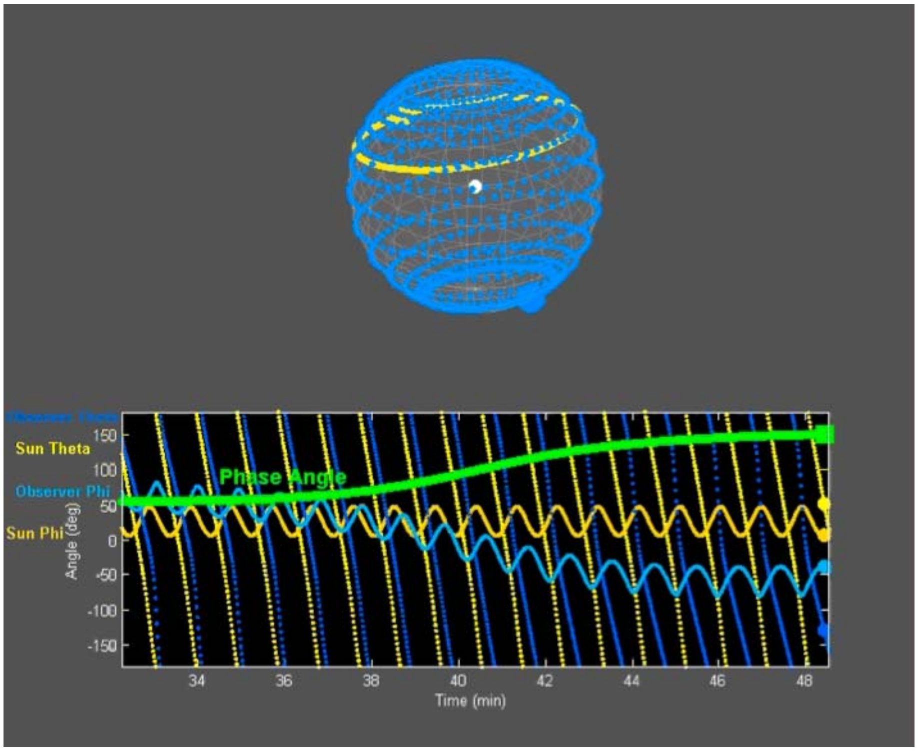


Figure 9: Tumbling RSO in LEO

Now look at the case of a tumbling RSO in LEO, Fig. 9. Recall that this coordinate system is attached to the RSO, so that we can keep track of which facets or surfaces of the RSO are either being illuminated, or are visible, or both. Note that the effective directions to both the Sun and to the observer are wobbling and appear to be rotating around the RSO. While the phase angle might show some correlation to the average value of one of the angles describing both the Sun and observer positions, the other angle is changing rapidly, so that a given surface goes from being illuminated to not illuminated over and over during the pass. The same is true for the visibility of a given surface.

Before we assign a score to phase angle utility for a tumbling RSO, we will present two simulations that may provide insight into the effects of this rapidly changing illumination geometry. Everything we have done so far has been independent of the shape of the object, we have just considered the directions to the Sun and to the observer in this body-centered coordinate system. We will now consider a particular shape, and look at the resulting brightness variations as the illumination geometry changes.

Our first example will be a single-axis spinning cube, with a different albedo for each surface. We will simulate this cube, flying in a surrogate orbit, and observe the brightness variation. For this simulation, we've chosen a spin rate fast enough to show the variations at a specific phase angle. That is, the phase angle is effectively constant over this simulated pass period. This is shown in Fig. 10.

