Super Resolved Harmonic Structure Function for Space Applications

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Abstract

Lockheed Martin IS&GS (Hawaii) has combined two novel signal processing algorithms in order to characterize non-resolved objects viewed by electro-optical sensors. The combined algorithm is termed the Super Resolved Harmonic Structure Function (SR-HSF). This paper introduces the SR-HSF algorithm and demonstrates its utility for creating "fingerprints" of space-based objects. The work presented here offers promise for enhancing the performance of several Missile Defense Agency sensor systems. An overview of the SR-HSF algorithm is initially presented. SR-HSF is shown to extract key space situational awareness (SSA) fingerprints from a minimal set of observations. Mathematical details of the SR-HSF algorithm are then described. SR-HSF has been shown to be both optimal and suited for real-time processing. This HSF method is extended to operate for cases where beating harmonic sets occur; i.e., the so-called Beat Structure Function (BSF). BSF processing results are then presented from both simulated data and unclassified data collected at AMOS from objects in space. These analytical results show that SR-HSF is capable of uniquely "tagging" an object with a minimal set of observations. The SR-HSF algorithm's capabilities provide important new considerations for sensor developers, SSA systems, and operators.

I. Introduction

This paper continues work presented at last years conference, Ref. [1]. The concept of the Harmonic Structure Function (HSF) was developed there and some initial applications were discussed. Here we show the application of an extended version of the HSF, the so-called Beat Structure Function (BSF). A sample of real-world data is introduced – a rocket booster captured by the Visible Imager (VISIM) on the 3.6-m Advanced Electro Optical System (AEOS) telescope. This data is used to construct a simple functional model. The functional model then allows us to vary a simulated observation window to test the impact of sample size versus spectral resolution versus observable features. In our analysis, we focus on the ability of the HSF and BSF to detect observables until lack of samples yields spectra with such limited resolution that observable features do not exist – in fact this is precisely how our detectors are tuned. We then introduce super-resolution algorithms which decrease the number of samples, and thus decrease the observation window, required to detect the observables of interest.

II. Phenomenology.

Our starting point is an observation of a booster by the VISIM on the 3.6-m AEOS telescope. Figure 1 shows the collected data. Clearly, there are modulations of multiple frequencies taking place.

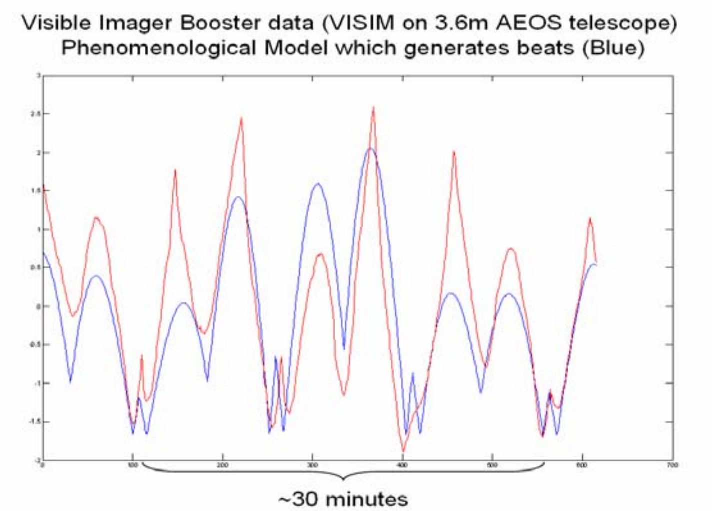

Figure 1. Rocket Body Data (Red). Model in Blue.

A simple model of the data is overlaid in blue in Figure 1, and is functionally:

$$\Psi = |\Psi_1(t)\cdot\Psi_2(t)| \text{ ; where } \Psi_1 = C_1\cdot\sin(2\pi f_1 t+\phi)+C_2\cdot\sin(4\pi f_1 t+\phi)$$
$$\text{and } \Psi_2 = C_3 + C_4\sin(2\pi f_2 t).$$

Though general, the model captures the essential features of the collected data. There is a smooth, in the sinusoidal sense, modulation of energy coupled with a 'less smooth', or non-sinusoidal, periodic modulation which gives rise to harmonics. In the spectral domain the coupling of these modulations yields a beating harmonic set, where the harmonics are generated by Ψ_1 and the beats are generated by the pure sinusoid Ψ_2. Figure 2 shows the spectrum of the model with harmonics shown in red and beats in blue.

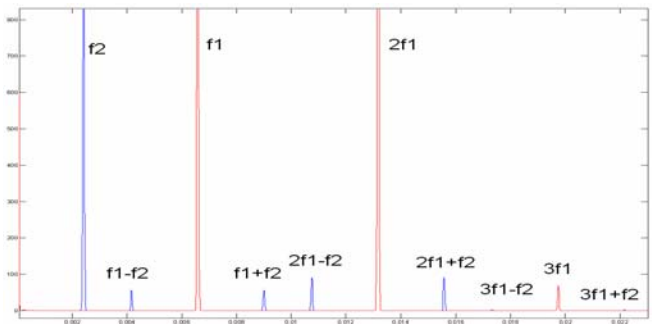

Figure 2. Spectrum of the Model.

