

# Derivation and Application of a Global Albedo Yielding an Optical Brightness to Physical Size Transformation Free of Systematic Errors

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## ABSTRACT

We have developed a technique for estimating the intrinsic size distribution of orbital debris objects via optical measurements alone. The process is predicated on apriori knowledge of the power-law size distribution of debris (as indicated by radar RCS measurements) and the log-normal distribution of optical albedos. Since the observed distribution of optical brightness is the convolution of the parent [size] population with the albedo distribution, it is a straightforward matter to transform a given distribution of optical brightness back to a size distribution by appropriate choice of a single albedo value. This is true because the integration of a power-law with a log-normal distribution yields a Gaussian-blurred power law distribution with identical power-law exponent. Application of a single albedo to this distribution recovers a simple power-law which is linearly offset from the original distribution by a constant whose value depends on the choice of the albedo. Significantly, there exists a unique “weighted-average” albedo which when applied to an observed brightness distribution yields zero offset and therefore recovers the original size distribution. For physically realistic power-laws of negative slope, the proper choice of weighted albedo effectively removes the biases caused by the large number of small objects that look anomalously “large” (bright) and the lower number of large objects looking anomalously “small” (dim).

Based on this comprehensive analysis a global value of 0.13 should be applied to all orbital debris albedo-based brightness-to-size transformations of debris objects regardless of data source. This represents a modification to the canonical value of 0.1 widely employed. Herein we present the empirical and mathematical arguments for this approach and by example apply it to a comprehensive set of photometric data acquired via NASA’s Liquid Mirror Telescopes during the 2000 observing season.

## 1. INTRODUCTION

Orbital object data acquired via radar and optical telescopes plays a crucial role in accurately defining the space environment. Radars actively probe the characteristics of small debris by directing electromagnetic energy at an object and measuring the reflected energy. Since object size is typically comparable to the wavelength of the radiation, the reflected signal is affected by the bulk electrical conductivity of the debris object as well as its shape and orientation. Optical measurements conversely use reflected solar radiation with wavelengths much smaller than the size of the objects. While shape and orientation are important, only the surface electrical properties of the debris material (i.e., the surface [simple] albedo) contribute to the reflected radiation field. As a result, these two methods are complementary in that they measure somewhat independent physical properties to estimate the same quantities – primarily debris size.

Short arc optical observations such as those obtained by NASA’s 3.0 meter Liquid Mirror Telescope (LMT) [1] give enough information (e.g., angular velocity) to estimate an Assumed Circular Orbit (ACO) and an associated range. This information, combined with the apparent brightness, can be used to estimate an “absolute” brightness (scaled to a fixed range and phase angle). This absolute brightness is what is used to estimate debris size [2]. However, shape and albedo variations make any given size estimate subject to large systematic and random errors. As has been shown with radar debris measurements [3,4], that does not preclude the ability to estimate the size of an individual object via an ensemble of measurements, or the size distribution of a large number of objects statistically.

After systematic errors have been eliminated (e.g., ACO and phase function errors) there remains a geometric albedo distribution that relates object size to absolute magnitude. Measurements acquired by the LMT [5] of a large subset

of tracked debris objects with sizes estimated from their radar cross sections (RCS) indicate that the random variations in the albedo (obtained by factoring RCS from brightness) follow a log-normal distribution. In addition, this distribution is independent of object size over the entire assessed range (a few centimeters to several meters). Note that this relation appears to hold only for debris - where the shapes and other properties are not primarily the result of human manufacture but rather of random processes. With this information in hand, it now becomes possible, via optical measurements alone, to estimate the actual size distribution of debris objects.

## 2. PHOTOMETRY AND SIZE ESTIMATION.

The photometric reduction of telescopic optical orbital object data normally involves the comparison of target fluxes with that of background stars of known brightness. Since astronomical objects used as fiduciary references have their brightness categorized using color filter photometry or low resolution spectroscopy, observations are normally conducted with broad or narrow-band colored filters. Using this calibration basis, an observed or apparent color-dependent magnitude ( $M_{app}$ ) is ascribed to the object of interest. For debris,  $M_{app}$  is usually derived for the visual band (V; typically 550 nm center, 125 nm width) or the red band (R; typically 650 nm center, 150 nm width).

By convention, LEO orbital debris observations are normalized to a reference range of 1000 km. The new absolute magnitude,  $M_{abs}$ , is given by:

$$M_{abs} = M_{app} - 5 * \log \left( \frac{\text{Range}_{km}}{1000_{km}} \right) \quad \text{Eq. 1}$$

where the range is the ACO derived orbital altitude.

