Photometric Color Conversions for Space Surveillance Sensors

Joseph Scott Stuart
MIT Lincoln Laboratory

ABSTRACT

In order to maximize sensitivity, optical space surveillance sensors use detectors that have good sensitivity over a wide region of the spectrum. For example, the CCD detectors for the Lincoln Near-Earth Asteroid Research (LINEAR) Project, which are nearly identical to the detectors of the Ground-based Electro-Optical Deep Space Surveillance System, have good sensitivity over the visible spectrum from 380 nanometers to beyond 1000 nanometers. However, photometric calibration of the intensities of objects (stars, satellites, asteroids, etc.) measured by these systems must be referenced to astronomical star catalogs that were measured over much narrower portions of the available spectrum. For example, the Sloan Digital Sky Survey (SDSS) Photometric Database contains photometric measurements in five bandpasses that are each about 150 nanometers wide. This paper will present a method for converting between photometric systems with different bandpasses. The method uses the measured response functions of the detectors of interest along with a model of the spectral transmissivity of the atmosphere (Stone, 1996), and a catalog of stellar spectra (Pickles, 1998) to derive polynomial functions that allow for the conversion of brightness measurements from astronomical catalogs to the bandpass of the sensor. The method has been extensively tested using data from the Lincoln Near-Earth Asteroid Research project in comparison with catalog measurements from the USNO B1.0 astrometric catalog, and the SDSS Photometric Database. Through OPAL (Optical Processing Architecture at Lincoln), this technique is being applied to ground-based and space-based sensors including the Space-Based Visible (SBV) system, the Space-Based Space Surveillance (SBSS) system, and the Space Surveillance Telescope (SST).

1. INTRODUCTION

Photometric calibration of space surveillance data from a wide-band optical sensor is problematic for many reasons. For ground-based sensors the primary difficulty derives from the variability in the optical properties of the atmosphere [1]. For all sensors, ground-based and space-based, the sensor itself changes with time (for a discussion off CCD noise sources, see [2]).

Ideally, one would compensate for changes in the sensor and the weather by simultaneously observing well-calibrated photometric reference stars along with the targets of interest. However, the currently available star catalogs lack either all-sky coverage (for example Sloan Digital Sky Survey, SDSS [3]) or accurate photometry (for example USNO-B1, [4]). Furthermore, typical optical space surveillance sensors use a wide bandpass covering as much of the optical and near-IR regime as possible to maximize sensitivity to objects illuminated by sunlight (Fig. 1). Even if a high-quality photometric reference catalog is available in the same region of sky as the desired targets, the bandpasses used in the catalog are unlikely to match the bandpass of a sensor. Such is the case for the bandpass of the Lincoln Near-Earth Asteroid Research (LINEAR) project (Fig. 1). In addition, the various catalogs have different bandpasses (Fig. 2) and much confusion can result if care is not taken in comparing photometry from one color system to photometry from another color system.

In this paper, I address the problem of comparing wide-band photometric observations from an optical sensor to multi-color, narrow-band data in standard star catalogs. The basic idea is to transform the multi-color photometric data from the star catalog into an approximation of the sensor bandpass so that the sensor specific photometric system can be defined and used consistently. The methodology is described in Section 2. Testing and verification of the method is presented in section 3.

This work is sponsored by the United States Air Force under Air Force Contract FA8721-05-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the author and are not necessarily endorsed by the United States Government.

