Defunct Satellites, Rotation Rates and the YORP Effect

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ABSTRACT

With the amount of space debris found in Earth orbit continuously increasing, it is imperative to understand the dynamics of these objects. A mechanism that has been shown to cause a secular change in the angular velocity of various asteroids is known as the Yarkovsky-OKeefe-Radzvieskii-Paddack (YORP) effect. This paper analyzes the theoretical YORP torque that could be acting on a defunct satellite found in Earth orbit. Observed rotational periods of defunct satellites are then used to estimate the magnitude of the YORP torque that would be required to obtain the observed behavior. The theoretical values are compared to those computed from observations. The implications of this effect for defunct satellites are discussed.

1. INTRODUCTION

Active satellites are not the only objects found in Earth orbit. In fact, as of October 2009, the U.S. Space Surveillance Network was tracking approximately 19,000 man-made objects larger than 10 cm in diameter that orbit the Earth. Of those, about 95% were considered to be pieces of debris, which is defined as any man-made object that is no longer active, including defunct satellites [1]. In addition, there are another estimated 300,000 objects measuring between 1 and 10 cm across [1]. Though the size may seem small, at high speeds even a small piece of debris can have great effects. The average impact velocity in LEO is 10 km/s. According to Pardini and Anselmo, at these velocities a piece of debris with a diameter of 1cm can disable a spacecraft, and objects between 10 and 20 cm can cause a spacecraft to break up [2]. With the growing amount of space debris in Earth orbit, understanding the dynamics governing space debris is becoming a crucial step in protecting assets in orbit.

A theory that has been used to explain the rotational dynamics of asteroids is known as the Yarkovsky-O'Keefe-Raszvieskii-Paddack (YORP) effect [3]. This effect has been credited for the secular change in angular velocity of various asteroids. As described in reference 4, one aspect of YORP is sunlight being absorbed and re-emitted as energy. The photons that are re-emitted create a net downward force on the body's surface. Another aspect of this effect is the transfer of angular momentum by the sunlight, which is reflected by the body's surface [5]. As a result of both of these factors, an overall torque is created on the body yielding a change in the angular velocity. The moments caused by YORP are rather small and therefore, YORP will have an impact on a body's rotational dynamics over long periods of time. While YORP has been extensively studied and observed for asteroids, it's effect on defunct satellites found in Earth orbit has not been investigated.

Many observations of space debris, in particular uncontrolled satellites, have been made. From these observations it has been found that the rotational period of defunct satellites found in geosynchronous earth orbit (GEO) experiences variations as a function of time [6]. These observations have shown that satellites that once were spin stabilized have periods ranging from 2 - 7 seconds, launch vehicles have periods between 5 - 15 seconds, and lastly a variety of satellites that had 3-axis stabilization have periods ranging from 15 - 500 seconds [6,7]. In addition, these observations have demonstrated that for some objects the rotational period will increase for a length of time and will subsequently decrease for another length of time [6]. As can be seen, there is a wide range of periods for similar objects. Additionally, other observations have shown the it is hard to determine the rotational period of some observed objects as light curves are rapidly changing. Ojakangas et. al. have done some studies analyzing the effects of solar radiation pressure on Earth orbiting objects in an effort to better understand light curves and observed data [8]. This study has found that solar radiation pressure has an effect on the angular momentum of a small Earth orbiting object [8].

The purpose of this paper is to analyze if the YORP effect could be a possible cause of the observed behavior of defunct satellites found in GEO. This paper shows initial work used to determine order of magnitude of the effect of YORP on defunct satellites. The YORP theory is used to estimate how much torque acts on a defunct satellite, on average over one year. Next, published observed rotational periods of defunct GEO satellites are used to find an estimate of how much YORP torque would be required to obtain the observed dynamics. The two values are then compared. For a more accurate or meaningful comparison, the two are normalized in the same manner. This comparison can show if YORP could be used to explain the observed behavior. A description of the evolution of the angular velocity and obliquity as a result of YORP is first given. This is followed by a method for computing a YORP coefficient from observed rotational periods. Next, the satellite model used for analysis is described and its computed Fourier coefficients are given. Then a comparison with estimated values from observed satellite is done. Lastly, a conclusion of the results is presented.

