Observations and Modeling of GEO Satellites at Large Phase Angles

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ABSTRACT

Satellites in geosynchronous earth orbit (GEO) are observed at large solar phase angles with a small-aperture telescope. A model is developed that describes the light reflected from the main satellite components; the model explains the apparent brightening of some satellites when they are observed at phase angles above approximately 100 degrees.

1. INTRODUCTION

In an earlier effort [1], satellites were observed at night-time from Earth over a range of phase angles, including the large phase angles available near twilight. (The solar phase angle here is the sun-satellite-earth angle whose vertex is at the satellite.) The motivation was to model the signatures to predict the brightness of the satellites in the daytime, when the phase angle is also large. The light curves—the apparent visual magnitude as a function of phase angle—and the empirical model fit to the data were reported in Ref. 1. Fig. 1 shows the data, the measured magnitudes, and a piecewise polynomial fit (black line) on which the empirical model was based.

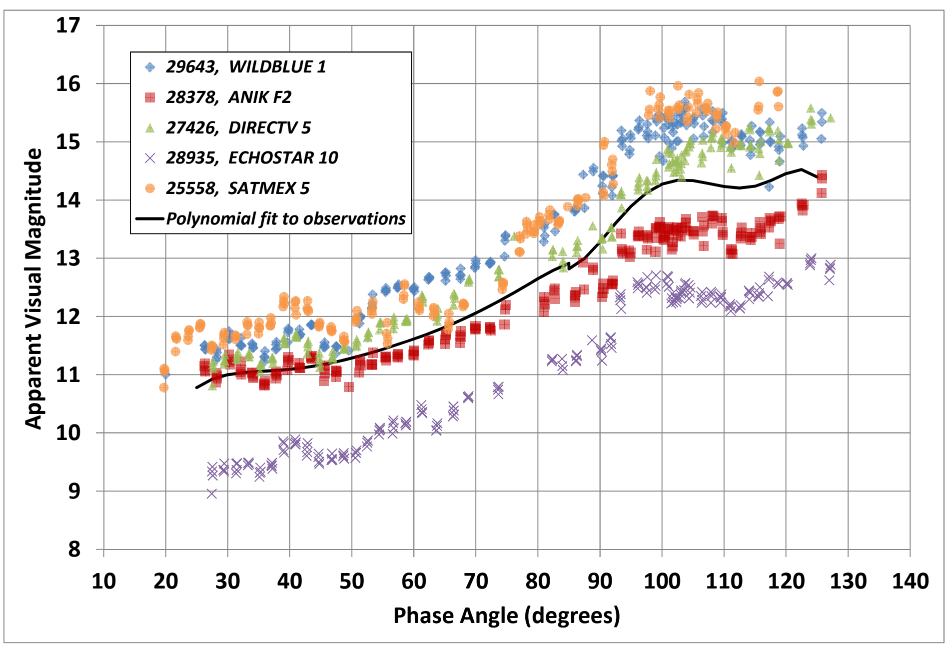

Fig. 1. Visual magnitudes of GEO satellites viewed from Maui and polynomial fit (black line) to the data [1].

The earlier work revealed a trend that is apparent in Fig. 1, namely, a flattening in the phase-angle dependence of the satellite's brightness, and even an apparent brightening of some satellites, at phase angles greater than approximately 100 degrees. A simple photometric model is presented that includes the contribution of earthshine to the illumination of the satellite at large phase angles; its features are generally consistent with the observations.

2. BACKGROUND

The photometric signature of a satellite is commonly modeled with the approximation that the satellite is a Lambertian (diffusely-reflecting) sphere. The fraction F_{diff} of incident solar flux reflected by the spherical satellite as a function of phase angle ϕ was derived by Vallerie [2] and is given as Eq. 1,

$$F_{diff}(a_0, r_{sat}, \phi) = \frac{2}{3} \cdot a_0 \cdot \frac{r_{sat}^2}{\pi R^2} \cdot \left(\sin(\phi) + (\pi - \phi)\cos(\phi)\right), \tag{1}$$

where a_0 is the satellite's albedo, r_{sat} is the radius of the spherical satellite, R is the range between the satellite and the observer, and ϕ is the phase angle. F_{diff} is often referred to as the phase function. The satellite's radius is approximated, perhaps poorly, with the assumption that the satellite's radar cross-section (RCS) is also its optical cross section, hence

$$r_{sat} = \sqrt{\frac{RCS}{\pi}}, \tag{2}$$

where RCS is the satellite's radar cross section in square meters.