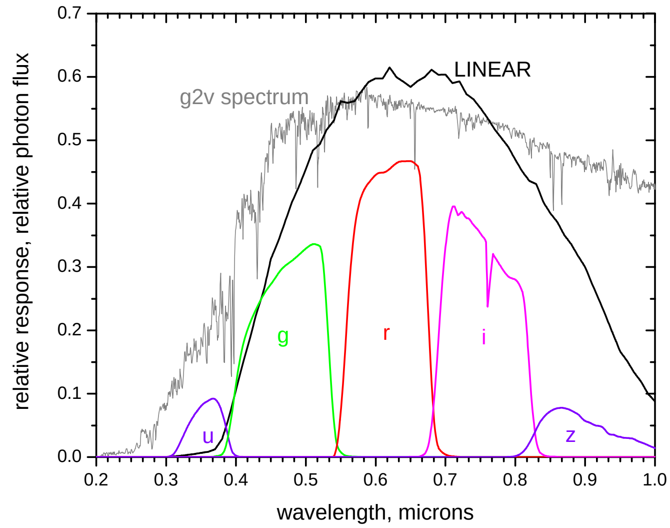

Fig. 1 Sensor Response Functions Compared. The photon flux from a g2V (sun-like) star (from [5]) is shown overlain with the sensor response function from a typical wide-band CCD sensor (LINEAR) and a standard set of catalog filters (SDSS ugriz). The sensor functions include the quantum efficiency of the CCD, the reflectivity and transmissivity of all optical components, and 1.3 atmospheres (atmosphere color response from [6]).

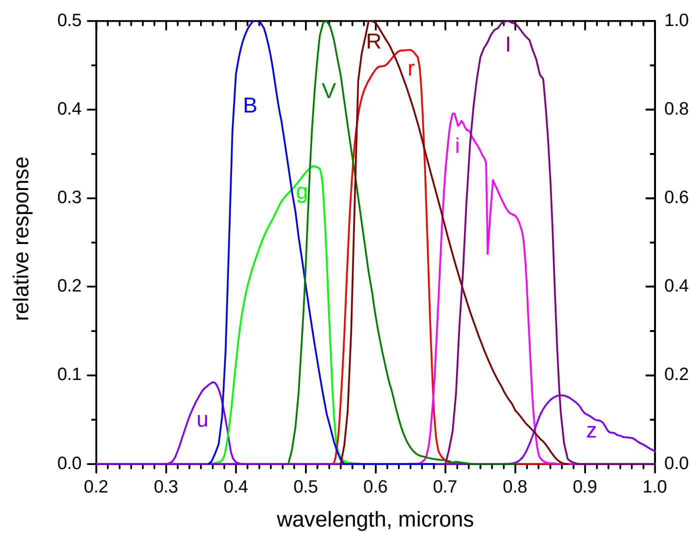

Fig. 2 Standard Response Functions, Johnson-Cousins BVRI [7,8] and SDSS ugriz [10].

2. METHODOLOGY

A photometric observation of a star or other resident space object with a wide-band optical system reduces a complex spectrum such as in Fig. 1 to a single intensity value at each observation time. Furthermore, that intensity value is usually impossible to convert to absolute physical units (such as photons per square meter per micron per second) from first principles due to unknown losses in the optical path (primarily the atmosphere). Therefore, measurements of the photon flux of a celestial object are typically converted to astronomical magnitudes via equation 1:

$$magnitude = -2.5\log_{10}(flux). \quad \textbf{Equation 1}$$

The measured magnitudes for reference stars are then compared to the accepted magnitudes of those same stars as recorded in photometric star catalogs. The unknown losses in the optical path are then characterized by an offset, called the zero-point, which is an additive term in the logarithmic space of magnitudes. Beyond the zero-point, any differences in the spectral response of the sensor and the catalog can be approximated by color terms that typically take the form of a linear or quadratic function of a color index. A color index is simply the difference between the magnitudes of a celestial object in two different filters. For example, B-V is the color index obtained by subtracting the V magnitude from the B magnitude and thus is a measure of how much brighter a source is in the V band than in the B band. Since magnitudes are the logarithm of the flux, a color index should be thought of a ratio of the brightness in the two bandpasses.

The spectral profile of the star Vega (or another star with spectrum very similar to Vega) is taken to be a standard reference for all astronomical photometric systems. The systems are defined such that Vega has a magnitude of 0 in all bandpasses, and thus Vega has color indices of 0 as well. Therefore the magnitude of a star in some bandpass is the logarithm of the ratio of the fluxes of the given star and Vega in the bandpass of interest. Likewise, a color index for a given celestial object is a comparison of the color of that object to the color of Vega. By tradition, the order of subtraction in the color index is typically taken so that stars that are redder than Vega have positive color indices. For example, the sun has B-V≈0.65 [11], so the sun is redder than Vega by a factor of 1.8 in the region sampled by the B and V filters.