2. ROTATIONAL DYNAMICS DUE TO YORP

This section provides a summary of the evolution of rotational dynamics of a body due to the YORP effect as described in reference 5. Only the equations necessary to apply the theory are presented in this paper rather than the derivation of each which can be found in reference 5. The theory, as presented in reference 5, is for a body in a heliocentric orbit. This is acceptable for this analysis. As a satellite orbits the Earth it will also orbit the Sun as the Earth itself completes an orbit around the Sun. The motion of the satellite about the Earth is small compared to the motion about the Sun and is therefore ignored for this study. As such, a defunct satellite is considered to be on the Earth's heliocentric orbit. First, a definition of the body's orientation that will be used throughout the paper is given. Next, the evolution of the body's angular velocity and solar inclination averaged over one Earth orbit period about the Sun.

2.1. Body Orientation

In order to analyze the YORP effect on a defunct satellite, a body-fixed frame must first be defined. The origin of the coordinate frame is located at the center of mass of the body and the body is assumed to be uniformly rotating about its maximum moment of inertia. Next, an orbit for the body is defined with the classical orbital elements. The orientation of the body's rotation pole is given by a right ascension, α, and declination, δ. In the body fixed frame, the sun will appear to orbit the body. The orbit's inclination or obliquity, right ascension of the ascending node and argument of perihelion can be found in the body-fixed frame. A derivation of each of these elements in the body-fixed frame can be found in reference 5. These orbital elements are then used to find the solar latitude, δ_s, and solar longitude, λ_s, in the body-fixed frame. The process for this is given in reference 5. With all of these parameters defined, the unit vector of the Sun's position in the body frame is then defined by Eq. 1. It should be noted that throughout the paper, the notation $\hat{x}$ will be used to denote a unit vector.

$$\hat{u} = \cos(\delta_s)\cos(\lambda_s)\hat{x} + \cos(\delta_s)\sin(\lambda_s)\hat{y} + \sin(\delta_s)\hat{z} \tag{1}$$

Figure 1 shows the geometry described here.

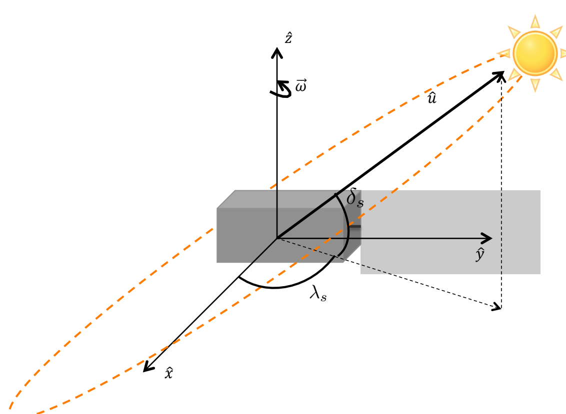

Figure 1: Geometry related to body orientation

2.2. Orbit Averaged Rotational Dynamics

Using the geometry given above we can find the average evolution of the angular velocity and the obliquity as YORP torques act on the body throughout one year. Recall that we have defined the body geometry in such a way that it is rotating about its z-axis. As described in reference 5, the YORP moment acting on the body can be expressed as a Fourier series, therefore, the evolution of the dynamics will be dependent on the Fourier coefficients that are used to describe the YORP torques. A full derivation of these averaged equations can be found in reference 5.

The evolution of the angular velocity and obliquity averaged over one orbital period around the Sun, is given by Eq. 2 and by Eq. 3, respectively. Note that Eq. 3, derived in 5, is corrected here by the addition of a $1/2$. Once again, recall that in Eq. 3, thermal lag is ignored. In both, Eq. 2 and Eq. 3, averaged Fourier coefficients are used.

$$\dot{\bar{\omega}}_z = \frac{G_1}{I_z a^2 \sqrt{1-e^2}} \bar{C}_{0,z} \tag{2}$$

$$\dot{\bar{i}}_s = \frac{G_1}{2\omega_z I_z a^2 \sqrt{1-e^2}} \left(\bar{C}_{1,x} + \bar{D}_{1,y} \right) \tag{3}$$

It is important to note that the dynamics of angular velocity and solar inclination are coupled together. The Fourier coefficients will vary as the inclination evolves, therefore, the rate of change of angular velocity is indirectly dependent on the evolution of the obliquity.