The apparent visual magnitude of the Lambertian sphere is then given by

$$m_V(\phi) = -26.74 - 2.5 \cdot \log\left(F_{diff}(\phi)\right) \tag{3}$$

where -26.74 is the apparent visual magnitude of the sun.

Because the Lambertian sphere model proved to be inadequate to describe the visual magnitudes measured in Fig. 1, an empirical model was developed [1] in which the constant albedo in Eq. 1 is replaced with a phase-dependent, piecewise polynomial, $a_{GEOsat}(\phi)$. The coefficients of $a_{GEOsat}(\phi)$ are listed in Table 1, where the units of phase angle in $a_{GEOsat}(\phi)$ are radians.

Table 1. Explicit forms for empirically-derived GEO satellite albedo.

Phase Angle, ϕ	***$a_{GEOsat}(\phi)$, with ϕ in radians***
$25° \le \phi < 100°$ $0.436 \le \phi < 1.745$ (rads)	$3.1765\,\phi^6 - 22.0968\,\phi^5 + 62.182\,\phi^4 - 90.0993\,\phi^3 + 70.3031\,\phi^2 - 27.9227\,\phi + 4.7373$
$100° \le \phi \le 150°$ $1.745 \le \phi \le 2.618$ (rads)	$0.510905\,\phi^3 - 2.72607\,\phi^2 + 4.96646\,\phi - 3.02085$

Fig. 3 shows the visual magnitudes from the empirical model and from the diffusely reflecting sphere. The satellite whose signature is modeled in these curves has a radius of 2.5 m and is located at a range of 36,000 km, the approximate altitude of a satellite in geostationary earth orbit (GEO). This range places the satellite directly overhead of an observer on the equator. An albedo of 0.2 has been chosen for the Lambertian sphere; this is consistent with Seitzer's assumed value of 0.175 for GEO debris [3], and with the value of 0.2 determined by Talent, et al. [4] from observations of GEO objects.

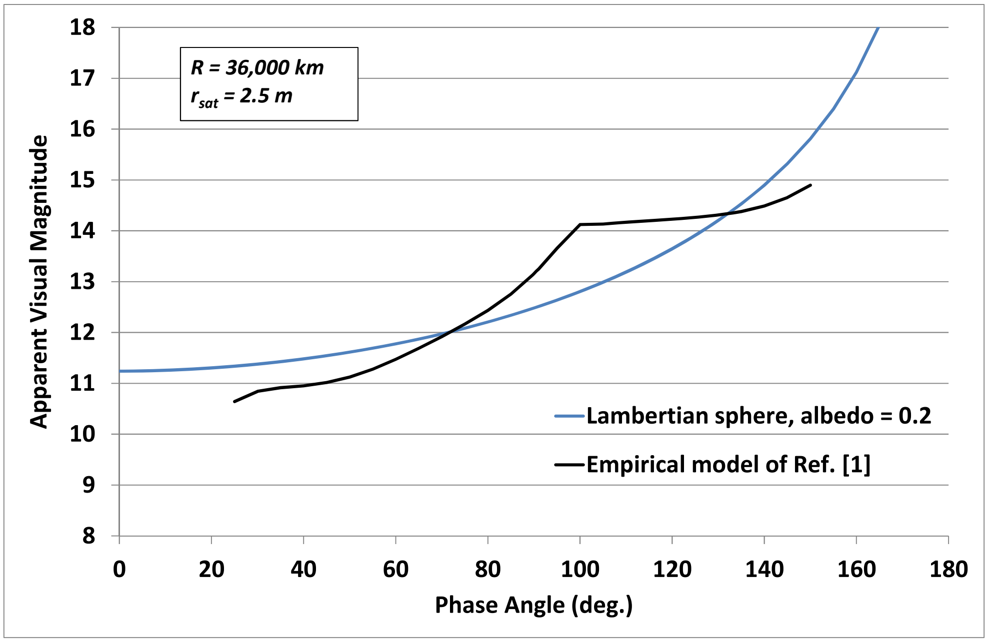

Fig. 2. Lambertian sphere model compared to the empirical model derived from data in Fig. 1.