The size of the debris object is estimated by computing the characteristic length,  $L_C$ , (e.g., diameter) of an equivalent specular sphere (isotropic phase function) having absolute magnitude,  $M_{abs}$ , from the equation

$$\log (L_C) = -0.2 (M_{abs} - M_{sun}) - 0.5 * \log \left( \frac{A}{16 R^2} \right) \quad \text{Eq. 2}$$

where  $A$  is the geometric albedo,  $R$  is the range, and  $M_{sun}$  is the apparent magnitude of the Sun (e.g., -26.74 in the Johnson V band centered at 550 nm.).

If an albedo for the object is assumed, the photometric brightness of the object can be converted into an optical cross section. Because the derived object size depends upon the square root of the chosen albedo, proper selection is critical.

## 3. SAMPLE DATA

Using LMT data [5], Fig. 1 illustrates the distribution of object brightness for SSN (Space Surveillance Network) Catalog correlated targets (CTs) as a function of RCS derived characteristic length ( $L_C$ ). The solid trend lines indicate the functional dependence for three different albedos - 0.01, 0.13 and 1.0. As expected, for constant size, object brightness increases with albedo. Similarly, for a given brightness, object size must increase as object albedo diminishes. The distribution itself (dashed line; least squares fit) has a different slope and is centered near an albedo of 0.1 (rather than 0.13) due to a combination of sensitivity roll-off and the skewed sampling (towards brighter detections) of smaller (and more numerous) objects relative to larger (and fewer) objects which themselves are evenly sampled. The next section describes a new approach to redressing this limitation - justifying the use of a 0.13 albedo and rectifying this apparent discrepancy.