Of concern here, then, is the question of how to use the color indices provided by a star catalog to obtain an approximate sensor-specific magnitude for a reference star in the absence of a detailed spectrum for the star. The primary method used to accomplish this is that of empirical color transformations as in [9]. In this case, a large number of measurements are made with the sensor of interest of reference stars for which magnitudes and colors are available in some star catalog. Then, color-color diagrams are plotted, and linear or quadratic functions are fit to the portions of the data that seem well behaved. Those limits for well-behaved data, and the resulting transformation equations are then used to select stars of suitable color from a star catalog and to transform the catalog magnitudes and colors into a sensor-specific magnitude system.

An alternative is presented here for when large amounts of observational data of reference stars from the sensor of interest are not available. This method makes use of "synthetic magnitudes," which are calculated from high-resolution stellar spectra [5], a detailed sensor response function, and a model of the spectral transmissivity of the atmosphere [6]. The stellar spectra are taken from the Pickles catalog [5], which contains average spectra for each of 131 different spectral types encompassing all normal spectral types and luminosity classes at solar metal abundances as well as metal-weak and metal-rich F-K dwarfs and G-K giants. Each library spectrum was formed by combining data from several sources overlapping in wavelength coverage. One subtlety to keep in mind when using this (or any) spectral library is the issue of energy-weighted flux versus photon-weighted flux. The Pickles spectra are normalized to 1 at a wavelength of 0.5556 microns so explicit units are not given. However, the input spectra were in energy-based units rather than photon-counting units. The spectral response functions of CCDs are typically specified in photon-counting units and so the Pickles spectra must be converted to photon units by multiplying each flux value by the corresponding wavelength and renormalizing. However, the spectral response functions of some photometric systems such as Johnson-Cousins UBVRI are typically specified in energy units. The Pickles spectra should not be converted to photon-counting units when they are used with energy-based spectral response functions.

To compute a synthetic magnitude for a given spectrum, equation 2 is used,

$$m_{i,b} = -2.5\log_{10}\left(\sum_{w} F_i(w)R_b(w)\lambda(w)\right) \qquad \textbf{Equation 2}$$

where $m_{i,b}$ is the synthetic magnitude for the i^{th} stellar spectrum and the b^{th} sensor response function, F_i is the flux (energy-weighted) for the i^{th} stellar spectrum, R_b is the response function for the sensor or catalog bandpass labeled b, λ is the wavelength and is included only when the sensor response function is in photon-counting units, and w iterates over the wavelength bins available in the catalog. Vega is an A0V star, so the synthetic magnitude for the A0V spectral class is used as a reference and is subtracted from the synthetic magnitudes of all other spectral types.

This results in the synthetic magnitudes for the A0V spectral class being zero for all bandpasses, which is another way of saying that the magnitude systems are placed on an A0V color standard. The A0V spectrum in the Pickles catalog differs from the actual spectrum of Vega by about 0.003 magnitudes in the Johnson-Cousins V band, so this definition is equivalent to saying that the magnitudes are on a Vega color standard to within about 0.003 magnitudes of uncertainty.

Once synthetic magnitudes have been calculated for all combinations of bandpass and spectral type in the Pickles catalog, and the synthetic magnitudes have been converted to a Vega color system by subtracting off the synthetic magnitude for the A0V spectral class, color indices can be computed. This is simply a subtraction of the synthetic magnitudes in each bandpass from a reference bandpass. For the reference bandpass, it is best to use the catalog bandpass that is closest to the sensor bandpass. For the LINEAR response function shown in Fig. 1, and indeed for most CCD-based wide-band sensors, the SDSS r bandpass or the Johnson-Cousins R bandpass is the best reference bandpass. This process is illustrated in Fig. 3.