The key feature of the empirical model is that the satellite flux does not fall off as steeply at large phase angles as the Lambertian sphere model predicts. On the other hand, while the empirical model describes the data, its usefulness is limited to the range and conditions over which the data were acquired. Most notably, there were no observations to support it beyond a phase angle of approximately 125°, hence extrapolation was necessary.

The model presented here also describes the data, but in addition, it provides insight to the physical origins of the photometric signals. With this insight, the relative importance of each component to the satellite's signature can be predicted, and useful range of the model can be extended.

3. MODEL CHARACTERISTICS

The satellites we have observed are far from spherical. Instead they are large, active GEO satellites, each typified by a long, planar solar panel structure (sometimes called "solar wings") with a box-like satellite bus near its center. Because the solar panels track the sun, the observer on the earth has an edge-on view as the phase angle approaches 90°, so the flux reflected from the solar panels is expected to drop to zero. Then as the phase angle passes 90°, the observer begins to see the back of the large solar panels, which are illuminated by earthshine and contribute to the satellite's signature.

The earthshine contribution to the satellite's illumination is modeled with the Earth serving as a diffusely-reflecting sphere illuminated by the sun. With the assumption that the observer is at the equator and the satellite is overhead, the problem simplifies to two dimensions. This earthshine model was proposed and described by Davies [5]. There are no spectral considerations; the earthshine is considered to have neutral albedo. As a Lambertian sphere, the flux that the earth presents to the satellite is given by Eq. 1, with r_{sat} replaced by the radius of the earth R_E, with the phase angle replaced by its supplement, and with the albedo replaced by the earth albedo a_E. The earth albedo is a parameter that can be varied; a reasonable starting value is 0.3.

The satellite model is consistent with the model in Ref. 5 in terms of its components. The main components of the satellite are its solar panels, assumed to be planar and sun-tracking, and its bus, assumed to be spherical. Both the solar panels and the bus are assumed to be Lambertian reflectors. In Ref. 5, the Area-Reflectivity (AR) product of each surface served as a parameter that could be varied to closely represent the measured magnitudes. Here we follow the same approach, but replace the general AR products with more specific terms in an attempt toward greater fidelity.

With these spacecraft components, the photometric signature of the satellite has the following contributions:

- Solar flux reflected from the satellite bus (Lambertian sphere);
- Solar flux reflected from the front (sun-facing) side of the solar panel (flat plate);
- Earthshine reflected from the satellite bus; and
- Earthshine reflected from the back of the solar panels.

Earthshine from the front of the solar panels is neglected.

4. MODEL PARAMETERS AND PHASE FUNCTIONS

The parameters of the model and some typical values are listed in Table 2. The albedo of the Earth varies widely from 0.02 for sun directly over water to 0.8 for snow-covered regions and more oblique angles of incidence. The value of 0.3 is commonly used as a global yearly average. The albedo of the solar panel back is assumed to be fairly high, while the front is assumed to be an more efficient absorber and hence fairly low in albedo. The albedo of the bus is the value cited in Sec. 2 from debris and spacecraft studies [3, 4].

Table 2. Model parameters

Parameter	Description	Value
a_E	Albedo of Earth	0.3 (average)
a_{back}	Albedo of back of solar panel	0.50
a_{front}	Albedo of front of solar panel	0.05
a_{bus}	Albedo of satellite bus	0.20
A	Area of solar panels	100 m^2
R_E	Radius of Earth	6371 km
r_{bus}	Radius of spherical satellite bus	1 m
R	Distance from observer to satellite	36,000 km

The fraction of sunlight reflected from the earth as earthshine to the satellite location is

$$F_E(a_E, R_E, \phi_E) = \frac{2}{3} \cdot a_E \cdot \frac{R_E^2}{\pi R^2} \cdot \left(\sin(\phi_E) + (\pi - \phi_E)\cos(\phi_E)\right), \tag{4}$$

where ϕ_E, the Earth-illumination angle, is given by

$$\phi_E = \pi - \phi. \tag{5}$$

The earthshine reflected from the back of the solar panels to the observer on Earth is nonzero only for phase angles greater than 90° and is proportional to the cosine of the angle θ that the observer's line of sight makes with the solar panel normal. This flux is given by Eq. 6, namely,

$$F_{back}(a_{back}, A, \theta) = a_{back} \cdot \frac{A}{\pi R^2} \cdot \cos(\theta) \cdot F_E. \tag{6}$$

Because the solar panel normal always points to the sun, $\theta = \pi - \phi$.

