Iteratively Reweighted Blind Deconvolution

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ABSTRACT

Traditional blind deconvolution techniques rely on a statistical model that relates the measured data to the pristine scene whose reconstruction is sought. If the data is not consistent with this forward model, then the reconstruction is badly degraded. We develop a way of making blind deconvolution robust to modeling errors by assigning a weight to each pixel of measured data and iteratively updating the weights. We show that this approach is effective in several realistic model-mismatch scenarios.

Keywords: B lind deconvolution, image reconstruction, robust estimation

1. INTRODUCTION

Astronomical imaging with large telescopes is difficult because atmospheric turbulence introduces a strong, rapidly-changing blur. This can be addressed in hardware (via adaptive optics, which tends to be complex and expensive) or in image processing software. In this paper, we focus on image processing techniques for mitigating atmospheric turbulence, in particular multiframe blind deconvolution (MFBD) [1, 2]. MFBD algorithms are generally formulated by regarding image reconstruction as a parameter estimation problem, where the parameters to be estimated are the pixels of the restored image. Additional "nuisance" parameters are needed to represent the state of the atmosphere at the time that each frame was measured. A maximum likelihood estimator is then constructed relating the image and time-sequence of atmospheric states to the measured data. The estimate (i.e., the reconstructed image) is computed using general-purpose numerical optimization techniques.

MFBD algorithms of this type can perform well provided that the estimator's model of the relationship between parameters and data is accurate. If the forward model is not consistent with the collected data, then the resulting image will be corrupted by artifacts and may be unrecognizable.

Before reviewing instances of this, let us establish notation and formulate a simple MFBD estimator. Let o be the two-dimensional image to be recovered. Assume that over a short time interval, the scene o remains constant while the atmospheric blur changes. Let d_i, $i = 1, \ldots, N$ be N camera frames collected during this interval. Then the relationship between o and d_i is

$$d_i = o * h_i + n_i, \tag{1}$$

where h_i is the atmospheric point-spread function corresponding to each camera frame, '$*$' represents convolution, and n_i captures all noise sources associated with the image measurement process. We will assume that n_i is a normally-distributed random variable with mean 0 and variance $o * h_i + \sigma^2$. This is intended to approximate a combination of shot noise (having variance $o * h_i$) and read noise (with variance σ^2).

Given the forward model in Eq. 1, we may formulate a maximum likelihood estimator

$$\{\hat{o}, \hat{h}_i\} = \arg\max_{o,h_i} \sum_{i,j} \frac{(o * h_i - d_i)_j^2}{(o * h_i)_j + \sigma^2}, \tag{2}$$

where i indexes measured image frames and j indexes pixels. For the examples shown in this paper, the optimization is performed using a L-BFGS-B [3], a quasi-Newton algorithm suitable for large-scale problems.

2. MODEL MISMATCH

This will fall apart if the data is in some way inconsistent with the model in Eqn. 1. Inspired by the iterated least-squares technique for robust linear regression [4], we now describe a simple robust MFBD. Define a weight w_{ij} on every pixel of every measured data frame. Initially, all the weights will be 1. Here is the procedure.

1. Evaluate a weighted ML estimate

$$\{\hat{o}, \hat{h}_i\} = \arg\max_{o,h_i} \sum_{i,j} w_{ij} \frac{(o * h_i - d)_j^2}{(o * h_i)_j + \sigma^2}. \tag{3}$$

2. Calculate a vector of normalized residuals

$$r_{ij} = \frac{(\hat{o} * \hat{h}_i - d_i)_j}{\sqrt{(\hat{o} * \hat{h}_i)_j + \sigma^2}}. \tag{4}$$

3. Calculate the median-absolute-deviation estimate of standard deviation:

$$\sigma_{\text{MAD}} = 1.48\,\text{median}(|r_{ij} - \text{median}(r_{ij})|). \tag{5}$$

4. Update the weights:

$$w_{ij} = \begin{cases} \left(1 - \left(\frac{r_{ij}}{5\,\sigma_{\text{MAD}}}\right)^2\right)^2 & \text{if } |r_{ij}| < 5\,\sigma_{\text{MAD}} \\ 0 & \text{else.} \end{cases} \tag{6}$$

5. Return to step 1 and repeat with the updated weights.

One could devise a termination condition based on the difference of the weights from one iteration to the next, but in practice, this procedure is usually observed to converge after three or four iterations.

Let us now apply this to several species of modeling error.

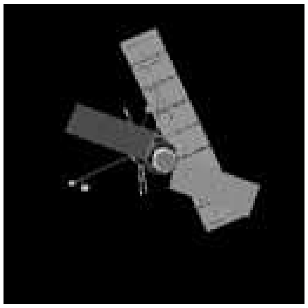
2.1 A changing scene

A basic assumption in the formulation of MFBD is that the scene o is the same for all data frames d_i. This is violated if, for example, an object in the scene briefly glints. This is a common circumstance when observing solar-illuminated man-made objects. Fig. 1(a) shows a CAD model of a satellite. In Fig. 1(b), the satellite image has been convolved with a simulated atmospheric PSF. In Figs. 1(c)–(d), random glints were simulated in the data by adding delta functions to the satellite model at random locations.