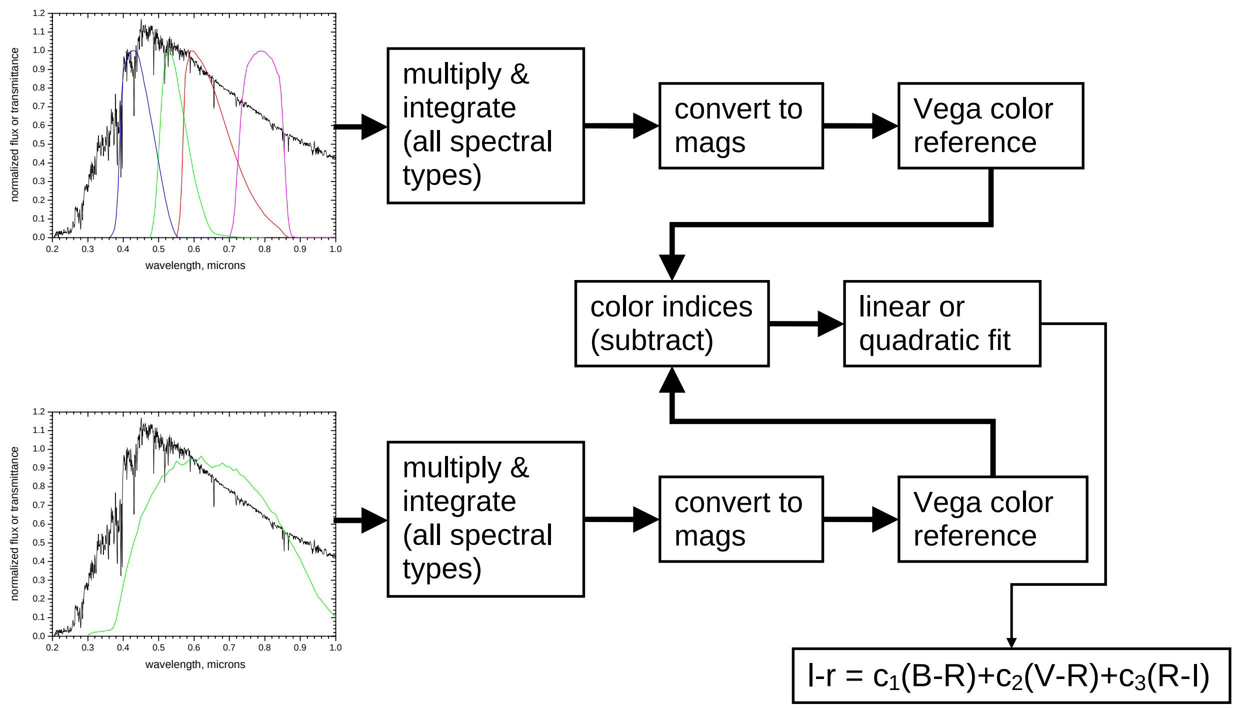

Fig. 3 Method for computing synthetic color conversion.

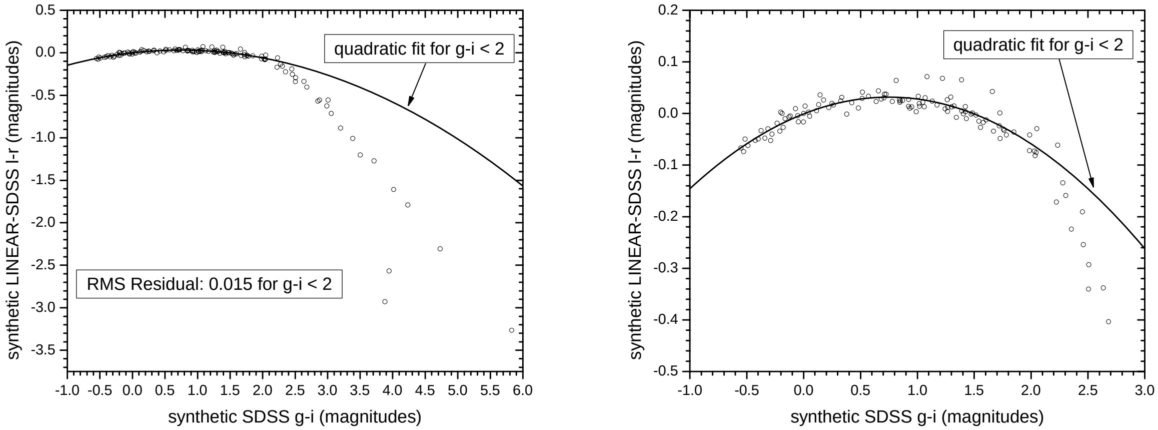

Fig. 4 Synthetic Colors Conversions for LINEAR from SDSS. The right panel shows a subset of the data depicted in the left panel, which shows the full range of data.

