Parametric Excitation of Very Low Frequency (VLF) Electromagnetic Whistler Waves and Interaction with Energetic Electrons in Radiation Belt

V. Sotnikov, T. Kim , J. Caplinger, D. Main
Air Force Research Laboratory, Wright-Patterson AFB, OH 45433

E. Mishin
Air Force Research Laboratory, Kirtland AFB, NM 87117

N. Gershenzon
T2Sys Inc., Beavercreek, OH 43432

T. Genoni, I. Paraschiv, D. Rose
Voss Scientific Inc, Albuquerque, NM 87107

Abstract

The concept of a parametric antenna in ionospheric plasma is analyzed. Such antennas are capable of exciting electromagnetic radiation fields, specifically the creation of whistler waves generated at the very low frequency (VLF) range, which are also capable of propagating large distances away from the source region. The mechanism of whistler wave generation is considered a parametric interaction of quasi-electrostatic low oblique resonance (LOR) oscillations excited by 1conventional loop antenna. The transformation of LOR waves on quasi-neutral density perturbations in the near field of an antenna gives rise to whistler waves on combination frequencies. It is shown in this work that the amplitude of these waves can considerably exceed the amplitude of whistler waves directly excited by a loop. Additionally, particle-in-cell (PIC) simulations, which demonstrate the excitation and spatial structure of VLF waves excited by a loop antenna, is presented. Possible applications including the wave-particle interactions to mitigate performance anomalies of Low Earth Orbit (LEO) satellites, active space experiments, communication via VLF waves, and modification experiments in the ionosphere will be discussed.

PA approved #: 88ABW-2015-5105

1. Introduction

The generation of VLF waves by antennas in plasma is an important topic because of the wide use of antennas in space and laboratory applications, both military and civilian. However, it is well known that the portion of the radiation field that goes directly into the excited electromagnetic spectrum of VLF waves – the whistler mode, is small (less than 3%) in comparison with the wave energy going into the quasi-electrostatic component – low oblique resonance (LOR) mode. For this reason, the efficiency of VLF antennas for generation of electromagnetic waves, which can propagate large distance from the source region, is very limited.

In the present paper we will analyze excitation of waves with frequencies ω several times above the lower hybrid resonance frequency, but below the electron cyclotron frequency i.e. :

$$\omega_{LH} < \omega < \omega_{ce} \tag{1}$$

where ω_{LH} is given by:

$$\omega_{LH}^2 = \frac{\omega_{pi}^2}{1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}} \tag{2}$$

ω_{ce} is an electron cyclotron frequency, ω_{pe} and ω_{pi} are electron and ion plasma frequencies correspondently.

Linear excitation of VLF waves by a loop antenna was investigated by many authors [1-3]. Under conditions (1) only one mode is excited in a cold plasma and main features of the radiation far away from the source can be understood from the plot analogous of wave refractive index surface. This plot can be obtained using the expression for the dispersion of VLF waves:

PA approved #: 88ABW-2015-5105

$$\omega^2 = \frac{\omega_{LH}^2}{(1+\frac{\omega_{pe}^2}{k^2c^2})^2}\frac{m_i}{m_e}\frac{k_z^2}{k^2} \tag{3}$$

In (3) k_z is the wave vector component along the magnetic field and $k_\perp$ is perpendicular component. By schematically plotting the wave vector component k_z against $k_\perp$ for a given ω one can obtain:

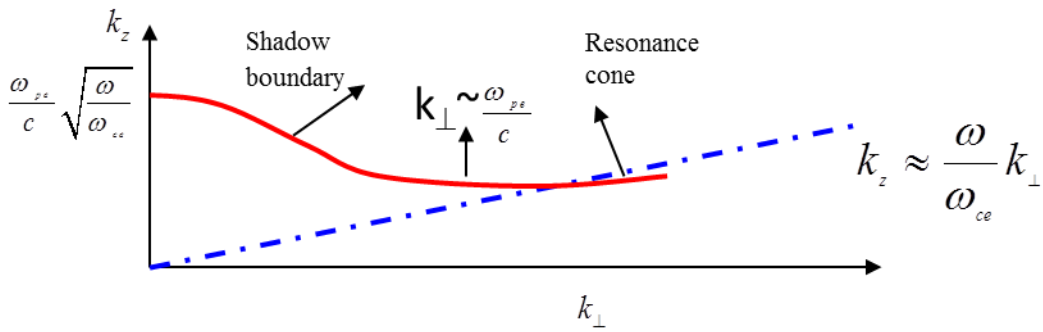


Figure 1: Wave number surface for a constant $\omega_{LH} < \omega << \omega_{ce}$ with three critical points.