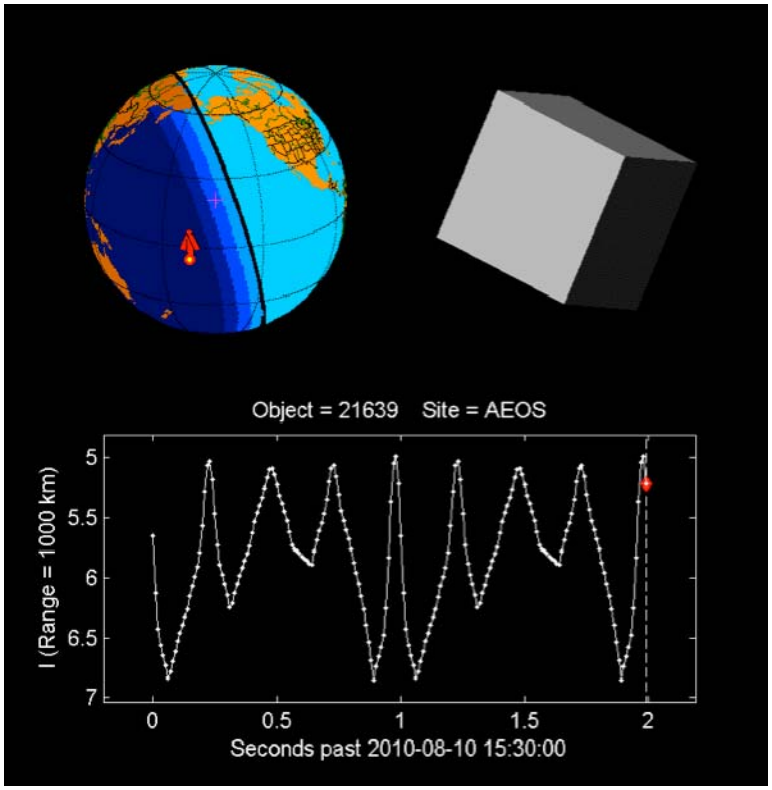


Figure 10: Simulation of irradiance from a spinning cube

Note that the irradiance varies by almost two visual magnitudes, all at effectively the same phase angle. If one were to take a measurement of this RSO at that phase angle, one could expect a range of values covering almost two visual magnitudes. Extrapolate that possible variation to any measurement taken during such a pass, and you can see how much uncertainty there will be in any single measurement. What is just as important is that for two successive measurements, there is no way to predict whether the second measurement will be greater or less than the first, without knowing the sampling rate of the data collection, and the physical properties of each surface of the RSO.

Our second example will also be a cube, but in this cases we will choose the same albedo for each surface, and we will perform a Monte Carlo simulation for the location of the Sun and the observer, in the RSO body-centered coordinate system. We will simulate 100,000 random directions for both the Sun and the observer, effectively giving a random distribution of illumination geometries. We show two representations of the results of the simulation. The first is simply a scatter plot (Fig. 11), while the second shows the density of points (Fig. 12). The axes on the plots are irradiance on the vertical axis, with phase angle on the horizontal axis. Both represent the same data set.

