

## Analysis of Faint Glints from Stabilized GEO Satellites

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### 1 SUMMARY

Ground-based telescopes routinely acquire temporal brightness measurements of satellites in geo-stationary and geo-synchronous orbit that provide valuable characterization information. For instance, GEO satellites that are not stabilized tend to rotate, and produce brightnesses that vary in time with frequencies corresponding to rotation rates. Temporal brightness patterns can also be exploited to characterize stabilized GEO satellites. For example, many operational GEO satellites have solar panels that glint when they reflect sunlight towards an observer in a mirror-like fashion. These well-known solar panel glints can be remarkably bright, often exceeding several stellar magnitudes in amplitude. Measured brightnesses and times of these glints can be exploited to estimate the size, segmentation, and alignment of the solar array, valuable information about the satellite's power generation and consumption capabilities. However, satellites can produce other glints in addition to those originating from solar panels. These glints can be much fainter, with amplitudes as small as 0.2 magnitudes. Several observations of GEO satellites show several such glints occurring during the span of a single night. Furthermore, many of these recur from night to night when observed from a single ground-based site, but with subtle, incremental changes in both peak times and brightnesses. These fainter glints must originate from reflective elements mounted on the satellite's main bus, solar panel structure, or other peripheral structures that might be stationary or moving with respect to the main bus. Our analysis indicates that such glints can be exploited for GEO satellite characterization.

### 2 INTRODUCTION

Ground-based optical and radar sites routinely acquire resolved images of satellites, yielding a great deal of knowledge about orbiting spacecraft. However, the important population of GEO satellites often cannot be resolved, and must be characterized using methods other than imagery. In this regard temporal photometry (i.e., measurements of whole-object brightness as a function of time) can be very valuable. For instance, GEO satellite rotational motion can be detected and characterized by analyzing periodic brightness variations [1-5]. Temporal brightness patterns can also be exploited to characterize stabilized GEO satellites [5-10]. In particular, glints that are produced by mirror-like reflections of Sunlight provide a great deal of information about the satellite and its components. The objective of this paper is to outline how sequences of faint glints from GEO satellites can be used for characterization.

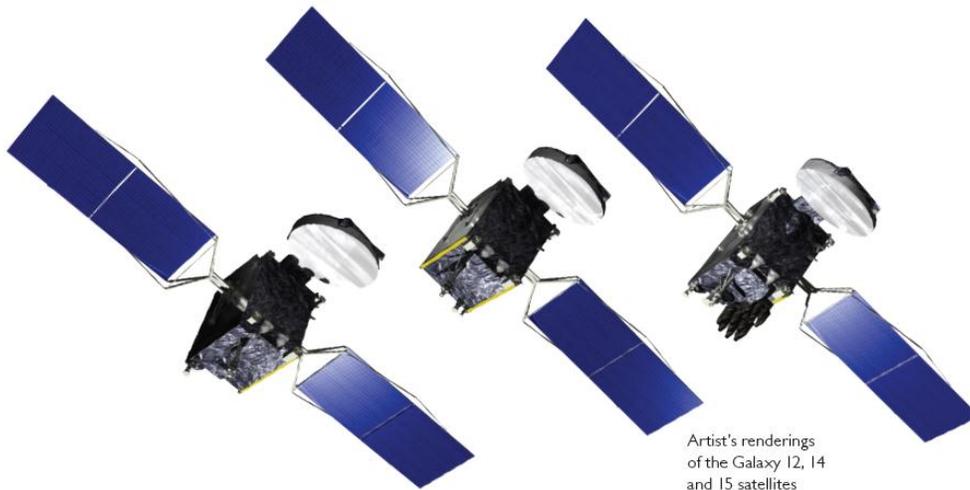
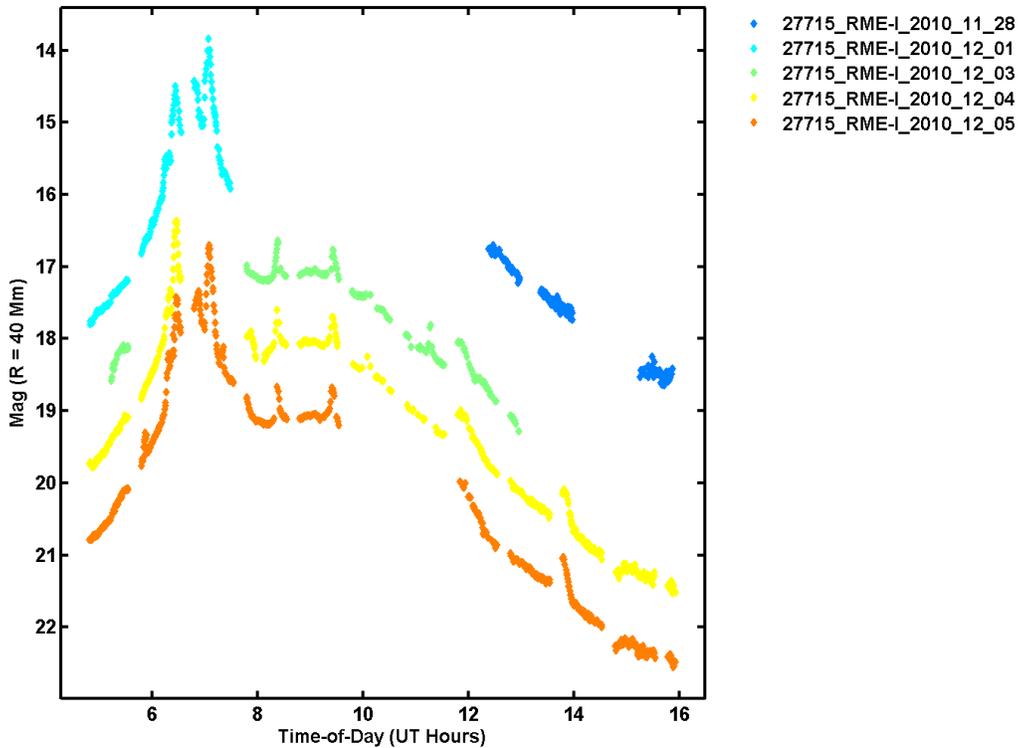