Fig. 4 is an example of a color-color plot using the synthetic magnitudes from the Pickles catalog along with the LINEAR and SDSS g and i bandpasses. The g-i color index on the horizontal axis spans the region of the LINEAR response function and therefore is a good color index to use for transforming catalog magnitudes to sensor-specific magnitudes. The g-i colors show increasing red stars to the right. The vertical axis in Fig. 4 gives the color index obtained by subtracting the SDSS r synthetic magnitudes from the LINEAR synthetic magnitudes. The SDSS r band is a good choice for the primary photometric reference for converting catalog magnitudes to sensor magnitudes because the r bandpass, without modification, is already a pretty good match to the LINEAR sensor bandpass, as can be seen from the nearly horizontal shape of most of the data on the left side of Fig. 4. The synthetic magnitudes with g-i < 2 are well behaved, and the l-r color index is well represented by a quadratic fit versus g-i. The RMS residual for that fit is 0.015 magnitudes. This means that the following function (Equation 3) can be used to convert SDSS catalog magnitudes to LINEAR sensor-specific magnitudes:

$$l = r + 0.08692(g-i) - 0.05797(g-i)^2 \qquad g-i<2 \qquad \textbf{Equation 3}$$

where *l* is the LINEAR magnitude of the catalog star, *g*,*r*,*i* are the SDSS catalog magnitudes, and the numbers are from the fit depicted in Fig. 4. This equation should only be applied to stars with *g-i*<2 since the fit is quite poor beyond that.

The form of Equation 3 means that the linear magnitude system is being defined in such a way that the LINEAR magnitude will be equal to the SDSS r magnitude for the A0V spectral type. As noted previously, this is nearly the same as a Vega color standard.

Fig. 4 shows a two dimensional projection of a higher-dimensional space. The SDSS catalog contains magnitudes in five bandpasses, all of which are relevant when converting to a sensor with a wide bandpass (Fig. 1). This allows for four color terms to be computed for SDSS catalog stars, and one color term for comparing SDSS magnitudes to sensor magnitudes. The process of fitting functions to synthetic or empirical magnitudes can be carried out directly in this five dimensional space. Plots are not given of the data in this space due to the difficulty of plotting a five dimensional function on two dimensional paper. However, a multidimensional quadratic function can be fit to the data. The SDSS r bandpass is chosen as the reference bandpass for all color terms, so the five dimensional space has LINEAR magnitude minus SDSS r magnitude for the dependent variable, and SDSS u-r, g-r, r-i, r-z as the four dependent variables. When this five dimensional fit is performed, some of the spectra in the g-i>2 space become well-behaved in the sense that they match the quadratic function, but there are still regions of five dimensional color space that are not well fit by a quadratic function.

In investigating this five dimensional color space, another benefit of the technique of using synthetic magnitudes, rather than empirically measured magnitudes, became apparent. The extremely low noise in the synthetic magnitudes made it much easier to visualize and explore the five dimensional color space to search for regions of outliers. In the end, though, we have chosen to continue limiting the stars using only the g-i<2 criteria for conversions to the LINEAR bandpass. Given that constraint, the technique of synthetic magnitudes yields the following function for converting from SDSS magnitudes to LINEAR magnitudes:

$$\begin{aligned} l = r &+ 0.005563(u-r) + 0.2885(g-r) - 0.2546(r-i) - 0.06946(r-z) \\ &- 0.001629(u-r)^2 - 0.07609(g-r)^2 - 0.02914(r-i)^2 - 0.09003(r-z)^2 \end{aligned} \qquad g-i<2$$

Equation 4

We now turn to the question of verifying and testing the method of color conversions.