3. INFERRED NORMALIZED COEFFICIENTS

The averaged YORP coefficient required to obtain an observed rotational period, only due to YORP, can be found if the rotational period, mass and moments of inertia are known for an observed object. This is done by simply solving Eq. 2 for $\bar{C}_{0,z}$ as shown in Eq. 4. It is important to note that obtaining the averaged coefficient in this manner assumes that the coefficient remains constant throughout time, that is to say that the obliquity does not change over time. Therefore, this only provides an order of magnitude estimate of the coefficient.

$$\bar{C}_{0,z} = \frac{\dot{\omega}_z I_z a^2 \sqrt{1-e^2}}{G_1} \tag{4}$$

It is of interest to normalize this value in order to compare inferred YORP coefficients across bodies. Following reference 5, we take the YORP coefficient and divide it by the moment of inertia about the z-axis, multiply it by the total mass, and divide by the largest dimension of the satellite, as shown in Eq. 5, where b is the largest dimension and M is the total mass.

$$\mathcal{C}_{0,z} = \frac{\bar{C}_{0,z} M}{I_z b} \tag{5}$$

4. YORP EFFECT AND OBSERVED ROTATIONAL DATA

To understand if YORP could be causing the observed rotational behavior, a model is developed to simulate an uncontrolled satellite. The YORP effect theory is then used on this model to compute the averaged Fourier coefficients describing the average torque that acts on the body over a year time period. Next, coefficients are computed to identify how much torque would be required to achieve the observed rotational rate. Both these sets of coefficients are normalized and their order of magnitude is compared. The following sections give a description of the model used and the results obtained.

4.1. Model Description

A satellite in Earth orbit will effectively orbit the Sun as the Earth itself completes an orbit around the Sun. The motion of the satellite about the Earth is small compared to the motion about the Sun and is therefore ignored for this study. Hence, for the purpose of this analysis, the defunct satellite is placed in an orbit around the Sun with the same orbital elements as the Earth's orbit about the Sun, with the exception of the inclination. The inclination of the orbit matches the tilt of the Earth's rotation axis. This will simulate the orbit traced out by an equatorial Earth orbiting satellite. The rotation pole of the defunct satellite is set to line up with that of Earth's.

The satellite Envisat is used as a model for this paper. Envisat was launched in 2002 and lost contact with Earth in April 2012, becoming a defunct satellite and hence a piece of debris. This satellite is an ideal model because of it's geometrical asymmetry. The size of the satellite frame is 10 m x 4 m x 4 m, in addition it has one solar array of size 15 m x 6 m [9]. The total mass of the spacecraft is 8200 kg which includes a total mass of 300 kg for fuel [9]. The satellite has the configuration shown in Figure 2.

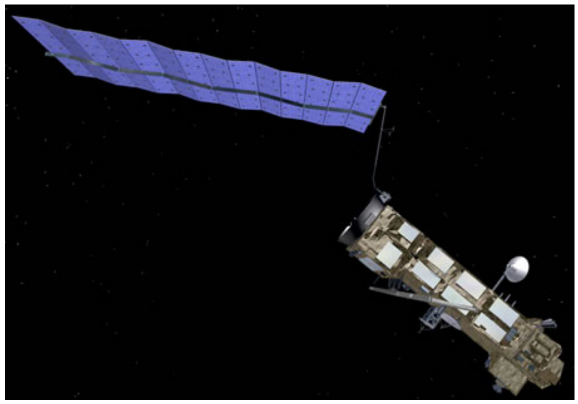

Figure 2: Envisat satellite

The satellite model used in this analysis has the same asymmetry of only one solar panel that is exhibited by Envisat, however it should be noted that the model does not have the exact same configuration as Envisat. Envisat was used as a guide so that the model used had realistic dimensions, mass and shape. The model used in this study is shown in Figure 3.

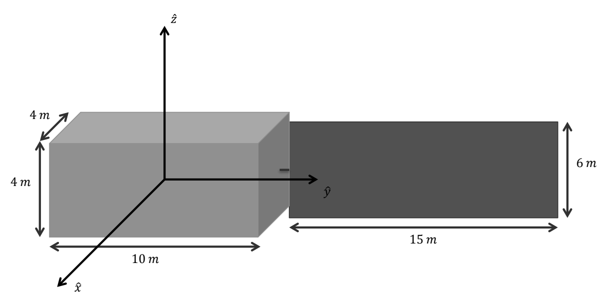

Figure 3: Defunct satellite model used in analysis

The center of mass of the satellite is assumed to be located at the origin of the body coordinate frame, as shown. The moments of inertia of the satellite model shown in Figure 3 are given in Table 1. It should be noted that when the moments of inertia were computed the mass of the fuel was subtracted from the total mass in order to better simulate a defunct satellite. The moment of inertia of the satellite bus was computed assuming a solid rectangular prism and the moment of inertia of the solar panel was computed assuming a thin plate. The total moment of inertia of the whole satellite was then found using the parallel axis theorem.