Figure 1. Artists renderings of the Galaxy 12, 14 and 15 geosynchronous satellites [11].

### 3 GLINTS FROM GEO SATELLITES

Many operational GEO satellites have solar panels that produce remarkably bright glints, often exceeding several stellar magnitudes in amplitude [9]. Measured brightnesses and times of these glints can be exploited to estimate the size, segmentation, and alignment of the solar array [6-8], valuable information about the satellite's power generation and consumption capabilities. GEO satellites can, however, produce other glints in addition to those

originating from solar panels. Some of these can be much fainter, with amplitudes as small as 0.2 magnitudes, and several GEOs have been seen to produce several such faint glints during the span of a single night [5]. Furthermore, these faint glints recur from night to night when observed from a single ground-based site, but with subtle, incremental changes in peak times and and/or brightnesses. These fainter glints must originate from reflective elements mounted on the satellite's main bus, solar panel structure, or other peripheral structures that might be stationary or moving with respect to the main bus.



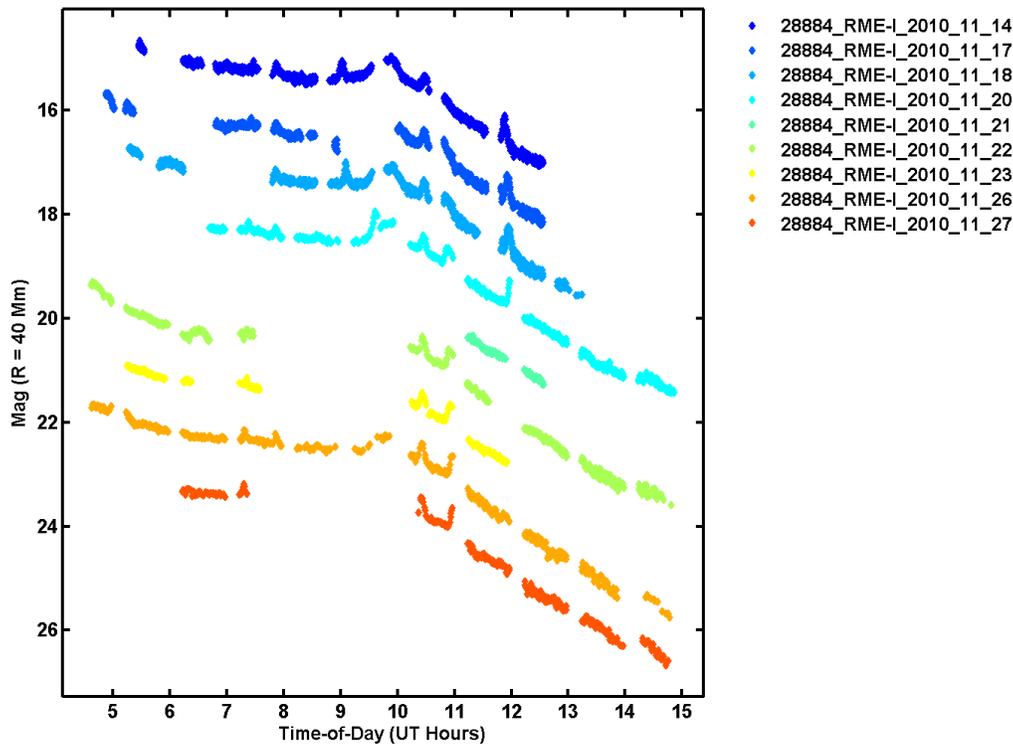
**Figure 2. Galaxy 12 (SCN 27715) photometric I-band measurements from the AMOS RME site, acquired during the period 2010 Nov 28 to Dec 05 (as color coded), with 1 magnitude vertical separations added between the successive nights.**

