HARMONIC ANALYSIS FOR OPTICALLY MODULATING BODIES USING THE HARMONIC STRUCTURE FUNCTION (HSF)

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Abstract

Lockheed Martin Hawaii presents a novel signal processing algorithm for focal plane array processing. We introduce the Harmonic Structure Function (HSF) and demonstrate its capability in detecting, classifying and counting rotating bodies in a *single pixel*. The work presented here is making a major impact in the Missile Defense Agency's Project Hercules Forward Based Sensor (FBS) group but the results presented here are shown in an unclassified form. First, the temporal phenomenology of optically modulating bodies is discussed. Next, the mathematical definition of the HSF and the natural extension from integral to discrete form is detailed. Simulations of rotating bodies and modulating reflectance used for analysis are then discussed. These simulations result in the construction of time series data for N rotating bodies with fundamental frequencies f_n in noisy backgrounds. The HSF is then used to analyze these simulations. This analysis yields important considerations for sensor developers and operators. Finally, the Generalized Harmonic Structure Function (GHSF) is illustrated. The GHSF is used to analyze more complicated situations where objects have multiple rotational degrees of freedom.

I. Introduction

This paper presents a general method for extracting temporal signals in multi-frame focal plane data for the purpose of counting, typing and discriminating optically modulating bodies. The so-called Harmonic Structure Function (HSF) is introduced and its use in focal plane analysis is sketched. The paper is organized as follows. In Section II, we discuss the phenomenology of optically modulating bodies. In Section III, we introduce and discuss the HSF. Section IV extends HSF. Section V concludes.

II. Phenomenology.

To understand the source of harmonic signatures from optically modulating objects, it is useful to consider a simple model of a slab rotating around two axes (three degrees provides further but straightforward complications). In this case, the passive, reflected signature is proportional to the normal area projected to the sensor, ala Fig. 1.

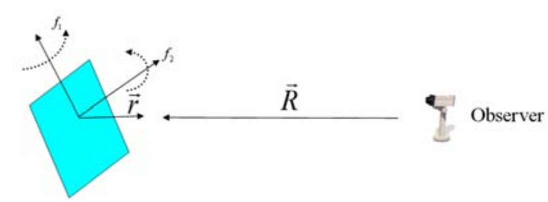

Fig. 1. Source to Sensor.

In an idealized version of the modulation, we get something like a signal, *S*, proportional to a sinusoidal modulation of the effective area, A_{eff} :

$$S \propto A_{effective} \sim A_0(\vec{r} \cdot \vec{R}) \sim A_0 \cdot \sin(f_1 t) \cdot \sin(f_2 t)$$

In general, however, these modulations will be non-sinusoidal, and instead periodic functions so that we have

$$S \propto A_0 \cdot \Psi(f_1 t) \cdot \Psi(f_2 t)$$

Where $\Psi(ft)$ is a periodic function which could look like, for example, a sawtooth or a square wave type function.

Let us now consider the simplest cases for the signal *S* created by these modulations. This occurs when f_1 or f_2 is small so that

$$S \propto A_0 \cdot C \cdot \Psi(f_2 t)$$

In this case, we, obviously, get a harmonic signature. From Fourier, when we expand a periodic series, we get the tower of harmonics of, in this case, f_2 , see Fig. 2.

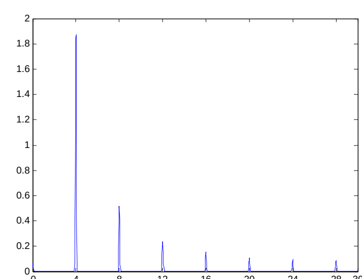

Fig. 2. Spectrum for the case where $\Psi(f_2 t)$ is a sawtooth with frequency of 4 Hz. Sample Rates are video.

The situation complicates considerably – not from the theoretical control, but from the obtained signal – when we have an appreciable f_1. Here, we get the well-known frequency beats. To see this, write the expansion in Fouriers for both periodic signals:

$$S \sim A_0 \cdot \Psi(f_1 t) \cdot \Psi(f_2 t) =$$
$$A_0 \cdot (c_1 \sin(f_1 t) + c_2 \sin(2 f_1 t) + \ldots)(d_1 \sin(f_2 t) + d_2 \sin(2 f_2 t) + \ldots)$$

and then use trigonometry relations to get:

$$S \sim (c_1 \sin(f_1 t) + c_2 \sin(f_1 t) + \ldots)(d_1 \sin(f_2 t) + d_2 \sin(f_2 t) + \ldots) =$$
$$c1 \cdot d1 \cdot \sin(f_1 t)\sin(f_2 t) + \ldots =$$
$$c1 \cdot d1 \cdot \cos((f_1 - f_2)t) + c1 \cdot d1 \cdot \cos((f_1 + f_2)t) + \ldots.$$

The signal, *S*, then becomes a complicated combinatoric conundrum. Moreover, there exists the possibility of aliasing, etc. so that the beats may wrap around the spectra further complicating matters. Furthermore, we may encounter the situation where multiple objects are ‘beating’ on a single pixel: here we get additive sets of complicated beat structures. Fig. 3. below shows an example spectrum for this case. In general, such signatures may provide applications, for example, SSA, with an opportunity to perform unique fingerprinting.