The calculation of the earthshine reflected back to Earth by the satellite bus is more complicated, but with the primary contribution at an angle $\theta = 0$, the flux is approximated as:

$$F_{bus,E}(a_{bus}, r_{sat}, 0) = \frac{2}{3} \cdot a_{bus} \cdot \frac{r_{sat}^{\;2}}{\pi R^2} \cdot F_E \,. \tag{7}$$

The solar contribution from the bus is in the usual form of flux from a diffuse sphere:

$$F_{bus,solar}(a_{bus}, r_{sat}, \phi) = \frac{2}{3} \cdot a_{bus} \cdot \frac{r_{sat}^2}{\pi R^2} \cdot \left(\sin(\phi) + (\pi - \phi)\cos(\phi)\right). \tag{8}$$

The solar flux reflected by the front of the solar arrays is nonzero only when ϕ is less than 90 degrees, and is given by

$$F_{front}(a_{front}, A, \phi) = a_{front} \cdot \frac{A}{\pi \cdot R^2} \cdot \cos(\phi). \tag{9}$$

The total flux from the satellite then is simply the sum, $F_{tot} = F_{front} + F_{back} + F_{bus,\,solar} + F_{bus,\,E}$, and the satellite's apparent magnitude is:

$$m_V(F_{tot}) = -26.74 - 2.5 \cdot \log(F_{tot}). \tag{10}$$

5. MODEL RESULTS

The apparent visual magnitude of a satellite with the parameters in Table 2 is plotted in Fig. 3. Comparisons to Fig. 1 and to the empirical model in Fig. 2 show good agreement in the general sense.

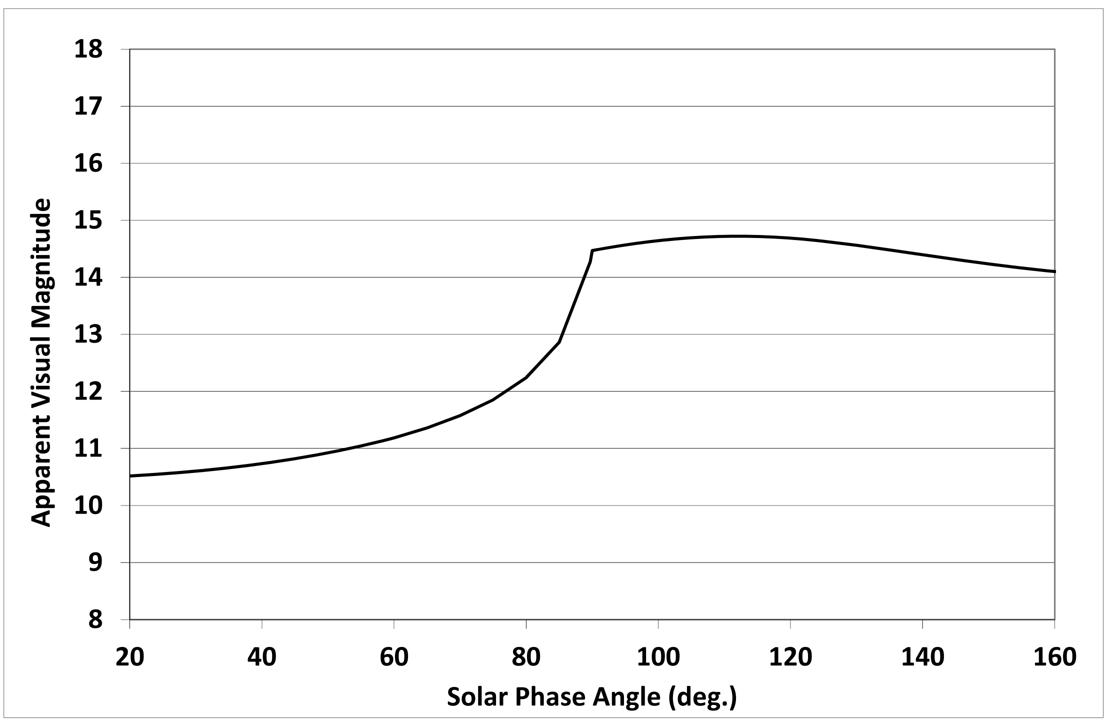

Fig. 3. Results of simple photometric model with earthshine

It is instructive to look at the individual contributions to the satellite's signature. Fig. 4 shows phase functions of each of the components plotted with the parameters of Table 2.