A great deal of the source power is radiated as a quasi-electrostatic Lower oblique resonance (LOR) waves [3] with $\frac{\omega_{pe}^2}{k^2c^2} << 1$. The real electromagnetic mode, the whistler wave, with $\frac{\omega_{pe}^2}{k^2c^2} >> 1$ is radiated in oblique directions up to an angle ~ 19.5^0, which is the shadow boundary determined by the long wavelength inflexion point and these waves are radiated comparatively weakly. For many ionosopheric applications it is important to increase the level of the radiated power which is going into the electromagnetic part of the excited wave spectrum – whistler waves.

Below we will analyze the efficiency of a parametric mechanism of transformation of quasi-electrostatic LOR waves excited by a loop antenna operating at frequency ω on density perturbations produced by a dipole antenna (low frequency source) with frequency Ω which excites ion-acoustic waves with frequency above the ion cyclotron frequency but well below the lower hybrid frequency. In this case whistlers will be excited on combination frequencies $\omega \pm \Omega$. The dipole is placed in the center of the loop and lies in its plane. Such an arrangement may be regarded as a parametric antenna for enhanced excitation of whistlers. The paper is organized as follows. In Section 2 we will present analytical analysis and estimates for the radiated power of a

PA approved #: 88ABW-2015-5105

parametric antenna and will compare it with the radiated power of a conventional loop antenna.

In Section 3 we will present particle-in-cell (PIC) simulation results on excitation of VLF waves by a conventional loop antenna and as well as parametric antenna. Section 4 will be devoted to the analysis of the interaction mechanism of parametrically excited whistler waves with energetic particles in the radiation belt.

2. Excitation of VLF waves by a loop antenna

In this Section we will discuss the distribution of an electric field excited by a loop antenna and will calculate total radiated power and power radiated into the electromagnetic part of the VLF wave spectrum – whistler waves. We will briefly discuss the cases when a loop plane is perpendicular and parallel to a loop plane. We will also calculate total radiated power and portion of radiation that goes directly into excited electromagnetic part of the VLF wave spectrum – whistler waves, for the case when a loop plane is parallel to an external magnetic field. This is important especially for active experiments in the ionosphere since whistler waves can propagate a great distance from the source region.

Logarithmic type singularities along the resonance cones are displayed when the plane of a loop is perpendicular to an external magnetic [1-2]. This is schematically represented in Figure 2.

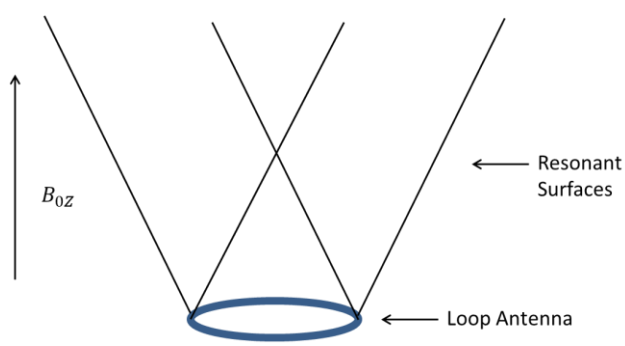


Figure 2: Resonant surfaces of a loop antenna. On these surfaces, the wave potential experiences logarithmic type singularity.

PA approved #: 88ABW-2015-5105

In another case, when magnetic field lies in a loop plane (see Figure 3) there are two resonance surfaces, one inside the other. The method of obtaining these surfaces was suggested in [3].

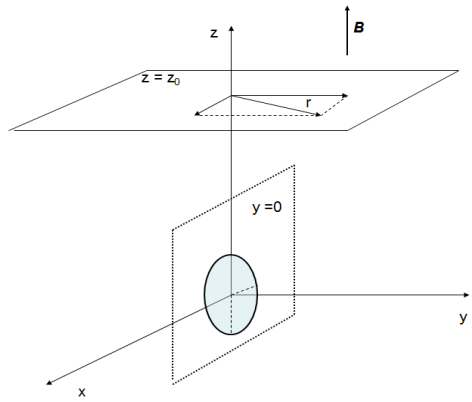


Figure 3: The coordinate system used. The loop antenna is in the y=0 plane and an external magnetic field is along the z axis.