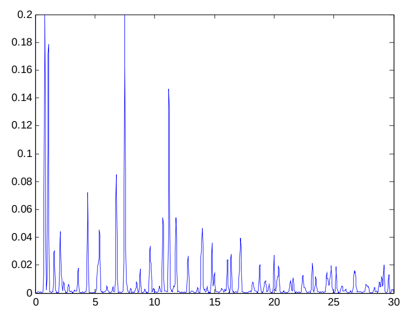

Fig. 3. Complicated Spectra created from two objects both ‘beating’ on a single pixel.

III. The Harmonic Structure Function.

For a given fundamental frequency, f_0, the harmonically related components will have frequencies nf_0 for $n > 1$, where n is the harmonic number and $n = 1$ is the fundamental. An integration of a signal, $s(f)$, is performed over frequency space and weighted such that only frequencies harmonically related to a given fundamental frequency will contribute to the integration. The integration is given by

$$\tilde{H}(f_0) = \int_0^\infty S(f)\left(\sum_{n=n_{\min}}^{n_{\max}} \delta(f - nf_0)\right)df$$
$$= \sum_{n=n_{\min}}^{n_{\max}} S(nf_0),$$

where $n_{\min}$ and $n_{\max}$ define the range of the harmonic components to be integrated over, and $\delta(f)$ is the Dirac delta function. Since the data are in discrete frequency bins, the integration must be performed as a sum over the corresponding bins. If $x[i]$ is the ith bin of the frequency spectrum and Δf is the frequency resolution of spectrum, then the sum over the discrete bins is given by

$$\tilde{H}(f_0) = \sum_{n=n_{\min}}^{n_{\max}} S[\mathrm{int}(nf_0/\Delta f)],$$

where int() truncates the argument to the nearest integer value toward zero. The sum, $\tilde{H}(f_0)$, is computed for frequencies in the specified fundamental frequency range, $f_{\min} \le f \le f_{\max}$. Since each fundamental frequency bin will be associated up to $n_{\max}$ bins at the $n_{\max}$ th harmonic, a fundamental frequency increment of at most $\Delta f / n_{\max}$ must be used to ensure a fine enough sampling of $\tilde{H}(f_0)$. This ensures that all possible harmonic sets corresponding to fundamental frequency and harmonic range $f_{\min} \le f \le f_{\max}$ and $n_{\min} \le n \le n_{\max}$ are considered.

In practice, the fundamental frequency range is quantized by

$$f_0[j] = f_{\min} + \frac{\Delta f}{n_{\max}}(j - \tfrac{1}{2}) \qquad 1 \le j \le j_{\max},$$

where

$$j_{\max} = \mathrm{int}\left(\frac{(f_{\max} - f_{\min})}{\Delta f} n_{\max} \right).$$

Then, for a given fundamental frequency index, j, and harmonic number, n, the function

$$H(n,j) = x\left[\mathrm{int}\left[n\left(\frac{f_{\min}}{\Delta f} + \frac{j - \frac{1}{2}}{n_{\max}} \right) \right] \right]$$

defines the harmonic structure function (HSF).

Harmonic Detection Example

Let A be an $m \times j_{\max}$ matrix, that is the outcome of applying HSF to a single spectral scan of the time-frequency spectrogram shown in Figure 1. Elements of the matrix are defined by

$$a_{ij} = \min(H(n_{\min} + i - 1, j), T) \qquad 1 \le i \le m, \qquad 1 \le j \le j_{\max},$$

where elements of the jth column form the harmonic set associated with fundamental frequency $f = f_{\min} + j\Delta f / n_{\max}$, the number of harmonics being considered is $m = n_{\max} - n_{\min} + 1$, and $\min(\cdot)$ limits the value of a single matrix element to at most T.

A weighted sum of the elements in each of the columns is given by $\mathbf{z} = \mathbf{bA}$, where $\mathbf{b}$ is an m-length row vector defined by

$$\mathbf{b} = [1/m \quad 1/m \quad \cdots \quad 1/m].$$

The resulting power-versus-frequency vector, z, yields a ‘fundamental’ spectrum where the large peak corresponds to the fundamental frequency of the harmonic structure. Note that currently equal weight is given to each harmonic component. If a specific harmonic set predominance is known a priori, the component weighting in the above equation should be adjusted accordingly. Figure 4 shows two examples of the HSF in action. Fig. 4a shows the output for the case with three harmonic signals. The upper left plot shows a single scan frequency spectrum. The upper right plot shows a ‘lofargram’ (stacked spectra with time on the y axis) of the signal. The lower plot is the stacked ‘fundamental’ spectrum obtained from HSF – in this case we see evidence of the three fundamentals. Fig. 4b is more complicated: here we have the case similar to Fig. 4a., but the fundamental frequency is increasing. Fig.4 explicitly shows the capability of HSF in counting multiple objects on a single pixel. HSF analysis could potentially be combined with spatial super-resolution algorithms for object counting applications.