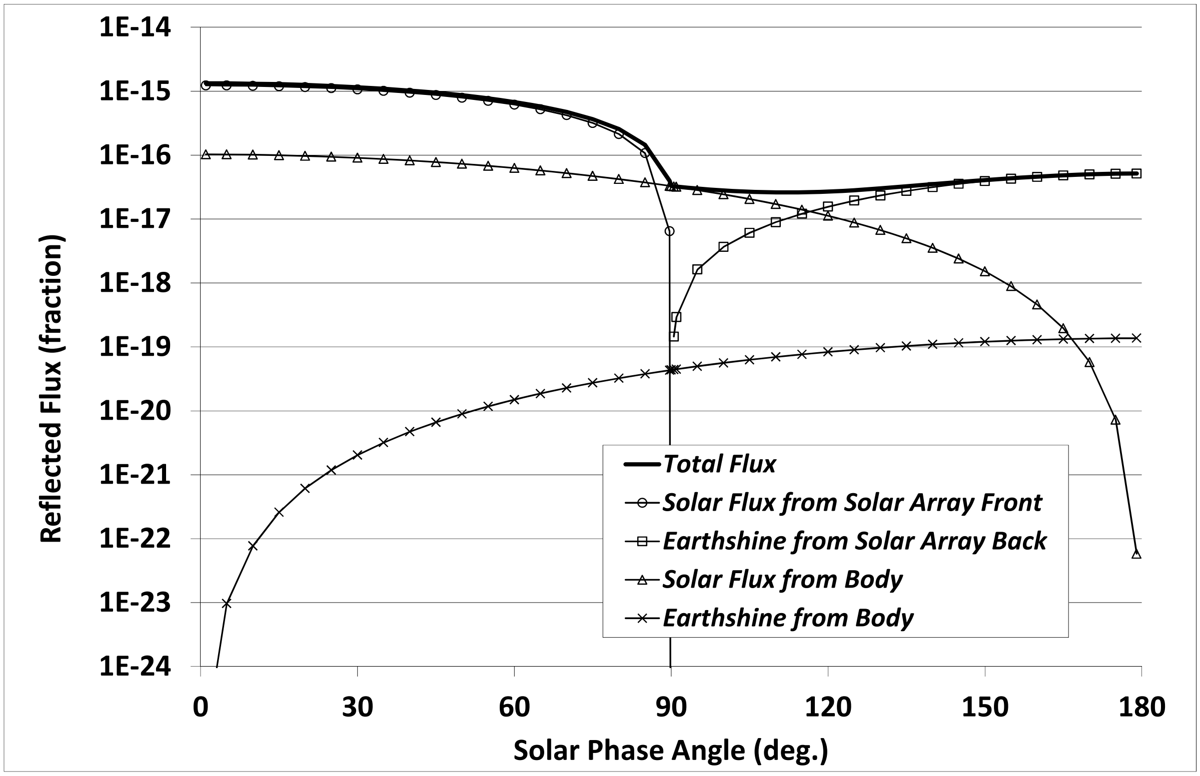

Fig. 4. Individual component contributions to the satellite's reflected flux

Fig. 4 shows that the relative importance of each contribution changes with phase angle. With the values in Table 2, the following behavior is evident. The sunlight reflected directly from the front of the solar arrays dominates the signal at phase angles below about 85 degrees. Between 90 degrees and 120 degrees, the solar flux from the satellite bus dominates. At phase angles greater than 120 degrees, the earthshine from the back of the solar arrays is the dominant contribution. Earthshine from the satellite bus is insignificant.

6. COMPARISON TO RECENT MEASUREMENTS

With small-aperture telescopes, observations of many GEO satellites were made in spring of 2013 for the program known as GEO Observations with Latitudinal Diversity Simultaneously (GOLDS). One of the satellites is DIRECTV 12, #36131, which was observed from the Remote Maui Experiment (RME) site in Kihei on Maui.