Internal and external resonance surfaces are plotted below, in figure 4. On these surfaces excited field experiences logarithmic type singularities.

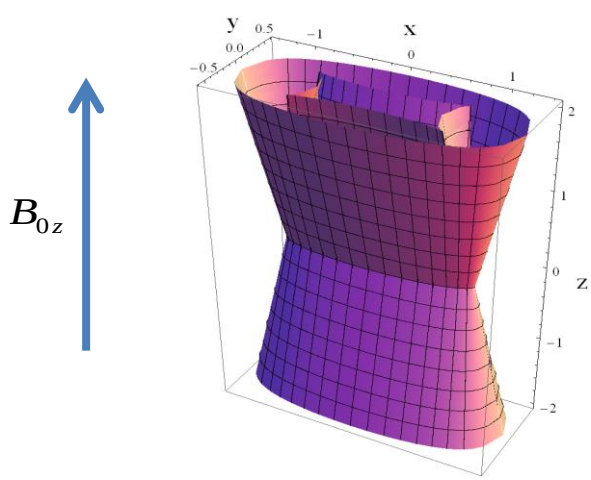


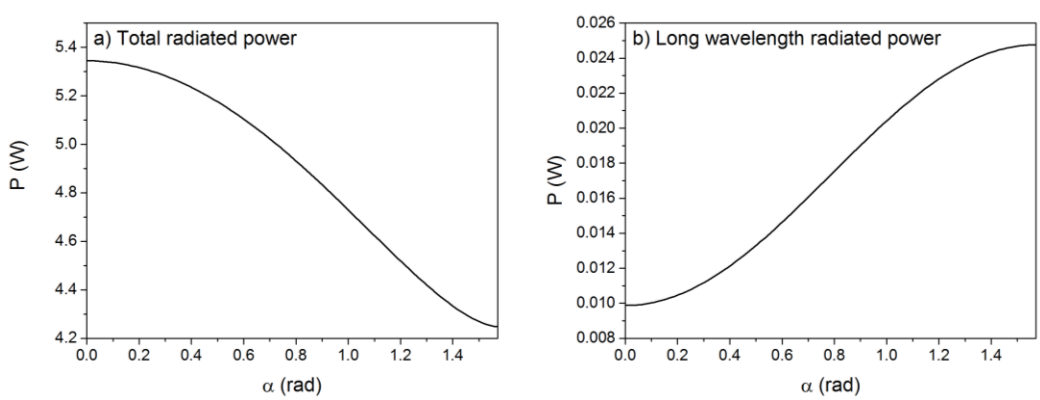
Figure 4: Example of two resonance surfaces (internal and external) of a loop antenna with an external magnetic field in a loop plane.

PA approved #: 88ABW-2015-5105

In order to calculate total radiated power and power which goes to the electromagnetic part of the VLF wave spectrum for whistler waves, we consider a time harmonic loop current source, $I_0 \exp[j\omega t]$, and begin with the expression for the radiated power [3]:

$$P = -\frac{1}{2}\,\mathrm{Re}\int \mathbf{J}_m(\mathbf{r}) \bullet \mathbf{H}^*(\mathbf{r})\, d^3r = -4\pi^3\,\mathrm{Re}\int \mathbf{m}(\mathbf{k}) \bullet \mathbf{H}^*(\mathbf{k}) d^3k \tag{4}$$

Where $\mathbf{m}(\mathbf{k})$ and $\mathbf{H}(\mathbf{k})$ are Fourier components of the external magnetization and the magnetic induction created by this current. The asterisk denotes the complex conjugate. A different expression for radiated power using current in a loop antenna and the electric field created by this current was used in [4]. Taking proper account of the resonance denominator in Eq. (63) of Ref. [3], we carried out calculations for the following set of plasma parameters which correspond to ionospheric conditions: for an electron density $n_e = 3\times10^4\ \mathrm{cm}^{-3}$ and for magnetic induction $B_0 = 0.3\ \mathrm{G}$. The value of the current in a loop antenna was $I_0 = 100\ \mathrm{A}$ and frequency $\omega = 6\times10^4\ \mathrm{s}^{-1}$. Our results are shown in Fig. 5. Figure 5(a) shows the total radiated power as a function of the angle between the antenna normal and the magnetic field. Figure 5(b) shows the long wavelength radiated power found by integrating in $k_\perp$ out to $\delta = k_\perp c/\omega_{pe} = 0.1$. We checked these results by also performing the calculations using the spherical coordinate formulation of Ref. [6]. In all cases we obtained agreement of the order of 0.1% or better, giving us confidence in the numerical methods used in our work.