3. VERIFICATION AND TESTING

Fig. 5 is similar to Fig. 4 above, except that the plots show measured magnitudes (from the LINEAR sensor or the SDSS catalog) rather than synthetic magnitudes. These data were taken from a single field (centered on Ra=10h 0m 20s, DEC = +20d 4m 3s) on a single night (UTC 2006 Feb 28) of LINEAR observations. The LINEAR magnitudes shown in Fig. 5 were obtained from LINEAR images with the SExtractor software [12] using standard aperture photometry techniques. The full-width at half-maximum for the optical point spread function was about 2 pixels for the images from which these data were obtained. The star measurements obtained from the LINEAR images were matched against the SDSS photometric catalog using only astrometric comparisons. The matching was accomplished with an iterative plate model fitter using a plate model function that included three terms that specifically match the optical distortion of the LINEAR telescope as well as a general second order polynomial to account for plate offset, rotation, scale, and differential refraction. The astrometric matching found 2613 stars for which the catalog positions matched the image positions to within 2.2 arcseconds (1 LINEAR pixel). Those stars were all retained and their photometry, in the form of color-color diagrams are shown in Fig. 5.

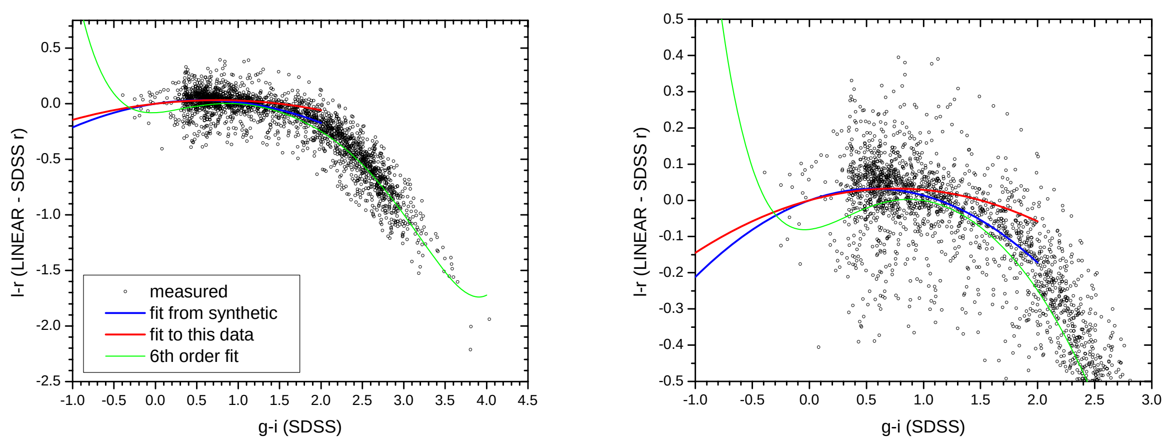

Fig. 5 Empirical Color Conversion for LINEAR from SDSS. The right panel shows a subset of the data depicted in the left panel, which shows the full range of data.

Fig. 5 matches Fig. 4 with the measured data points showing a similar quadratic trend for g-i<2, and significant divergence for g-i>2. The quadratic function that best fits (in a linear-least-squares sense) the measured data for g-i<2 is shown in blue and may be compared with the quadratic function shown in red, which is taken directly from the synthetic magnitudes shown in Fig. 4. The transformation function obtained from synthetic magnitudes is a very good match to the one obtained directly from the data. At the extreme end of the range of applicability (g-i=2) the two transfer functions differ by 0.11 magnitudes. In the region of solar type stars [g-i=1.02], the empirical and synthetic color transformations differ by only 0.016 magnitudes. The root-mean-square (RMS) residual for the empirical fit to the data shown in Fig. 5 is 0.111 magnitudes for all points with g-i<2. The RMS difference between the data points shown in Fig. 5 (for g-i<2) and the synthetic color transformation (red) is 0.118 magnitudes. This is a small difference between the two functions when compared to typical uncertainties in wide bandpass optical photometry, as can be seen in the vertical scatter of the measured magnitudes in Fig. 5.