### 3.1 Observations of Faint Glints from GEO Satellites

Intelsat's Galaxy 12 and Galaxy 15 satellites (see Figure 1) have both been observed to produce faint glints [5]. Figure 1 shows artist renderings of these two satellites, plus their nearly identical sister Galaxy 14, which use Orbital's *Star-2*<sup>TM</sup> design, featuring a roughly cubical bus (measuring 1.75 m × 1.7 m × 1.8 m) and two solar panels (each measuring approximately 1.6 m × 4 m) extending on axles from the bus. For nominal operations, these satellite buses typically maintain an Earth-facing attitude and the solar panels articulate about an axis approximately aligned with Earth's N/S pole direction, in order to maintain optimal sunward orientation. Each of the three sister satellites carries a payload of multiple C-band transponders to perform their commercial communications tasks. However, Galaxy 15 also possesses a series of L-band transmitters to broadcast GPS navigation data for the FAA's Geostationary Communications and Control Segment mission [11].

Figures 2 and 3 show observations of Galaxy 12 and Galaxy 15, respectively, that include multiple examples of faint glints [5]. These figures show long-duration brightness signatures over several nights in late 2010. Each plot shows the whole-object brightness on the vertical axis vs. time of day on the horizontal axis. The brightnesses were measured from the AMOS Remote Maui Experiment (RME) observing site using the photometric I-band (wavelength  $\approx$  800 nm), and are plotted as stellar magnitude normalized to an observer-to-object range of 40,000 km. The measurements were acquired during the time period that Galaxy 15 had lost the ability to re-position itself (due to an earlier anomaly) causing it to drift in longitude through the GEO belt (see [5] for details). During this period Galaxy 15 could no longer perform its original commercial mission for Intelsat, so its sister satellite, Galaxy 12, was re-positioned for that purpose. While Galaxy 15 was freely drifting, both satellites were observed intensively using small telescopes in both Maui and New Mexico [5], including multiple efforts to obtain long-duration (i.e., all-night) signatures for each.

Figure 2 shows observations of the Galaxy 12 satellite conducted during six nearly consecutive nights in late 2010 using a 0.4 m telescope at the RME site [5]. The measurements were acquired after Galaxy 12 had been re-located to Galaxy 15's pre-anomaly position, and during a period of time when Galaxy 12 was known to be functioning normally in all respects. These signatures provide overlapping coverage over multiple nights, and clearly indicate the consistent and largely repeatable nature of the brightness signatures. This consistency confirms that during this period, this satellite maintained a stabilized attitude, which naturally produces such night-to-night repeatability for GEO satellites observed from a single ground-based site. In fact, these signatures are so repeatable that they plot on top of one another from night to night as shown originally in [5]; they're re-plotted here with 1 magnitude separations added for successive nights, in order to emphasize the repeatability of the glint features. These Galaxy 12 signatures clearly show a series of bright glints apparent between 06:00 and 07:00 UT, but several faint glint features as well. Some, but not all, of these faint glints repeat from night to night, such as those near 08:20, 09:25, 12:00, and 14:00 UT.



**Figure 3. Galaxy 15 (SCN 28884) photometric I-band measurements from the AMOS RME site, acquired during the period 2010 Nov 14 to Nov 27 (as color coded), with 1 magnitude vertical separations added between the successive nights.**