PA approved #: 88ABW-2015-5105

Figure 5: (a) Total radiated power versus angle between the antenna loop normal and the magnetic field. (b) Radiated power at long wavelength (δ≤0.1) versus angle between the antenna normal and the magnetic field.

3. Parametric excitation of electromagnetic whistler waves

The equation for analysis of nonlinear interaction of VLF waves with ion-acoustic (IA) waves in a magnetized plasma was derived in [7]. Electric and magnetic fields in a magnetosonic type VLF wave were represented through a scalar potential φ and a vector potential $\mathbf{A}$ with the Coulomb gauge, $\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$ and $\mathbf{B} = \nabla \times \mathbf{A}$ with $\nabla \cdot \mathbf{B} = 0$. For analysis of parametric excitation of electromagnetic whistler waves due to transformation of LOR waves with frequency ω excited by a loop antenna, on IA oscillations excited by another low frequency source with frequency Ω

($\omega_{ci} << \Omega << \omega$) we can modify the equation derived in [7]. Whistlers are excited on combination frequencies $\omega_{\pm} = \omega \pm \Omega$, and we are interested in the excitation of whistlers on frequencies from the range $\omega_{\pm} \sim (2 \div 10)\omega_{LH}$. After modifying equation (7) we can write for the parametrically excited whistler wave's Fourier component potential $\varphi_{\mathbf{k}_-}$ with $\omega_- = \omega - \Omega$ and $\mathbf{k}_- = \mathbf{k} - \mathbf{k}_s$:

$$\frac{\partial^2}{\partial t^2}\varphi_{NL}^t + \frac{c^4}{\omega_{pe}^2}\Delta\frac{\partial^2}{\partial k_z^2}\varphi_{NL}^t = -\frac{1}{n_0}\frac{1}{\omega_{ce}}\frac{\omega_{pe}^2}{c^2}\frac{\partial}{\partial t}[\nabla\delta n_s^*, \nabla\varphi^l] \tag{5}$$

In equation (5) φ_{NL}^t is the potential associated with a parametrically excited whistler mode, φ^l is the potential of LOR mode excited by a loop antenna and δn_s is ion-acoustic type density perturbation excited by a dipole antenna. Equation (5) can be used to analyze the generation of whistlers due to parametric interaction of LOR waves with low frequency ion-acoustic type density perturbations excited by a dipole antenna. Using previously obtained expressions for φ^l and δn_s for the case when an external magnetic field is perpendicular to the loop plane, we arrive to the following expression for components of the electric feild in the plane perpendicular to an external magnetic field:

PA approved #: 88ABW-2015-5105

$$E^t_{x,NL} = -\frac{1}{12\sqrt{2}}\frac{1}{en_0}\frac{\omega\_\omega\omega_{pe}^2\Omega^2}{\omega_{ce}c^4 \mathrm{v}_s^3} R_a^2 d_0 J_a J_d \frac{\sin\Phi}{r}(i\sin\Phi - \frac{z}{r}\cos\Phi) \tag{6}$$

$$E^t_{y,NL} = -\frac{1}{12\sqrt{2}}\frac{1}{en_0}\frac{\omega\_\omega\omega_{pe}^2\Omega^2}{\omega_{ce}c^4 \mathrm{v}_s^3} R_a^2 d_0 J_a J_d \frac{\sin\Phi}{r}(-i\cos\Phi - \frac{z}{r}\sin\Phi) \tag{7}$$

Knowledge of parametrically excited electric fields in a whistler mode allows us to proceed with the calculation of power radiated by parametrically excited nonlinear current $\mathbf{J}_{NL}(\mathbf{r},\mathbf{t})$:

$$\mathbf{J}_{NL}(\mathbf{r},\mathbf{t}) = -e\delta n_s \mathbf{v}_{LOR} \tag{8}$$

In (8) δn_s is the ion-acoustic type density perturbation excited by a low frequency dipole and $\mathbf{v}_{LOR}$ is the speed of electrons in the presence of LOR wave excited by a loop antenna. To find δn_s we can use equation for excitation of ion-acoustic waves by a dipole antenna. In the simplest case of a point dipole antenna with the current density in the dipole given by $\delta\mathbf{j}_c = J_d d_0 \delta(\mathbf{r})\mathbf{e}_x$ we have:

$$\delta \mathrm{n}_s(\mathrm{x},\mathrm{y},\mathrm{z},\mathrm{t}) = \frac{J_a d_0 \Omega}{4\pi e \mathrm{V}_s^2}\frac{x}{R^3}(\mathrm{i} - \frac{\Omega R}{V_s})\exp\{i[\Omega(t - R/\mathrm{V}_s)]\} \tag{9}$$

where $R = [\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2]^{1/2}$.