Table 1: Moment of Inertia of Envisat

$\mathbf{I_x (kg\ m^2)}$	$\mathbf{I_y (kg\ m^2)}$	$\mathbf{I_z (kg\ m^2)}$
582133.333	55033.333	605433.333

Because the satellite's body is symmetric, it is likely that no torque will be created as a result of YORP, therefore, when computing the YORP torque on the body, only the solar panel is considered. It is important to note, however, that for the computation of the evolution of angular velocity, the total mass of the body and total moment of inertia about the z-axis will be used. To model the solar panel of this spacecraft, two facets are used to simulate the front and back of the array. The facet used for the front of the solar panel will be called facet 1 and the facet for the back of the array will be called facet 2. The optical properties of both facets are given by Table 2. The reflectance and emissivity of the solar panel were obtained from reference 10.

Table 2: Optical Properties of Facets

	Facet 1	Facet 2
ρ	0.21	0.82
s	0.2	0.2
B	2/3	2/3
ϵ_f	0.81	0.85
ϵ_b	0.85	0.81

The geometrical properties of each facet are given by Table 3, where $\hat{n}$ is the normal vector and $\vec{r}$ is the unit vector which points from the center of mass to the center of each facet.

Table 3: Geometrical Properties of Facets

$\hat{n}$	$[1,0,0]^T$	$[-1,0,0]^T$
$\hat{r}$	$[0,1,0]^T$	$[0,1,0]^T$

Using this model description, the averaged YORP moment coefficients are found. The relationships between $\bar{C}_0$ and $\bar{C}_1$ and obliquity are shown in Figures 4 and 5, respectively. The relationship between the $\bar{D}_1$ coefficient and obliquity is not shown because all components of the coefficient are zero at all solar latitudes.

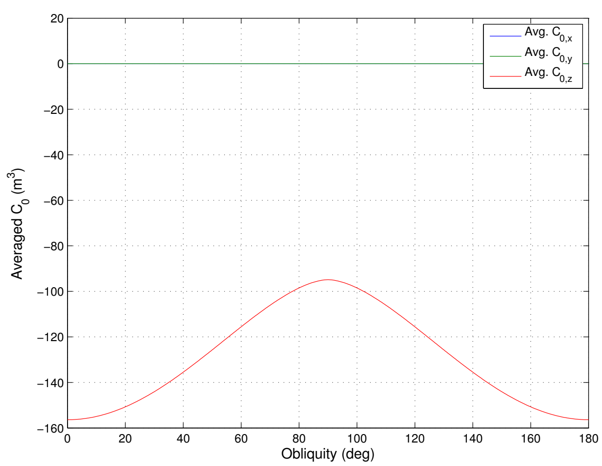

Figure 4: $\bar{C}_0$ coefficient as a function of obliquity

4.2. Inferred Coefficient Comparison

YORP coefficients can be computed for satellites whose rotational period has been observed using the method outlined in section 3. An example of this is povided for the GEO satellite Gorizont-11. The satellite was observed in 1994 and was found to have a rotational period of 189.4 seconds [7]. This is a 3-axis stabilized spacecraft and is expected to have began rotating under external torques at the end of its lifetime [7]. For this computation, it is assumed that all Gorizont family spacecraft are of equal mass and equal dimensions. The mass of this satellite is 2150 kg and has dimensions of 5.4 m x 3.3 m x 9.5 m [11]. This satellite was launched in 1985 and had an operational life span of 3

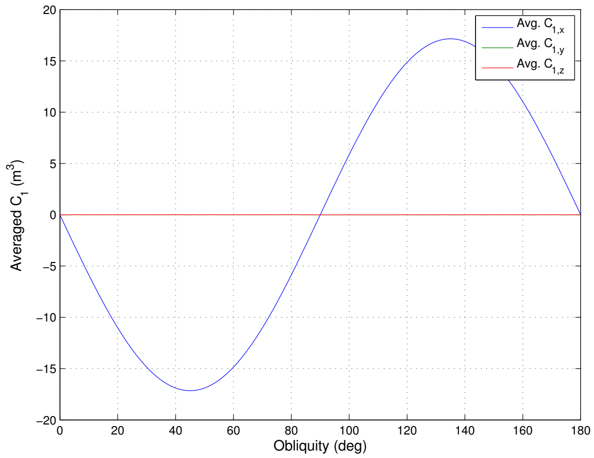

Figure 5: $\bar{C}_1$ coefficient as a function of obliquity

years [11]. This mission was considered successful and it became a defunct satellite in 1988. Hence the satellite must have reached its observed rotational period of 189.4 seconds in a time span of six years. This corresponds to a rate of change of the angular velocity, $\dot{\omega}$, during that time of 1.7521e-10 rad/sec^2. To find the moment of inertia of the satellite, it is assumed that it is a solid cube with the dimensions (w x d x h) and mass stated above. The moment of inertia about the z-axis is then found to be 21394 $kg\ m^2$. Using Eq. 4, the averaged $C_{0,z}$ coefficient is found to be 0.83878 m^3. It is relevant to compare this quantity with the computed quantity for our Envisat-like example. For this comparison the YORP coefficients are made into dimensionless numbers so that they can easily be compared between bodies. Doing so for the Envisat-like example gives a value of -8e-2, while doing so for the above Gorizont example gives a value of 9e-3.