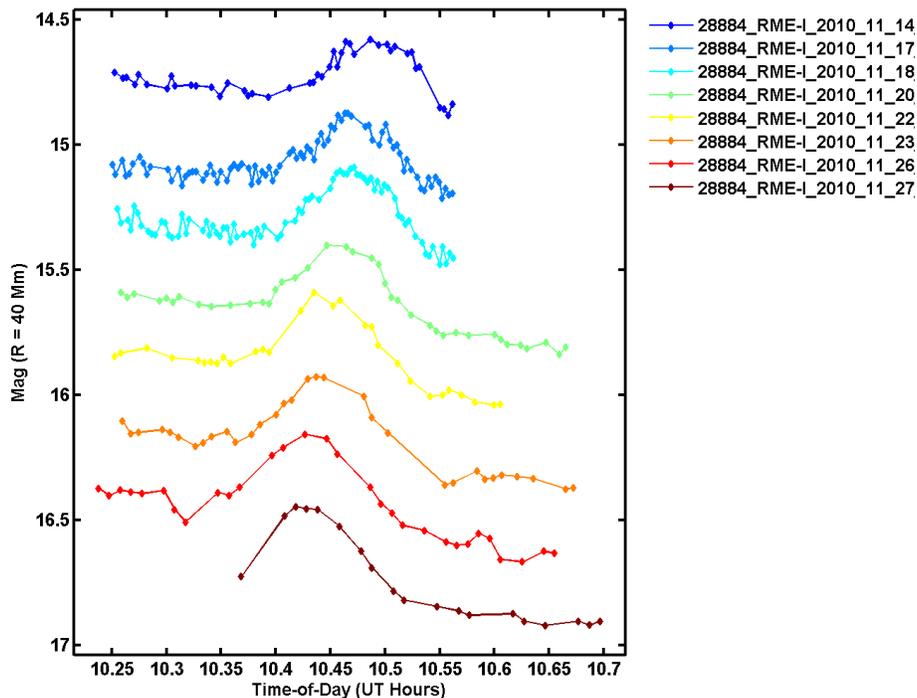
Figure 3 shows Galaxy 15 data acquired from the same RME site during nine nights distributed over a two-week interval in 2010. The observations were conducted while Galaxy 15 was slowly drifting through the GEO belt, but maintaining a stabilized attitude [see 5]. Again, the overlapping signatures clearly indicate consistent and largely repeatable night-to-night brightness patterns, confirming the attitude stabilization status of this freely-drifting satellite [5]. These nine Galaxy 15 signatures also show multiple examples of faint glints. Again, some but not all of these faint glints repeat from night to night, including one notable sequence of glints near 10:30 UT that appear with roughly the same amplitude in all eight nights when observations were acquired at that UT time.

While many of these faint glints persist night after night over the several day period spanned by the observations in Figures 2 and 3, as least some appear, disappear, and/or change. For instance, on Figure 2 near 13:40 UT no glint feature was detected from RME on 2010 Nov 28 (dark blue points), but was detected several days later for two days in a row, on Nov 04 and 05 (yellow and orange points, respectively). The two figures show other similar examples of faint glints changing as well. These examples suggest that, while faint glints measured from a single ground-based site may remain largely repeatable over periods of a few days, they can also appear, disappear and/or change as well [5]. These changes likely result from night-to-night changes in illumination, controlled by the seasonal changes in the Sun's position with respect to the observer and object. However, the fact that many (if not most) of these faint glints do repeat multiple times at very nearly the same time of night strongly suggests that they originate

from the same component on the satellite. This component could be located either on the cubical main bus, or on the articulating solar panel.

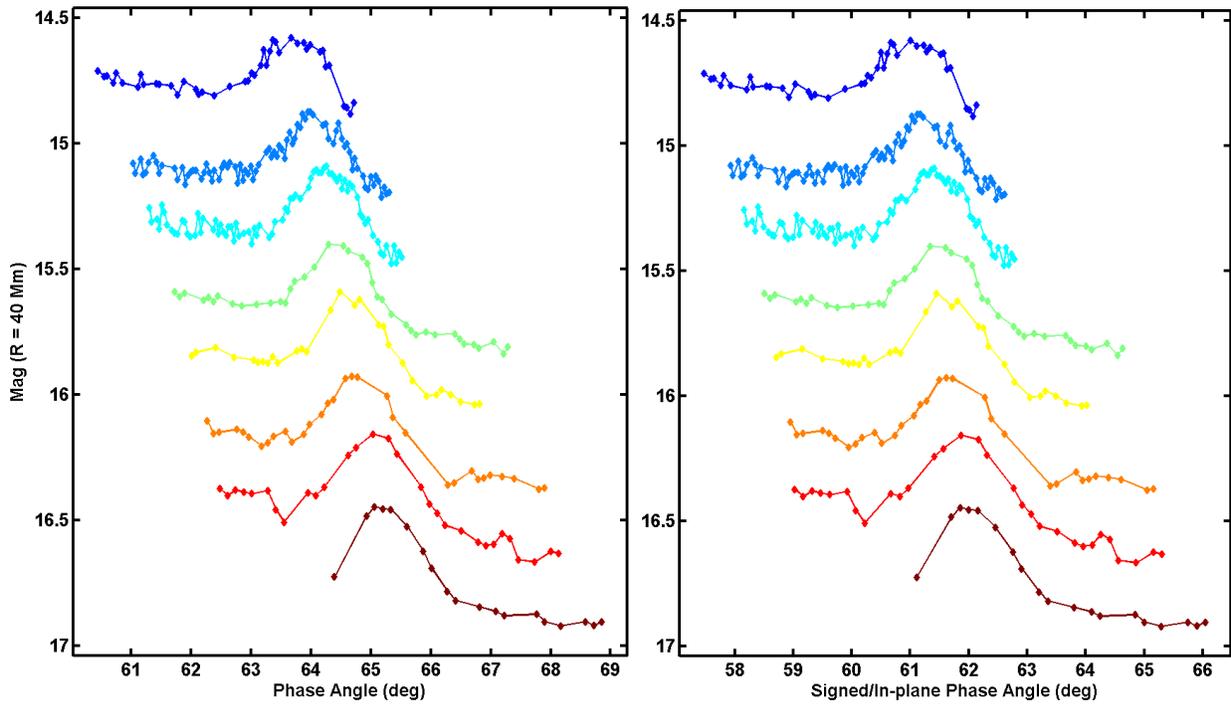
### 3.2 A Sequence of Faint Glints from Galaxy 15