To find velocity $\mathbf{v}_{LOR}$ which appears due to the presence of an electric LOR wave field excited by a loop antenna we can use the electron equation of motion in a drift approximation. Resulting expressions for $\mathbf{v}_{LOR}$ are:

$$\mathbf{v}_{LOR} = c\frac{\mathbf{E}_{LOR}\times\mathbf{B}_0}{B_0^2} \tag{10}$$

where $\mathbf{E}_{LOR}$ is the field of quasielectrostatic LOR waves excited by a loop antenna.

Now the expression for the radiated power of parametrically excited whistler waves can be written as:

PA approved #: 88ABW-2015-5105

$$P_{NL} = -\frac{1}{2}\mathrm{Re}\int \mathbf{j}_{NL}^{*} \cdot \mathbf{E}_{NL} d^3 r \tag{11}$$

Using the same set of plasma parameters fro (11) as before one can obtain the value of power radiated into an electromagnetic part of VLF wave spectrum – whistler waves. Taking the current in a low frequency dipole antenna $J_d = 4\ A$, the value of radiated power for the case when the loop plane is perpendicular to an external magnetic field is $P_{NL} = 0.11$ Watts. The value of radiated power (at the angle $\alpha = 0$) of a single loop antenna can be found to be $P_L = 0.01$ Watts. Increasing the current in a dipole antenna to $J_d = 10$ A for parametrically excited radiated power we have $P_{NL} = 0.65$ Watts. These examples shows that parametric mechanism of excitation is very effective and produces much higher radiated power output in electromagnetic part of the VLF wave spectrum.

4. Parametric excitation of whistler waves by a parametric antenna: LSP simulation results

A well-developed particle-in-cell plasma simulation code called Large Scale Plasma (LSP) [8] was used to perform 3D simulations of VLF field excitation by a conventional loop antenna and by a parametric antenna. We will present 3D simulation results on generation of electromagnetic and quasi-electrostatic VLF waves by a loop antenna and by a parametric antenna. The particle-in-cell code, LSP, provides a variety of boundary conditions (periodic, outlet, PML, conducting and others) and also utilizes both explicit and implicit algorithms for evolving the particles in time and solving for the self-consistent fields. Since the evolution of the plasma and fields occurs on time scales much greater than the so-called CFL (Courant-Friedrichs-Lewy) condition and also on spatial scales much larger than the electron Debye length, we will take advantage of the implicit algorithms by using spatial scales much larger than the Debye length and time steps several times the CFL-constrained time step. This, along with the use of high performance computing, allowed for a large volume (1680x1680x1000 Meters) simulation over long simulation times (30 µs), which would have been overly computationally expensive otherwise.

In the case of 3D simulations the ring antenna is placed in the center of the simulation domain, with its plane perpendicular to the z-axis. The applied background magnetic field of 0.3 Gauss is directed along the z-axis as well. For the boundaries, perfectly-matched layers (PML) were used in order to minimize reflections. The initial plasma density was $10^5\ cm^{-3}$. The loop antenna in both the single loop case and the parametric antenna case was excited with 100 A current oscillating with the frequency $\omega = 1.31\times10^6$ rad/s , For chosen plasma and magnetic field parameters this frequency is ten times larger

PA approved #: 88ABW-2015-5105

than lower hybrid frequency, i.e. $\omega = 10\omega_{LH}$. Electron cyclotron frequency is equal to $\omega_{ce} = 5.27\times10^{6}$ rad/s The low frequency dipole antenna for excitation of ion-acoustic waves in the case of parametric excitation was driven by a current $I_d = 2$ A oscillating at the frequency $\Omega = 8.54\times10^{4}$ rad/s. This frequency is well above the ion cyclotron frequency $\omega_{ci} = 2.64\times10^{3}$ rad/s in the system. The table below gives several simulation parameters for the large 3D simulation performed for analysis of efficiency of whistler wave excitation.