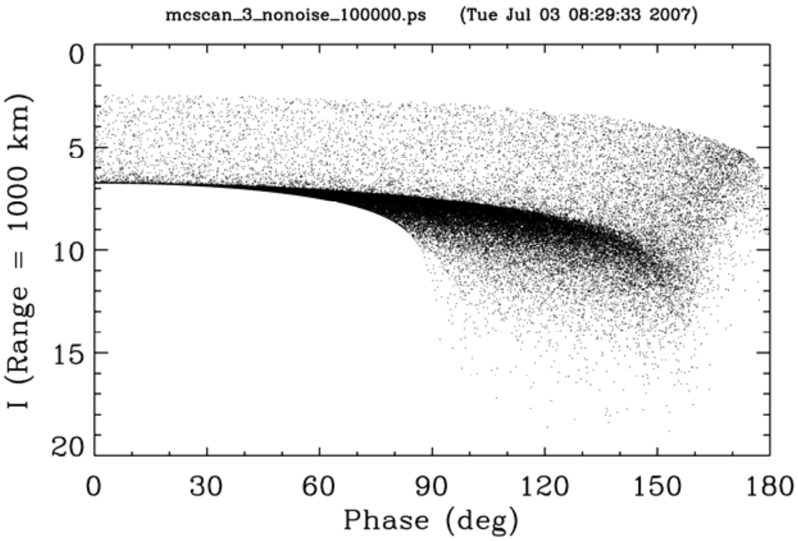


Figure 11: Scatter plot of Monte Carlo simulation of a cube

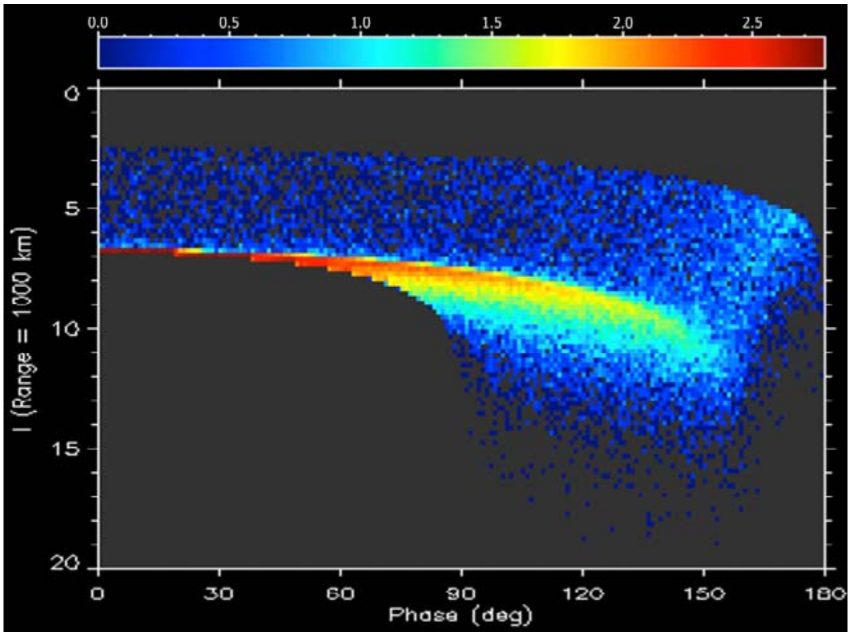


Figure 12: Distribution plot of Monte Carlo simulation of a cube

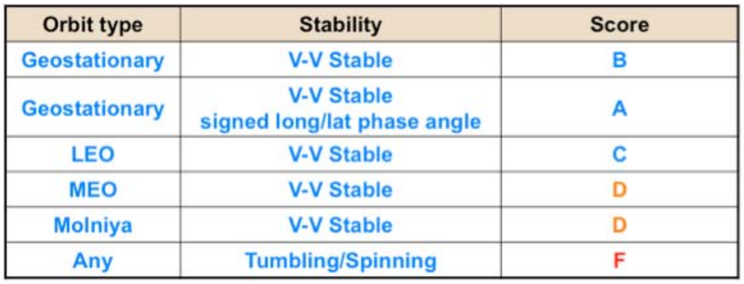
Note in both Fig. 11 and Fig. 12 the wide variation in irradiance possible for any given phase angle. If one were to randomly choose an irradiance from this plot for each phase angle for which a measurement could be taken to generate a light curve, one would expect considerable variation within that light curve. The data would look noisy, even though no noise was introduced in this simulation. If one were to repeat this for the same set of phase angles, the subsequent light curve would likely be quite different than the first. This is representative of the fact that for each measurement, the illumination conditions in the body reference system are not known, resulting in apparently random light curves.

Note that in both this example, as well as the previous example, we have chosen a fairly benign RSO shape. One would expect that as the RSO shape becomes more complex, the variation in possible irradiances for a given phase angle would increase.

It should also be noted that the shape of these Monte Carlo simulations, both the envelope, the average value for each phase angle, and the variation at each phase angle, is dependent on the shape of the RSO, as well as the albedo-area product of each of the surfaces of the RSO.

Based on these simulations, and the large variation of visual magnitudes for a very simple shape, with no noise in the data, we give a failing grade for the utility of phase angle for any tumbling RSO. The same can be said for a spinning RSO, since the illumination geometry provides a wide range of possible irradiances for any given phase angle. So our final grading chart for orbit type and stability is shown in Table 2.

Table 2: Final grades for stability and orbit regimes



Orbit type	Stability	Score
Geostationary	V-V Stable	B
Geostationary	V-V Stable signed long/lat phase angle	A
LEO	V-V Stable	C
MEO	V-V Stable	D
Molniya	V-V Stable	D
Any	Tumbling/Spinning	F

There may be some disagreement about the absolute grades, but they should be appropriate in a relative, differential sense.

4. CONCLUSIONS AND RECOMMENDATIONS

Phase angle disregards important illumination geometry, which has a dramatic effect on the irradiance measurements.

The advantages of phase angle as a tool for the analysis of light curves are that it is easy to calculate, it is perfect for diffuse spheres (by definition), and it is very useful for stable objects in geo or near geo.

The limitations of phase angle are that its utility degrades for stable objects in other orbit regimes, and its utility for rotating or tumbling objects is pretty bad.

Phase angle can be quite useful for use in the analysis of light curves from stable objects in geosynchronous orbits, particularly if signed longitudinal and latitudinal phase angle is used. Its utility decreases outside of this orbit class, even for stable objects. Phase angle fails for objects that are spinning or tumbling, with the possible exception of pathological cases (e.g., a spinning cylinder).

Phase angle is really only useful for stable objects, and its use for those still requires caution. Our recommendation is that you be cautious as well as rigorous in any analysis that uses phase angle.

5. REFERENCES

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