Figure 4 shows an expanded zoomed-in view of the previously-mentioned series of eight glints produced by Galaxy 15 — the most extensive sequence of faint glints observed for these satellites. This plot shows the signatures as function of time-of-day, but with a reduced time span of approximately 10:10 to 10:45 UT, which brackets all of the glints in the sequence. The plot also uses a smaller night-to-night magnitude offset, in order to emphasize the sequential, night-to-night evolution of the glints. Each glint in the sequence appeared in every observation spanning that UT period throughout the entire 14-day observation period of 2010 Nov 14-27. Notably, all eight glints have very nearly the same duration ( $\approx 10$  minutes), and the same brightness amplitude ( $\approx 0.2$  magnitudes). However, the eight glints do not peak at the same times. The first glint observed during the 14-day period (shown plotted in blue in Figure 4) peaks in brightness at  $\approx 10:30$  UT, and as time progresses the peak times of the glints clearly shift to earlier UT values, so that by the end of the sequence the latest glint peaks at  $\approx 10:25$  UT.



**Figure 4. A sequence of eight glints detected from Galaxy 15 (SCN 28884) during the period 2010 Nov 14 to Nov 27 (as color coded), plotted as a function of time-of-day, with 0.25 magnitude vertical separations added between the successive nights.**

The  $\approx 5$  minute shift in glint peak-times apparent in Figure 4 must result from night-to-night changes in observation and illumination conditions. One possibility is that this time-shifting simply reflects changes in the solar phase angle or the closely-related “signed/in-plane” phase angle, used previously by GEO observers to account for illumination changes [6-10, 14, 15]. Figure 5 shows the eight glints re-plotted in terms of the normal solar phase angle (left panel) and the signed/in-plane phase angle (right panel). The normal solar phase angle measures the Sun-satellite-observer angle, and varies between  $0^\circ$  for “full-moon” type illumination to  $180^\circ$  for “new moon” illumination. The signed/in-plane phase angle measures the E/W longitudinal component of the Sun-satellite-observer angle (i.e., the component subtended in Earth’s equatorial plane), and can have both positive and negative values (see [16] and references therein for more details). This latter angle has been found especially useful in accounting for the night-to-night shifts in glints from solar panels. Specifically, when solar panel glints are plotted in terms of the signed/in-plane phase angle their peaks tend to line up at the same value. Clearly from Figure 5, neither the normal phase angle nor the signed/in-plane phase angle aligns these eight peaks, and therefore do not help explain why the glints shift in time. However, as discussed below, a more detailed study of the geometric conditions that create glints can be used to define other angles that can be more useful in this regard.



**Figure 5.** The eight-glint Galaxy 15 sequence detected during 2010 Nov 14 to Nov 27 (as color coded in Figure 3), plotted as a function of the solar phase angle (left) and the signed/in-plane (i.e., E/W longitudinal) phase angle (right), again with 0.25 magnitude vertical separations added between the successive nights. Note that the glint peaks do not line up as a function of either of these commonly-used phase angles.