Simulation Time	$2.15e4\ ns$
Dipole Antenna Periods Simulated	3.49
Particles per cell	8
Total Particles	$5.64e9$
Processors	512
X, Y Size	$1680\ m$
Z Size	$1000\ m$
Computation Time	$\sim 3\ Weeks$

Table 1: 3D PIC simulation Parameters

Two simulations were performed, one with a simple loop and one with a parametric antenna consisting of a loop and a dipole antenna. These simulations were otherwise identical. The resulting electric and magnetic field magnitudes were qualitatively and quantitatively very similar when considering all wavelengths. However, the differences are more apparent when k-space is filtered to separate electromagnetic dominant wavelengths from electrostatic dominant wavelengths. The isometric plots below show the electromagnetic part of the excited VLF wave spectrum.

PA approved #: 88ABW-2015-5105

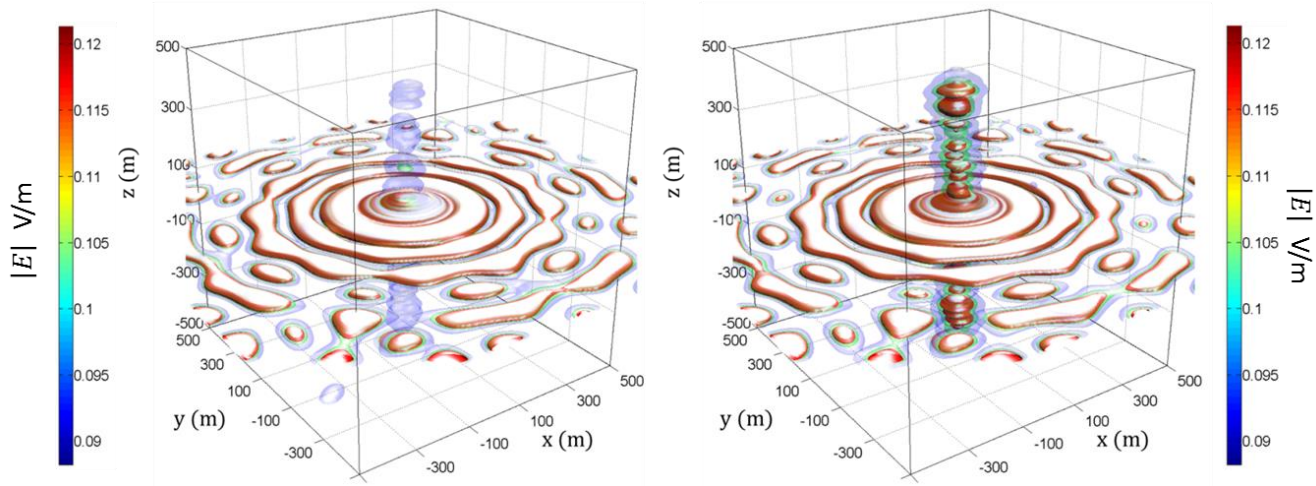


Figure 6: Whistler wave electric field magnitude for a loop antenna (left) and a parametric antenna (right). Three isometric surfaces are shown with $|E| = 0.12\,, 0.11\,, 0.09\; V/m$.

Using obtained simulation data we can calculate electromagnetic part of radiated power in the case of a single loop antenna as well as parametric antenna. To do so like in the previous sections, we can use (4) and (11). Simulation results for a single loop antenna with a loop plane perpendicular to an external magnetic field are as follows: for a single loop antenna for a long wavelength $(k_{\perp} < 0.25\frac{\omega_{pe}}{c})$ electromagnetic part of the radiated power we found $P_L = 0.175$ Watts and for the power of parametrically excited whistler waves we have $P_{NL} = 2.313$ Watts. This implies that for a given choice of initial plasma parameters and antenna currents the parametric antenna radiates approximately 13 times more power into the electromagnetic part of the VLF wave spectrum.

5. Precipitation of radiation belt electrons by whistler waves

Whistler waves interact with radiation belt (RB) electrons via cyclotron resonance (e.g., [8, 9]):

$$\omega - k_{||} v_{||} = n\gamma^{-1}\omega_{ce} \tag{12}$$

PA approved #: 88ABW-2015-5105

where $v_{||}$ is the electron velocity parallel to the magnetic field line, n is the harmonic resonance number, and γ is the relativistic Lorentz factor. Figure 7 illustrates VLF-RB electron interaction.