Fig. 5 shows the R magnitudes—which are generally brighter than visual magnitudes—of DIRECTV 12 as a function of phase angle. These data were acquired between 31 January and 14 February of this year. The photometric model for the satellite in Fig. 5 has the same parameters as in Table 2 and Fig. 4. Agreement is quite good for phase angles above 90°, even with the simplifying assumptions of the model, and without adjustments to the parameters. For phase angles below 90°, the model predicts a brighter satellite than the measurements reveal, which indicates that the front surface of the solar arrays may not be modeled correctly relative to the other components for this satellite.

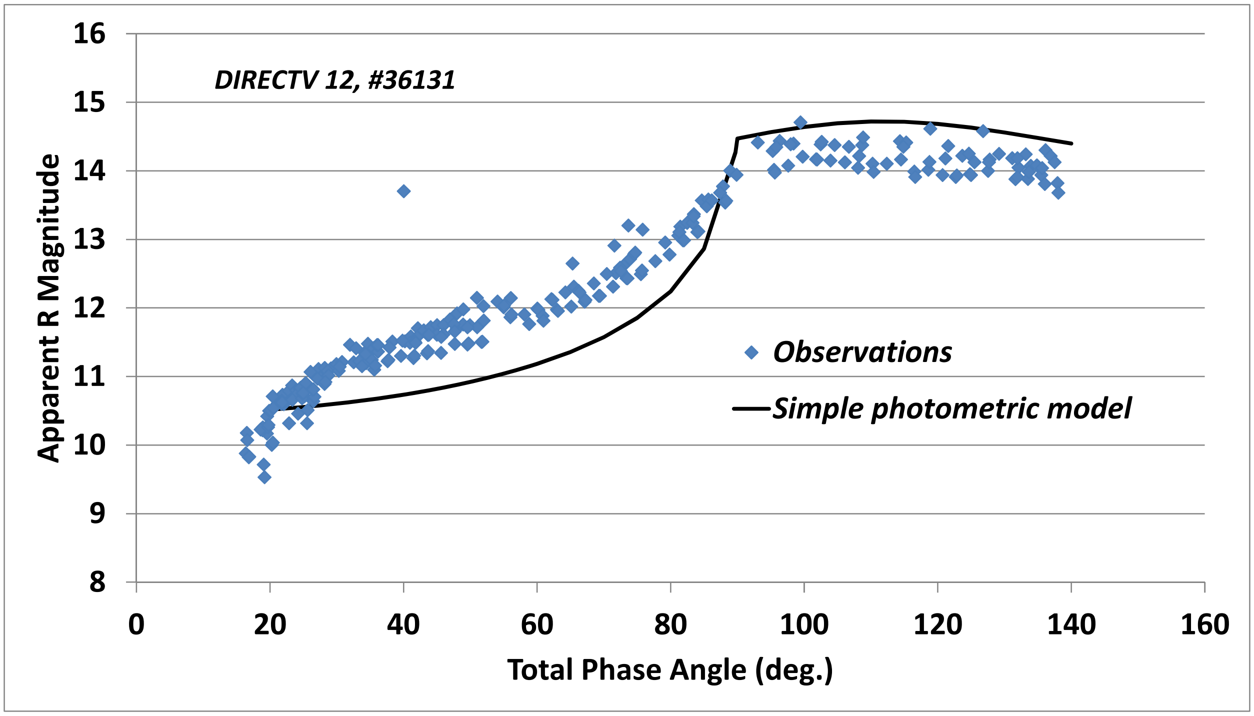

Fig. 5. Measured R magnitudes for DIRECTV 12 compared to modeled visual magnitudes.

7. CONCLUSIONS

In order to explain the photometric signatures of GEO satellites observed at large phase angle, a simple photometric model has been presented. It incorporates earthshine as a reflection of sunlight from a spherical, Lambertian Earth to the satellite, and it models the satellite with two basic components: a large planar solar array that tracks the sun, and a spherical satellite bus, both diffusely reflecting.

Comparisons to the previous empirical model and to recently acquired data are favorable. The simple photometric model allows each contribution to the satellite's reflected flux to be examined independently and provides insight to the physical origins of the satellite's signature.

8. ACKNOWLEDGMENTS

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9. REFERENCES

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