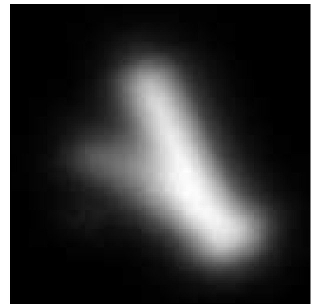
Reconstructing o with "regular MFBD" (i.e. Eqn. 2) yields Fig. 2(a), in which artifacts are clearly visible. Applying iteratively reweighted MFBD yields Fig. 2(b), which is clearly superior. By the end of the third iteration, those portions of the image frames contaminated by glints have been assigned zero weight. The reconstruction therefore does not show any glints.

2.2 Mismodeled noise

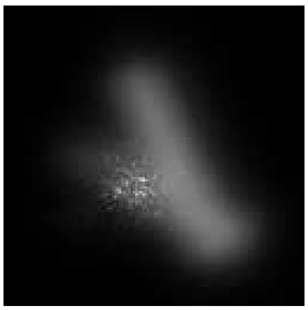
Iteratively reweighted MFBD is also applicable to mismodeled noise. To demonstrate this, simulated data is generated using the same scene and PSFs as before, but salt-and-pepper noise is added to 10% of the pixels selected at random. The regular MFBD and iteratively reweighted MFBD reconstructions are shown in Fig. 3. Since the salt-and-pepper noise does not conform to the model (i.e. normally distributed with zero mean and variance $o * h_i + \sigma^2$), regular MFBD is unable to produce a high-quality reconstruction. The iteratively reweighted MFBD weights the noise-corrupted pixels to zero and successfully recovers the scene.



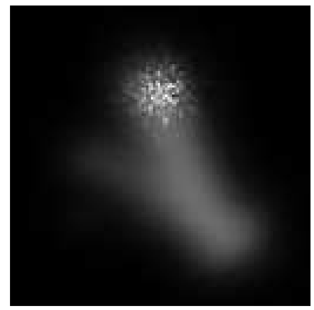
(a) Pristine image



(b) Blurred image

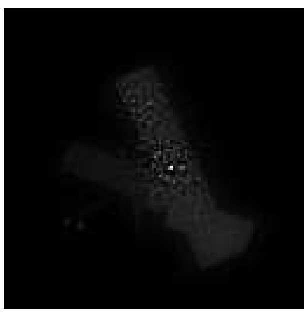


(c) Blurred glinting image

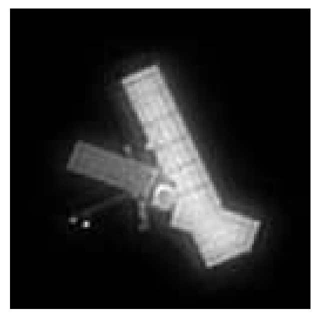


(d) Blurred glinting image

Figure 1. Pristine image and blurred (and glinting) measurements.



(a) Regular MFBD



(b) Iterated MFBD

Figure 2. Reconstruction of glinting data illustrated in Fig. 1.

2.3 Edge artifacts

The convolution in Eqn. 1 is implemented by multiplying FFTs of the image and PSF arrays and then calculating the inverse FFT. This means that the contribution from a pixel near one edge can wrap around to the other side of the predicted image if the PSF is too big. Wrap-around can be avoided by zero-padding both grids prior to convolution. If padding is not performed, then the forward model is not consistent with the data, and artifacts will occur in a naïve reconstruction.

This is illustrated in Fig. 4. In the simulated dataset, the object drifts partially out of the field of view. In the standard maximum likelihood reconstruction with no array padding in the convolutions, the predicted image circularly wraps. This makes it hard to match the abrupt edge in the observed data. The result, shown in Fig. 4(a) is a reconstruction with an edge artifact. When the iteratively reweighted algorithm is applied, still with no array padding, a clean reconstruction is obtained (Fig. 4(b)). The algorithm achieves this by deweighting the pixels near the edge in the image frames where the object drifts out of the field of view.

(a) Regular MFBD (b) Iterated MFBD

Figure 3. Reconstructed data with mismodeled noise.

(a) Regular MFBD (b) Iterated MFBD

Figure 4. Reconstructed data with edge effects.

3. CONCLUSIONS

We have demonstrated that traditional MFBD algorithms based on maximum likelihood estimation are sensitive to the sort of modeling errors that one frequently encounters in real-world applications. In robust estimation theory, this sensitivity is quantified in terms of the asymptotic breakdown point, which is the fraction of the measurements that can be corrupted without introducing an arbitrarily large change in the estimate. The breakdown point of traditional MFBD is zero: as one measured pixel tends to infinity, the norm of the estimated image will tend to infinity as well.

The iteratively reweighted MFBD developed here has a breakdown point greater than zero, for if a single pixel is corrupted, it will receive a weight of zero. We have presented several types of model mismatch for which traditional MFBD fails and iteratively reweighted MFBD succeeds. The technique is simple and mostly external to the MFBD itself, so it may be easily retrofitted onto existing MFBD implementations.

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