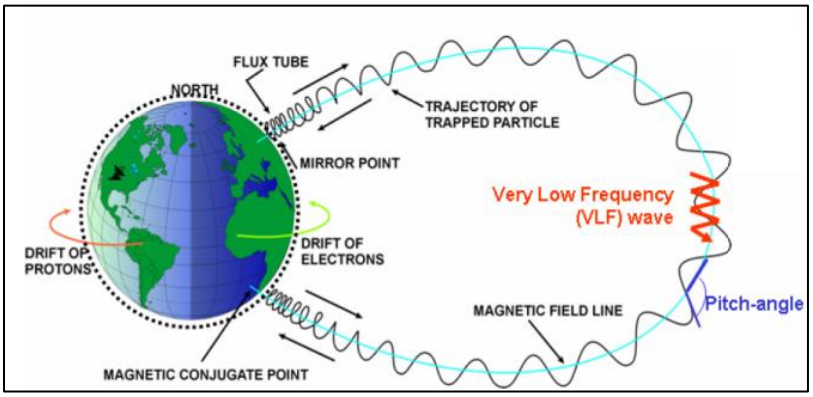


Figure 7. Schematic of VLF wave particle interaction

The energy of electrons resonant with a wave of a given frequency is strongly dependent on magnetic field strength and plasma density in the interaction region. For example, ~300 keV electrons are in primary resonance (n = -1) with 22 kHz whistler-mode waves for B_0 ~ 1600 nT and plasma density n_c~ 80 cm^{-3}, typical of the plasma sphere at R~2.5 R_E. These lower energy electrons give up perpendicular energy to the waves reducing their magnetic pitch angle and shifting them toward the loss cone. MeV electrons are in higher order resonance (n = -3) with 22 kHz waves at the equator, and also at off-equatorial latitude at n=-1.

For a broadband spectrum, $\Delta\omega \gg \omega_t = \sqrt{k\omega_{ce}\delta B_w/B_0}$, where ω_t is the trapping angular frequency, the interaction of resonant electrons with whistler waves of the amplitude δB_w is described by quasilinear equation (e.g., [8]). Figure 8 shows bounce-averaged quasi-linear diffusion coefficients D_{ij} in units of inverse days, for electrons at R=4.9R_E, with energy and equatorial pitch angle values as indicated [10]. The cold plasma density n_c, assumed constant along dipole magnetic field lines, was such that the ratio of the electron plasma and cyclotron frequencies

s 7.5. As conventional, the distribution of wave-normal angles θ_k was taken to be a Gaussian in tan(θ_k), with peak at 60° and cutoffs at 45° and 70°, and the width specified by 45°. Similarly, a truncated Gaussian was used for the distribution of wave frequency, with peak at 0.2f_{ce} and cutoffs

PA approved #: 88ABW-2015-5105

at $0.1f_{ce}$ and $0.3f_{ce}$ with bandwidth $0.1f_{ce}$. The wave population was assumed to be present within 35° of the magnetic equator. The magnetic amplitude δB_w = 40 pT was used.

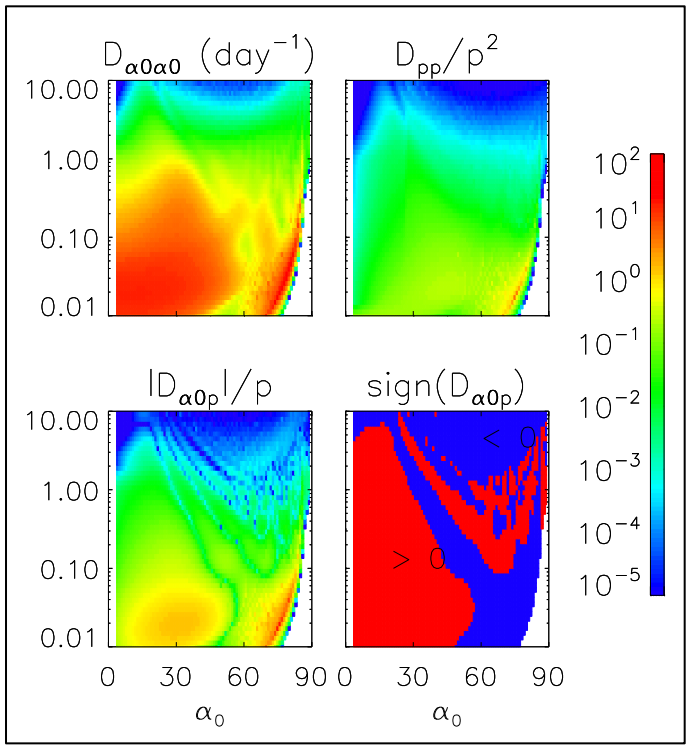


Figure 8. Bounce-averaged quasi-linear diffusion coefficients, in units of inverse days, for electrons at L=4.9, with energy in MeV and equatorial pitch angle values as indicated.