#### 4 ANALYSIS OF GLINTS FOR GEO SATELLITE CHARACTERIZATION

For a component of a satellite to produce a glint, it must reflect sunlight in a specular or near-specular reflection fashion. Depending on the component's shape, this kind of mirror-like reflection generally requires that a specific geometrical constraint be satisfied. In other words, the observation/illumination geometry that can potentially produce glints represents a rather tightly constrained condition. Investigating the nature of this "specular reflection condition" for different shapes provides a means to exploit sequences of glints for characterization to constrain the nature and orientation of the reflective component on the GEO satellite.

##### 4.1 Geometrical Glint Conditions: The Phase-Angle Bisector

Different shapes have different specular reflection conditions [12]. For instance, a large flat surface, such as a solar panel, glints when the angle of incoming sunlight equals (or nearly equals) the outgoing angle towards an observer. Another way to express this specular condition is that the normal to the facet nearly coincides with the "phase-angle bisector" (PAB), which is defined as the unit direction vector midway between the satellite-to-Sun and satellite-to-observer directions:

$$\mathbf{b}(t) \equiv \frac{\mathbf{o}(t) + \mathbf{s}(t)}{|\mathbf{o}(t) + \mathbf{s}(t)|} \quad (1)$$

Here,  $\mathbf{o}(t)$  denotes the satellite-to-observer unit direction vector and  $\mathbf{s}(t)$  the satellite-to-Sun unit direction vector, both specified in an inertial reference frame (e.g., J2000). These two unit vectors vary in time due to the changing relative positions of the satellite, observer, and Sun. A perfect glint with peak brightness occurring at a time  $t_g$  indicates that, at that time, the facet normal and the phase-angle bisector perfectly coincide:

$$\mathbf{n}(t_g) \cdot \mathbf{b}(t_g) = 1 \quad (2)$$

where  $\mathbf{n}(t)$  denotes the unit vector normal to the facet specified the inertial reference frame, which varies in time in response to changes in the orientation of the satellite and/or its articulating components. For most materials,

significantly enhanced reflection also occurs when the facet normal and the phase-angle bisector make a close approach to one another, so that they nearly coincide and the dot product in eq. (2) approximates one.

Notably, satellite sub-components with other shapes have different specular conditions, which generally can also be conveniently expressed in terms of the phase-angle bisector. For instance, a cylinder glints at peak brightness when its axis lies perpendicular to the phase-angle bisector:

$$\mathbf{c}(t_g) \cdot \mathbf{b}(t_g) = 0 \quad (3)$$

where  $\mathbf{c}(t)$  denotes the time-dependent unit vector along the cylinder axis specified in the inertial reference frame. Other shapes (e.g., cones, parabolic dishes) may have more mathematically complex specular conditions, but share the same fundamental property that their orientation with respect to the phase-angle bisector determines when glints occur.

After a glint is observed, the phase-angle bisector orientation at the time of peak brightness of the event can be calculated relatively easily using the known positions of the observer, satellite, and Sun along with eq. (1). This yields the vector  $\mathbf{b}_g = \mathbf{b}(t_g)$ . Knowing the orientation of this phase-angle bisector can potentially constrain the orientation of the glinting component through specular conditions like those given in eqs. (2) and (3). For instance, many GEOs glint strongly as the satellites skirt just outside the Earth's shadow, because at these times the phase-angle bisector very nearly coincides with the vector normal to a sun-tracking solar panel, and eq. (2) is approximately satisfied.

## 4.2 Phase-angle Bisectors Specified in the Satellite Body and Solar Panel Reference Frames

All direction vectors discussed so far have been specified in the inertial reference frame. However, the glinting satellite components very likely do not have a fixed orientation in this frame. In fact, components mounted on the main bus of the GEO satellite generally remained fixed in the satellite's "body reference frame". Similarly, components mounted on the articulating solar panel structure generally remained fixed in the "panel reference frame". These two frames are distinct from one another, and neither is equivalent to the inertial reference frame.

Converting phase-angle bisectors into the satellite's body reference frame (BRF) requires knowledge of the stabilization scheme used to maintain the attitude of the satellite's main bus [13]. Modern 3-axis stabilized satellites have attitude and control systems that most commonly employ "align/constrain" vector pairs to determine a satellite's on-orbit attitude profile. For instance, GEO communications satellites commonly *align* one body axis along the nadir direction (to ensure constant ground contact) and *constrain* another as much as possible to lie along Earth's spin axis (i.e., the z-axis of the J2000 inertial reference frame). This common GEO align/constrain scheme is often referred to as a "nadir/inertial" stabilization. Less commonly, GEO satellites also employ other align/constrain schemes, such as "nadir/solar" or "solar/inertial".