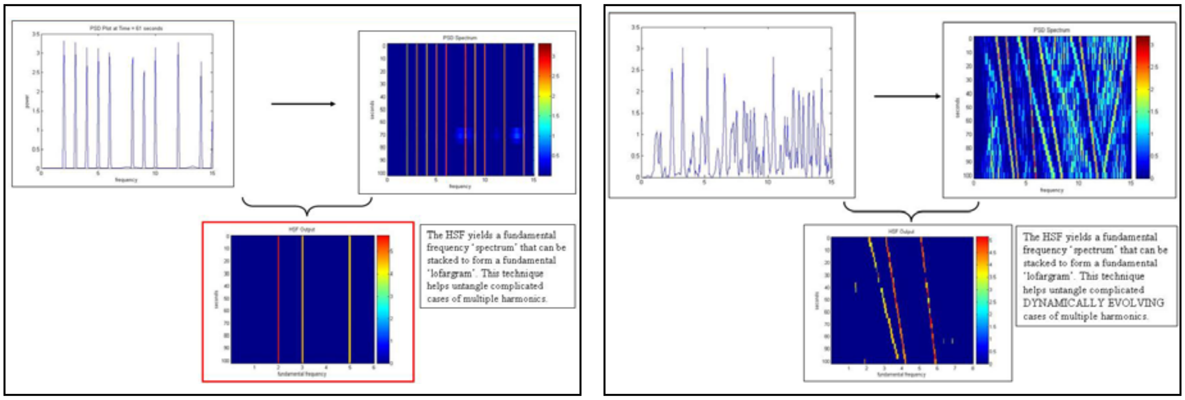

Fig. 4a. and 4b. Examples of the HSF.

IV. The Generalized Harmonic Structure Function.

The HSF presented above does the job when we are dealing with the case where we only have one appreciable frequency of rotation. When we have multiple degrees of freedom in play, the HSF needs to be modified. In this case, we have the Generalized Harmonic or Beated Structure Function (GHSF). Such a modification is straightforward. The idea is to be able to 'template match' (an example template is shown in red in Fig. 5a overlaid with a beated spectrum in blue.) received signatures to generate a test statistic. In the example spectrum of Fig. 5a, Fig. 5b shows the GHSF image output. The bright spot in Fig. 5b corresponds to the case where the spectrum matches the beated spectrum for ***f1*** and ***f2***.

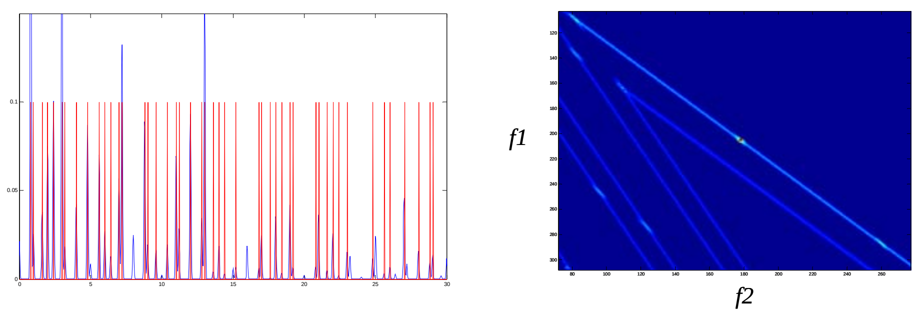

Fig. 5a and 5b. Beated Spectra and the output of the GHSF.

V. Conclusion.

The signal processing methods presented here represent novel and optimal ways for exploiting harmonic and beated signatures in the presence of noise in a variety of situations ranging from EO/IR focal plane applications to acoustics. The methods presented have found success in a variety of DoD applications. It is the author's opinion that such methods may have application to the SSA problem. In conclusion, it is interesting to look at the case where we have a large number of modulating bodies on a single pixel of a focal plane. In this case, it can be shown that the temporal signal received is composed of a large number of beating objects and the resulting spectra is a Gaussian shaped distribution centered at DC.

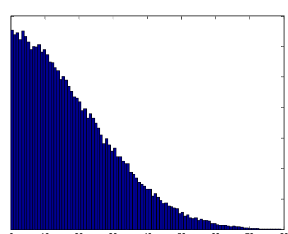

Fig 6. Gaussian shaped distribution of multiple beating objects.