III. The Harmonic Structure Function.

The detection of such modulating functions is performed using HSF. For a given fundamental frequency, f_0, the harmonically-related components will have frequencies nf_0 for $n > 1$, where n is the harmonic number and $n = 1$ is the fundamental. An integration of a signal, $s(f)$, is performed over frequency space and weighted such that only frequencies harmonically related to a given fundamental frequency will contribute to the integration. The integration is given by

$$\tilde{H}(f_0) = \int_0^{\infty} S(f)\left(\sum_{n=n_{\min}}^{n_{\max}} \delta(f - nf_0)\right) df$$

$$= \sum_{n=n_{\min}}^{n_{\max}} S(nf_0),$$

where $n_{\min}$ and $n_{\max}$ define the range of the harmonic components to be integrated over, and $\delta(f)$ is the Dirac delta function. Since the data are in discrete frequency bins, the integration must be performed as a sum over the corresponding bins. If $x[i]$ is the i$^{\text{th}}$ bin of the frequency spectrum and Δf is the frequency resolution of spectrum, then the sum over the discrete bins is given by

$$H(n, j) = x\left[\operatorname{int}\left[n\left(\frac{f_{\min}}{\Delta f} + \frac{j - \frac{1}{2}}{n_{\max}}\right)\right]\right]$$

which defines the harmonic structure function (HSF). The HSF is essentially the population of the matrix via spectrogram scans, as shown in Figure 3. Subsequent HSF detections are performed via *non-trivial* schemes that integrate the HSF matrix over harmonic number. Such schemes handle sidelobe and harmonic rejection tasks which are required to generate optimal detection metrics.

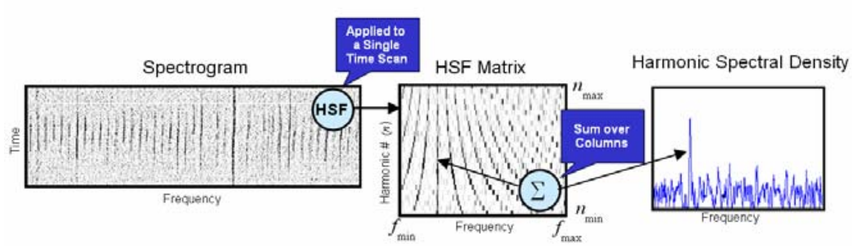

Figure 3. The HSF Matrix.

In Figure 3, a simple sum is used for the integration rather than other schemes which can also be employed.

Harmonic Detection Example

Figure 4 shows HSF working on the model, with harmonics only; i.e. $f_2 = 0$, up to the point where the observation window required for HSF to perform its detections is just large enough to achieve 'good enough' spectral resolution — this is a particular area of interest. Such a result has to be scaled to the frequency of the modulations; i.e. number of total modulations in the sample. For our particular case the model parameters are fixed to give relatively good correlation to the collected data, where there are roughly ten low frequency modulations in thirty minutes. The plot in Figure 4 is the FFT of the model (Figure 1) for roughly 2000 (blue) and 2 (red) low frequency cycles. The right plot of Figure 4 shows the corresponding HSF detections of the spectrums in blue and red. Generating the FFT with less data than ~2 or 3 cycles does not give spectral resolution which can support detection of harmonics.

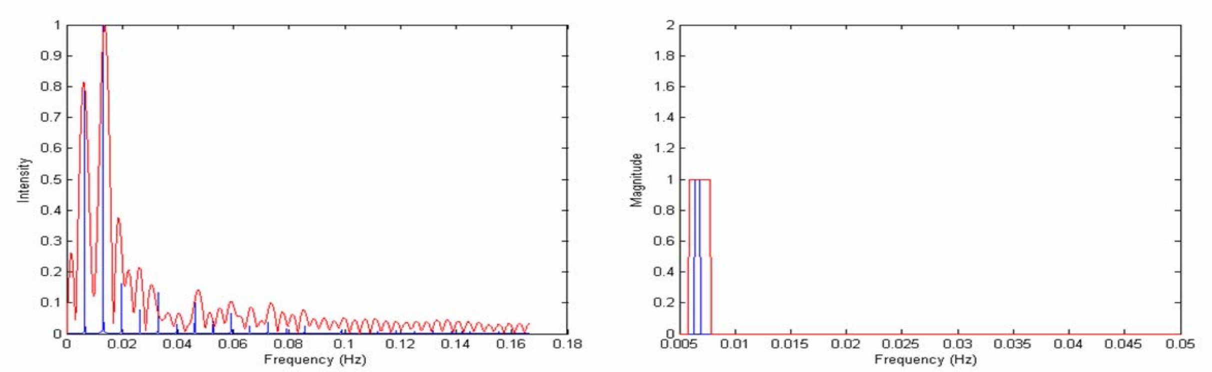

Figure 4. Resolved (blue) and Partially Resolved Spectra (red) are Detected by HSF (bottom).