### 4.2.1 The Body Reference Frame for a Nadir/Inertial Stabilization

Vectors can be converted from the inertial reference frame into the body reference frame by using the body's time-dependent "attitude matrix",  $\mathbf{A}(t)$ , which is a 3×3 rotation matrix [12, 13]. For instance, the phase-angle bisector vector in the body reference frame can be written as the product of this matrix and the inertial frame bisector

$$\mathbf{b}'(t) = [\mathbf{A}(t)] [\mathbf{b}(t)] \quad (4)$$

Here the prime superscript denotes vectors specified in the body frame, and no superscript denotes inertial frame vectors. Reference [13] provides a detailed description of how align/constrain vector pairs can be used to derive  $\mathbf{A}(t)$ . This discussion assumes for convenience that the stabilization scheme aligns the body's x-axis with the inertial nadir direction, and constrains the body's z-axis of the body with the inertial z-axis. In this case, the phase-angle bisector in the body frame can be expressed using two time-dependent angles as follows

$$\mathbf{b}'(t) = \begin{bmatrix} \cos(\phi'(t)) \sin(\theta'(t)) \\ \sin(\phi'(t)) \sin(\theta'(t)) \\ \cos(\theta'(t)) \end{bmatrix} \quad (5)$$

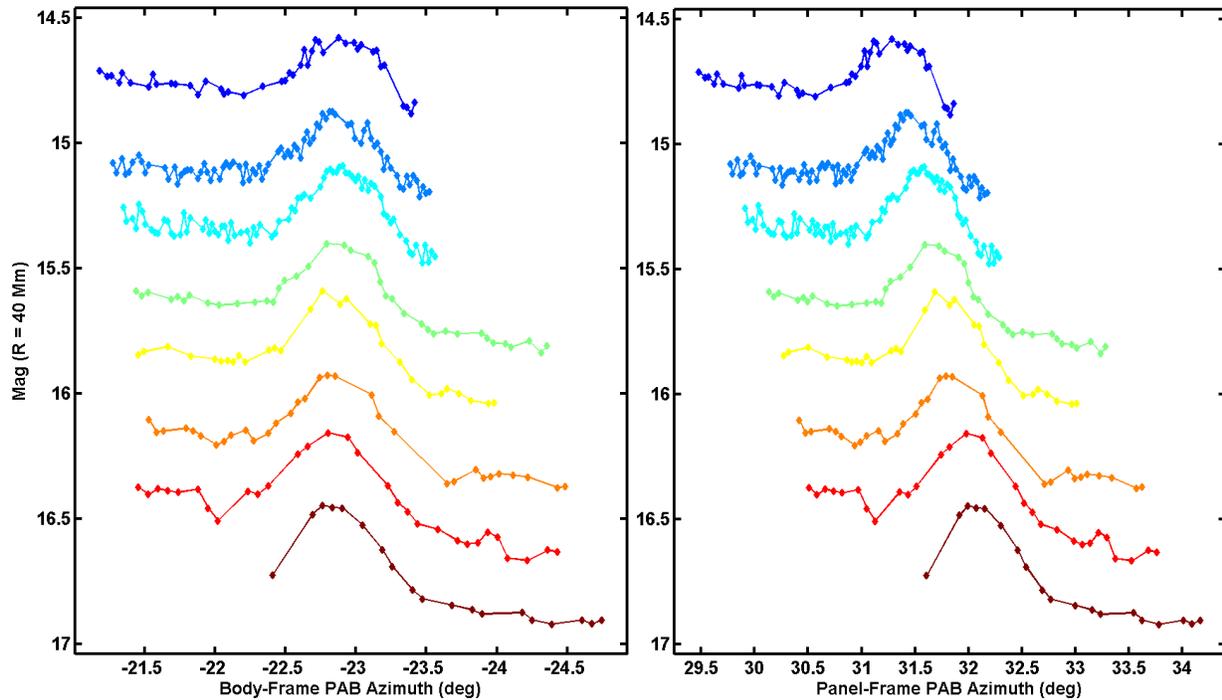
where  $\phi'(t)$  denotes the azimuthal angle and  $\theta'(t)$  the axial angle of the phase-angle bisector in the body frame.