It is of some interest to see if polynomials of order higher than two would allow for the color transformations to be extended to g-i>2. Polynomials of order 3 through 6 where fit to the full range of data shown on the left side of Fig. 5. Polynomials of odd order were indistinguishable from the next lower order polynomial of even order. None of the higher order polynomials were able to provide a reasonable fit to the full range of data. As an example, the polynomial of order 6 is shown in Fig. 5 in green. We see very poor fit with much higher residuals than in using a quadratic restricted to g-i<2. Approximately 60% of the stars in the SDSS photometric catalog have g-i<2, so this is not a serious restriction. Furthermore, the stars with g-i<2 are similar to the sun in color and thus provide better references for space surveillance operations that are observing targets illuminated by the sun.

The five dimensional form of the color conversion was also tested against the LINEAR data. Again, the observed LINEAR magnitudes and SDSS catalog magnitudes for real stars produce a quadratic function that is quite close to the function found from the synthetic magnitudes. Using the same data as depicted in Fig. 5, but fitting a quadratic in five dimensional color space, the RMS residual between the measured magnitudes and the best-fitting five-dimensional quadratic function was 0.058 magnitudes. This compares very favorably to the RMS residual between the measured magnitudes and the five dimensional quadratic function obtained from the synthetic magnitudes, which was 0.055 magnitudes.

A similar verification procedure was carried out to test the technique of synthetic color transformations by comparing the SDSS catalog to the Hubble Guide Star Photometric Catalog (HGSPC) (version 2.4) [13]. Several (30) clusters in the HGSPC were selected, and the matching regions were extracted from the SDSS catalog. Astrometric positions were used to match the stars from the two catalog. The SDSS catalog contains five bandpasses (ugriz), and the HGSPC contains 3 (Johnson-Cousins BVR), so a large number of combinations are possible, and several of these were explored to verify the synthetic magnitude technique. For brevity, one is presented here in Fig 6. This transformation from V-R to g-r was chosen so as to allow a comparison to [3] which performed a similar analysis of empirical color transformations between the SDSS catalog and several other magnitude systems including the Johnson-Cousins BVRI system.

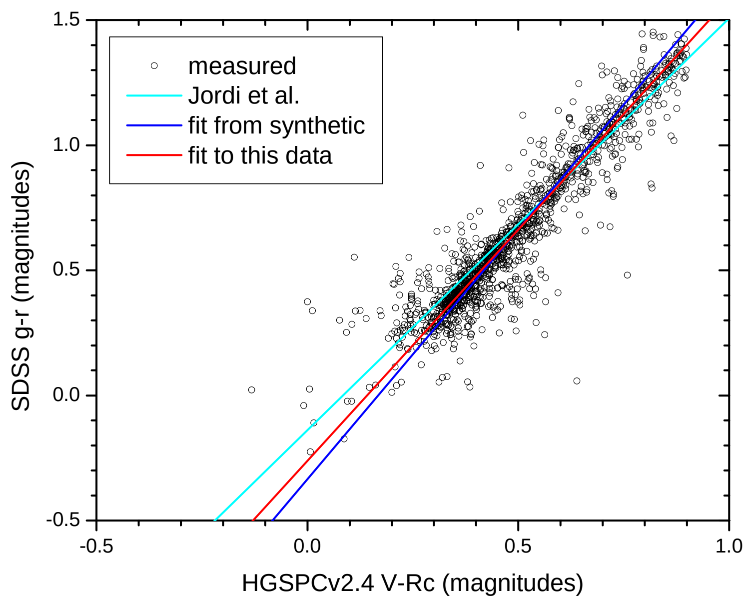

Fig. 6 Empirical Color Transformations for HGSPC and SDSS.

The color-color diagram for SDSS g-r versus HGSPC V-R is shown Fig. 6, with data points representing the color term for the 1360 stars matched in this small subset of the SDSS and Hubble catalogs. Since the V and R bandpasses are similar to the g and r bandpasses (Fig. 2), and all four bandpasses are considerably narrower than the LINEAR bandpass, a first order polynomial suffices. The lines plotted in Fig. 6 show three different versions of the color transformation function: the one obtained from the synthetic magnitude method described here, a direct fit to the plotted data, and the transformation function found in [3]. All three of these transformations fit the data reasonably well and are all close to each other. The transformation function from [3] was determined from actual star measurements, though with a completely independent set of stars and observations than what is shown in Fig. 6. The difference between the fit obtained here, and that obtained in [3] is representative of the differences one can obtain in empirical color transformations when different instruments are used or a different set of stars is chosen. It is reassuring that the synthetic magnitude method obtains a transformation equation that differs from the empirically determined transformations by a similar or smaller amount than the empirically determined transformations differ from each other.