This same analysis is done for a number of other defunct GEO satellites and the results are shown in Table 5. The observational data for each satellite is also shown in Table 5. The satellite defunct dates in this table were obtained from reference 12 and the observation dates and observed rotational periods were obtained from reference 7. The physical parameters that were used to compute the moments of inertia of the defunct satellites are shown in Table 4. These values were obtained from references 11 and 12. For the satellites with no radius value the moments of inertia were computed by assuming a solid cube and for the satellites with a radius the moment of inertias were computed assuming a solid cylinder. It should be noted that the dimensional data for all satellites is used not as an exact number, rather as an order of magnitude estimate of the true dimensions of each satellite.

Table 4: Physical Properties for Defunct GEO Satellites

Satellite Name	**Mass(kg)**	**Length(m)**	**Width(m)**	**Depth(m)**	**Radius(m)**
Gorizont 9	2110	5.45	3.3	9.46	N.A.
Gorizont 11	2110	5.45	3.3	9.46	N.A.
Gorizont 14	2110	5.45	3.3	9.46	N.A.
Gorizont 16	2110	5.45	3.3	9.46	N.A.
Ekran 2	1970	4	N.A.	N.A.	2
Raduga 10	1954	5.5	N.A.	N.A.	2.5
Raduga 12	1954	5.5	N.A.	N.A.	2.5
Raduga 14	1954	5.5	N.A.	N.A.	2.5
Raduga 20	1954	5.5	N.A.	N.A.	2.5
Luch	2400	8.5	11	16	N.A.

As can be seen from Table 5, the normalized coefficients for all satellites are all in the same order of magnitude as the Envisat-like example. Though this work does not prove that YORP is in fact causing the observed rotational behavior of defunct satellites, it indicates that YORP could, at the very least, be a major contributing factor yielding the

Table 5: Observation Data and Normalized Coefficient for Defunct GEO Satellites

Satellite Name	**Defunct Date**	**Observation Date**	**Observed Rotational Period (sec)**	$\mathcal{C}_{\mathbf{0,z}}$
Gorizont 9	1987	2/22/1996 - 11/24/2003	24.33 - 41.50	4.5e-2 - 1.6e-2
Gorizont 11	1988	12/7/1994	189.4	9e-3
Gorizont 14	1990	12/20/1998	85.50	1.5e-2
Gorizont 16	1991	3/29/1998	89.95	1.6e-2
Ekran 2	1978	12/14/1995 - 11/21/2003	68.96 - 69.93	1.8e-2 - 1.3e-2
Raduga 10	1987	12/18/1990 - 1/25/1998	15.20 - 24.70	0.2 - 3.4e-2
Raduga 12	1985	3/27/1995 - 3/20/1999	128.80 - 37.80	7e-3 - 1.7e-2
Raduga 14	1987	12/5/1994 - 3/20/1999	46.28 - 53.61	2.8e-2 - 1.4e-2
Raduga 20	1991	10/16/1991 - 10/17/1991	495.43	1.9e-2
Luch	1990	2/25/1987 - 4/27/1995	21.60 - 21.90	6.1e-2

observed rotational periods. This further motivates the need for more observations of defunct satellites as many light curves cannot be resolved. Furthermore, observational data should aim to estimate optical properties of the observed objects. It should also be noted that these normalized coefficients are of the same order of magnitude as those found for asteroidal shapes [5].

5. CONCLUSION

The YORP effect, which has been observed to caused a change in an asteroid's angular velocity, is applied to a defunct satellite model. The year averaged dynamics describing the evolution of angular velocity and obliquity as a result of this effect are described. The averaged Fourier coefficients describing the average torque acting on an Envisat-like satellite were computed. The year averaged dynamic equations were used to compute the required $\bar{C}_{0,z}$ so that an observed defunct satellite would have the observed rotational period. This was done for several satellites whose rotational period has previously been observed. The inferred coefficients were normalized and compared to the previously computed coefficient for the Envisat-like model. The results show that YORP could be used to explain the rotational behavior observed. However, more research is needed to prove that YROP is causing the rapid and varying rotational periods that are observed. The theory needs to be further compared with observations for a full validation.

6. ACKNOWLEDGMENT

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