#### 4.2.2 The Solar Body Reference Frame for a Nadir/Inertial Stabilization

Typically, nadir/inertial stabilized GEOs have solar panel structures that articulate about the body axis that lies normal to the orbital plane (in the case discussed here, the body's z-axis). This articulation defines the solar panel reference frame, which moves with respect to the body frame. Vectors can be converted into this frame as follows

$$\mathbf{b}''(t) = [\mathbf{P}(t)] [\mathbf{b}'(t)] = [\mathbf{P}(t)] [\mathbf{A}(t)] [\mathbf{b}(t)] \quad (6)$$

where the double-prime superscript denotes panel-frame vectors. Once again, reference [13] provides a detailed description of how to derive the rotation matrix  $\mathbf{P}(t)$ . The vector  $\mathbf{b}''(t)$  can be expressed in the same form as eq. (5) using  $\phi''(t)$  and  $\theta''(t)$ , the azimuthal and the axial angles of the phase-angle bisector in the panel frame.



**Figure 6.** The eight-glint Galaxy 15 sequence detected during 2010 Nov 14 to Nov 27 (as color coded in Figure 3), plotted as a function of body-frame phase-angle bisector azimuth (left) and solar-panel-frame phase-angle bisector azimuth (right), with 0.25 magnitude vertical separations added between the successive nights. The fact that the peaks align in terms of the body-frame PAB angle on the left (and not in terms of the panel-frame PAB angle on the right) strongly suggests that this glint originates from a body-mounted component on the satellite (and not a panel-mounted component).

#### 4.3 Plotting Glint Sequences in Terms of Phase-Angle Bisector Angles

As mentioned previously, components mounted on the main bus of the GEO satellite generally remained fixed in the satellite's body frame, and those mounted on the solar panel structure fixed in the panel frame. Thus, if a body-mounted component were responsible for the a faint glint sequence such as that shown in Figure 4, then one would expect that the body-frame phase-angle bisector orientations at the peak-times of the glints would be the same or very similar. Similarly, glinting panel-mounted components would correspond to the same or similar panel-frame PAB orientations. It turns out that for low-inclination and low-eccentricity GEOs, night-to-night variations in the PAB orientations are often largely dominated by changes in the azimuthal angles,  $\phi'(t)$  and  $\phi''(t)$ , for the body and panel reference frames respectively. Thus, instead of plotting the glints in terms of time as has been done so far, replotting the brightnesses in terms of these azimuthal angles can potentially provide a graphical test to evaluate whether the glinting component is mounted on the body or panel structure.

Figure 6 shows the glint sequence re-plotted in terms of  $\phi'(t)$ , the body-frame PAB azimuth, on the left panel, and  $\phi''(t)$ , the panel-frame PAB azimuth on the right panel. The plot on the right, in terms of panel-frame angle, actually seems to make the peaks less aligned than they originally were in terms of time. However, the glints align extremely well when plotted in terms of the body-frame angle (left), which strongly suggests that the structural

component responsible for this glint sequence is mounted on the main bus of the Galaxy 15 satellite. Moreover, this specific structural component glints most brightly for PAB body-frame azimuths of about -22.8 degrees, which constrains its orientation and/or its position on the satellite bus.

As mentioned previously, for GEO satellites, night-to-night variations in the PAB orientations are often dominated by changes in the azimuthal angles,  $\phi'(t)$  and  $\phi''(t)$ , for the body and panel reference frames respectively. Night-to-night shifts in these angles naturally explain why the glint sequence shifts systematically in time. However, the axial angles,  $\theta'(t)$  and  $\theta''(t)$ , also change from night to night, but generally much more slowly than the azimuths. One cannot dismiss the possibility that changes in the axial angles can also lead to shifts in the glint peak-times and/or peak-amplitudes. So, in general, both sets of angles must be considered when analyzing glint sequences. However, the left panel of Figure 6 suggests that night-to-night shifts for this particular sequence are predominantly controlled by changes in the body-frame PAB azimuth,  $\phi'(t)$ .

## 5 CONCLUSIONS AND FUTURE WORK

Observations of the Galaxy 12 and 15 satellites conducted in late 2010 indicate that these two GEO satellites occasionally produce faint glints that repeat from night to night. In one particular sequence from the Galaxy 15 satellite, a glint occurred at very nearly the same time of day for all eight nights when the satellite was observed at that time during a 14-day observation period in late 2010. These eight glints all had very nearly the same duration ( $\approx 10$  minutes) and the same brightness amplitude ( $\approx 0.2$  magnitudes), but their peak-times progressively shifted by  $\approx 5$  minutes during the 14-day period. This temporal shifting results from night-to-night changes in observation and illumination conditions, but cannot be explained by either the unsigned or signed/in-plane phase angles commonly used when analyzing satellite photometry. However, the night-to-night shifts can be eliminated by replotting the data in terms of the body-frame phase-angle bisector azimuth, strongly suggesting that, in this case, the glinting structural component is mounted on the satellite's main bus, rather than its solar panel.

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