4. SUMMARY

A general technique for transforming photometric measurements of stars and celestial objects from one color system to another has been presented. This technique is generally applicable to a variety of optical space surveillance sensors, and will allow those sensors to make optimal use of photometric measurements in star catalogs that were obtained in bandpasses other than those of the sensor itself. The technique of synthetic magnitudes is simple to compute, is based upon the publicly available library of stellar spectra published by Pickles [5], and requires only the spectral library and the spectral response functions of the sensor and catalog bandpasses of interest.

This technique of color transformations via synthetic magnitudes has been validated by comparing the synthetic transformations with empirically determined transformations for the LINEAR system, the ugriz bandpasses of the SDSS, and the Johnson-Cousins BVR bandpasses of the HGSPC. This technique has a number of operational advantages over empirically determined color transformation equations. This technique may be employed without actual observational data. For some systems, it may be difficult or inconvenient to obtain a large quantity of observations of high-quality reference stars. Furthermore, this technique can be done before a given system is operational. This allows for the color diagrams to be explored and for regions of color space in which the transformations are well-behaved to be identified before a system is operational. Then, a suitable star catalog can be identified, culled of stars with unusable color terms, and the remaining stars can be built into a catalog that is ready to use with magnitudes already calculated in the instrumental bandpass.

5. REFERENCES

1. Angione, R.J., *Atmospheric Constraints on Precision Photometry*, in CCD Precision Photometry Workshop, eds. Craine, E.R., Tucker, R.A., Barnes, J., Astronomical Society of the Pacific Conference Series, V 189, 1999.
2. Gilliland, R.L., *Details of Noise Sources and Reduction Processes*, in Astronomical CCD Observing and Reduction Techniques, ed. Howell, S.B., ASP Conference Series, V 23, 1992.
3. Adelman-McCarth, J.K., et al., *The Sixth Data Release of the Sloan Digital Sky Survey*, submitted to Astrophysical Journal Supplement, preprint available as arXiv:0707.3413.
4. Monet, G.D., et al., *The USNO-B Catalog*, The Astronomical Journal, 125:981-993, 2003.
5. Pickles, A.J., *A Stellar Spectral Flux Library: 1150-25000 Å*, Publications of the Astronomical Society of the Pacific, 110:863-878, 1998.
6. Stone, R.C., *An Accurate Method for Computing Atmospheric Refraction*, Publications of the Astronomical Society of the Pacific, 108:1051-1058, 1996.
7. Bessell, M.S., *UBVRI Passbands*, Pub. of the Astronomical Society of the Pacific, 102:1181-1199, 1990.
8. Landolt, A.U., *Standardization in the Classical UBVRI Photometric System*, in The Future of Photometric, Spectrophotometric, and Polarimetric Standardization, ed. Sterken, C., Astronomical Society of the Pacific Conference Series, V 364, 2007.
9. Jordi, K., Grebel, E.K., Ammon, K., *Empirical Color Transformations Between SDSS Photometry and Other Photometric Systems*, Astronomy & Astrophysics, 2006.
10. Fukugita, M., et al., *The Sloan Digital Sky Survey Photometric System*, The Astronomical Journal, V11.4, 1996.
11. Cox, A.N., editor, *Allen's Astrophysical Quantities, Fourth Edition*, Springer-Verlag, 2000.
12. Bertin, E., *SExtractor Software version 2.5*, produced by the TERAPIX data center located at the Institut d'Astrophysique de Paris, http://terapix.iap.fr/, 2007.
13. Bucciarelli, B., et al., *An all-sky set of (B)-V-R photometric calibrators for Scmidt surveys. GSPC2.1: First release*, Astronomy and Astrophysics, v368, p335-346, 2001.