IV. The Generalized Harmonic Structure Function.

To properly treat the data and model (with $f_2 \neq 0$) presented above, the HSF must be generalized to the case where two frequencies of motion are considered. To accomplish this we expand from the HSF

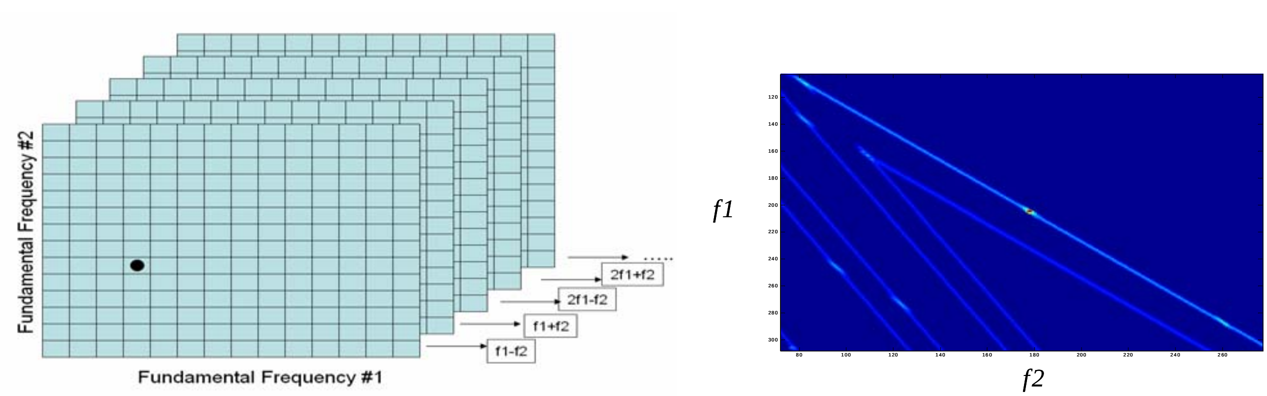

Figure 5a and 5b. BSF Cube and Integration of the BSF Cube when beating harmonics exist.

matrix to the BSF cube, as is shown in Figure 5a. The BSF cube is constructed in such a way that each 'sheet' (representing either a beat or harmonic) in the cube (Figure 5a) will contribute to the overall integration of the cube to form a detection for a particular set of frequencies. Figure 5b shows an example detection generated from the integration of the BSF cube of our model (see Figure 2) with non-zero f_2. Here, a bright point indicates a 'detection'.

V. Super-resolution Aided HSF and BSF.

FFT and other known spectral techniques suffer when insufficient observations are available. Therefore, to deal with such cases, we have enlisted the aid of a Lockheed Martin patented super-resolution technique, Reference [2]. Figure 7 shows the spectrum via super-resolution and FFT. Studies of the use of super-resolved spectrum versus an FFT show that roughly 30 to 50% fewer samples are required for generating positive detection of harmonics and/or beating harmonic sets.

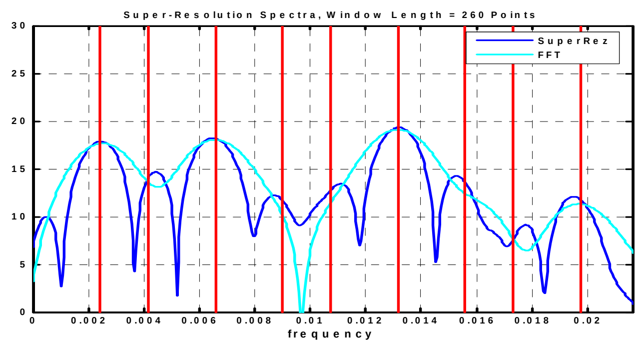

Figure 7. Super-resolved Spectra (dark blue) Plotted Alongside Unresolved Spectra (light blue). Red Lines Denote the Expected Peak Locations of Harmonics and Beats.

VI. Conclusion

This paper highlights two key concepts. One, there exists a mathematical technique to automatically detect signals with harmonic and beating harmonic structure. Two, these techniques, when coupled with super-resolution algorithms, allow for detections of harmonics and beating structures with small observation windows. Such a capability may also find relevance for SSA applications to perform fingerprinting, tagging, or feature added tracking, as well as detecting, classifying and counting rotating bodies in a *single pixel* of a sensor's focal plane array.

References

[1]. Dikeman, R.David and Seto, Scot "Harmonic Analysis for Optically Modulating Bodies using the Harmonic Structure Function (HSF)". Presented at AMOS 2006.

[2]. Abatzoglou T., Lam L., Malek H. and Reagan J., "Real Time Super-resolution Signal Processing," Patent #5,748,507. May 5, 1998.