As the diffusion coefficients simply scale with δB_w^2 the results of Figure 8 can be easily renormalized. The results are for a single field line; for a drift average, they should be reduced by the appropriate fraction of MLT on which the waves are present. At any rate, the values of D_{ij} are significant, as seen by comparison to similar calculations of diffusion by plasmaspheric hiss in plumes [9, Fig. 5]. Actually, VLF-caused precipitation depletes the ~1-MeV electron population in ~3 days (the timespan of major storms. Less energetic (≤0.1-MeV) electrons are depleted in ~2–3 hours. The amplitudes and spatial distribution of the whistler waves propagating from the generation region at R_0 to the plasmasphere/radiation belt can be easily estimated from the conservation of their Poynting flux. For ducted propagation, waves are confined to L -shells such

PA approved #: 88ABW-2015-5105

that the wave frequency is less than or equal to half the minimum electron gyrofrequency along the field line [11]. Non-ducted propagating waves can reach all the way to the plasmapause in the "antenna" hemisphere, piling up along the caustic and showing up on relatively low L-shells in the conjugate hemisphere. In the former case, the amplitude reduces as $\sqrt{R_0/R}$. Taking $R = aR_E$ (the Earth's radius) and $\delta B_w(R)$ = 40 pT gives $\delta B_w(R_0) \approx 40\sqrt{a}$ pT. In the latter case, the amplitude reduces as $\sqrt{S_0}/R$, where S_0 is the surface area of the generation region. Taking $\sqrt{S_0} = 6.3l_0$ km gives $\delta B_w(R_0) \approx 40a/l_0$ nT.

References

[1] R. K. Fisher and R.W. Gould, Phys. Fluids 14, 857, 1972.

[2] V. I. Karpman, Fizika Plasmy 12, 836, 1986.

[3] V. I. Sotnikov, G. I. Solovyev, M. Ashour-Abdalla, D. Schriver and V. Fiala, "Structure of the near zone electric field and the power radiated from a VLF antenna in the ionosphere," Radio Sci. **28**, 1087 (1993).

[4] T. N. C. Wang and T. F. Bell, "Radiation resistance of a short dipole immersed in a cold magnetosonic medium," Radio Sci. **4**, 167, 1969.

[5] V. Fiala, E. N. Kruchina and V. I. Sotnikov, "Whistler excitation by transformation of lower oblique waves on density perturbations in the vicinity of a VLF antenna," Plasma Phys. Control Fusion **29** (B10), 1511 (1987).

[6] T. N. C. Wang and T. F. Bell, "VLF/ELF radiation patterns of arbitrarily oriented electric and magnetic dipoles in a cold lossless multicomponent magnetoplasma," J. Geophys. Res. **7**, 1174 (1972).

[7] R.Z. Sagdeev, V.I. Sotnikov, V.D. Shapiro, V.I. Shevchenko, "To the theory of magnetosonic turbulence", JETP Letters, Vol. 20, No 26, p. 582, 1977.

PA approved #: 88ABW-2015-5105

[8] Welch, D. R., V. Rose, M. E. Cueno, R. B. Campbell, and T. A. Mehlorn. "Integrated Simulation of the generation and transport of proton beams from laser-target interaction." *Physics of Plasmas* 14 2006

[8] Albert, J. M., "Evaluation of quasi-linear diffusion coefficients for whistler mode waves in a plasma with arbitrary density ratio", J. Geophys. Res., 110, A03218, doi:10.1029/2004JA010844, 2007.

[9] Li, W., Y. Shprits, and R. Thorne, "Dynamic evolution of energetic outer zone electrons due to wave-particle interactions during storms", J. Geophys. Res., 112, A10220, doi:10.1029/2007JA012368, 2007.

[10] Mishin, E., J. Albert, and O. Santolik, "SAID/SAPS-related VLF waves and the outer radiation belt boundary", Geophys. Res. Lett., 38, L21101, doi:10.1029/2011GL049613, 2011.

[11] P. Kulkarni, U. S. Inan, T. F. Bell, J. Bortnik, Precipitation signatures of ground-based VLF transmitters, J. Geophys., 113, A07214, doi:10.1029/2007JA012569, 2008.

PA approved #: 88ABW-2015-5105