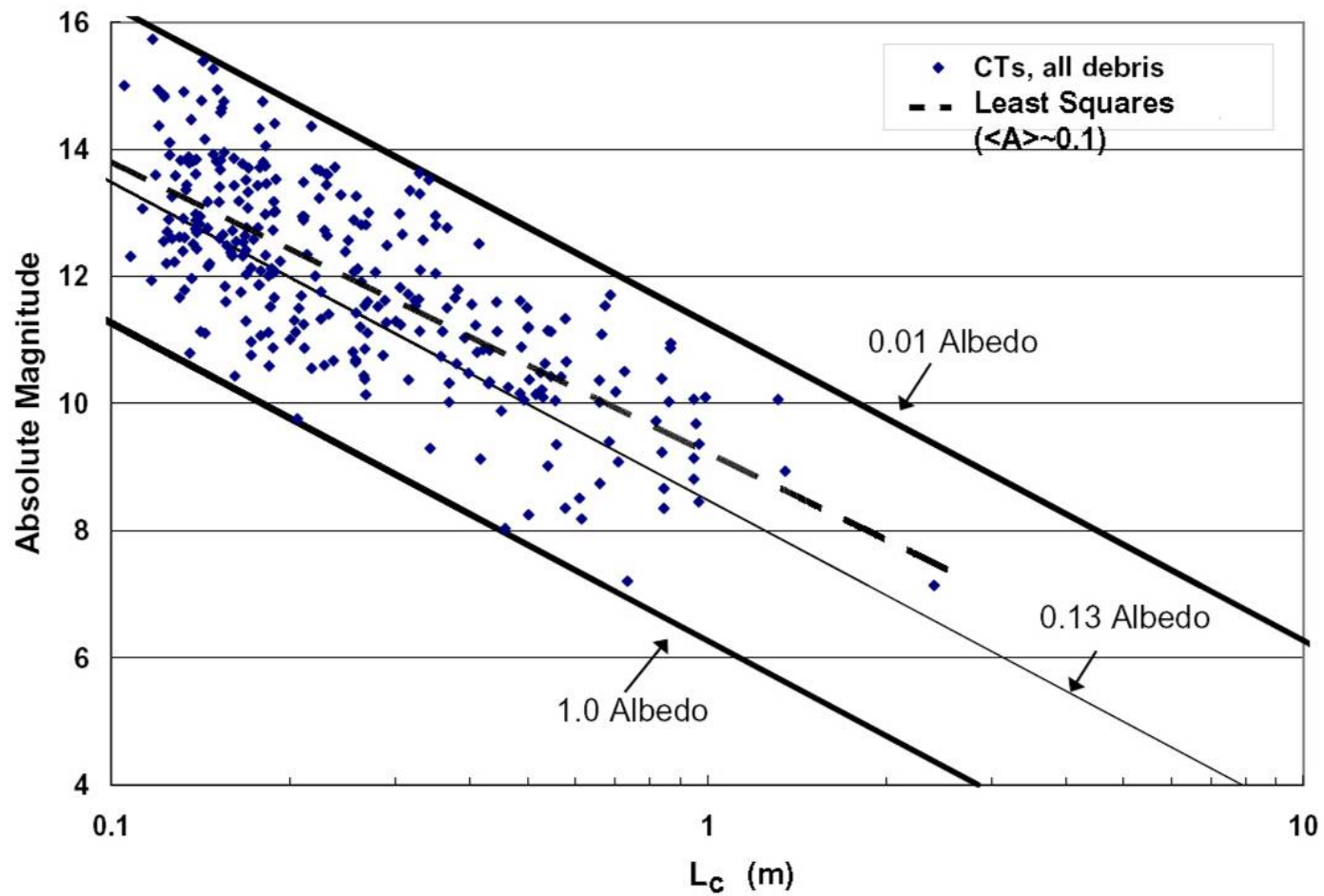


Fig. 1. All LMT observed CT debris objects [5]. (LC is RCS derived).

#### 4. DERIVATION OF A GLOBAL ALBEDO

The observed distribution of object magnitudes (and by corollary, range and phase corrected intrinsic brightnesses) must ultimately be transformed to a distribution of object sizes free of any systematic bias. Although there are certainly errors introduced by the application of ACO range estimates, a Lambertian phase function, and photometric measurements themselves, the errors are either well characterized (e.g., range bias due to non-zero eccentricity) or are randomly distributed and generally size independent (e.g., specular reflections from debris). This leaves as pivotal the judicious choice of the albedo used to transform the observed brightness distribution back to the intrinsic size distribution of the parent population.

There are two characteristics of the debris population that make this process tractable:

1) the size distribution of debris (the parent population) obeys a simple Power-Law [6]

and

2) the distribution of debris albedos is observed to be log-normal (Fig. 2).

The combination of these attributes means that there exists a unique global albedo that when applied to an observed brightness distribution recovers the parent size distribution. This is true because the convolution of a power-law with a log-normal distribution is de-convolved to a power law of the same form by application of a constant global albedo to the entire distribution. While a power law with the parent's exponent is recovered upon application of this global albedo, an offset is produced – the magnitude of which determines the extent to which small (or large) objects in the parent population are erroneously transformed to objects of alternate size. It is this transformation which results in differential or cumulative size distributions that are contaminated with small (and large) object bias.

Fig. 3. illustrates the fundamental principle underlying the process – that the observed number distribution of object brightness  $N(M_{app})dM_{app}$  is a convolution (Equation 3) of a parent size power-law ( $N(D_0)$ ) with a log-normal albedo spread  $P(A|A_0)$  – yielding a Gaussian-blurred size distribution. This figure reveals the hidden structure behind the familiar number versus magnitude plots or, analogously, number versus size plots (previous Figure 1). Understanding this structure is the key to extracting the intrinsic parent size distribution.

$$N(M_{app})dM_{app} = \int_D P(M_{app} | D_0) \cdot N(D_0) \cdot dD_0 \quad \text{Eq. 3}$$

where  $P(M_{app} | D_0) \propto P(A | A_0) \cdot \frac{D_0^2}{4} A$  is the probability (P) of measuring apparent magnitude  $M_{app}$  given actual object diameter  $D_0$ , median geometric albedo  $A_0$ , and actual albedo  $A$  (an element of the log-normal distribution in  $A$ ).

Applying a constant albedo value to a brightness distribution yields a diameter distribution. The accuracy of that distribution (i.e., the extent to which it is systematically biased) depends on the choice of albedo – which itself depends upon the power-law slope  $\beta$ . Fig. 4 illustrates the distribution of perceived object size ( $D$ ) relative to the parent (or true) object size ( $D_0$ ) resulting from the transformation of a brightness distribution to a size distribution. This is simply a re-casting of Fig. 3 into the size domain. The Gaussian blur is now in size rather than magnitude.

Naturally, the underlying structure – in the form of a Gaussian blur - in magnitude or size is not a directly observable quality. Telescopic observations effectively integrate slices through constant apparent magnitude – or - after albedo transformation – through constant derived diameter. That integral is itself a power-law and that is what is directly measured. This observed power-law will have zero systematic offset, and thereby match the parent, if the transformational albedo is chosen properly. Furthermore, because the offset is independent of diameter, the bias introduced by poor albedo choice is constant over all size ranges. For the orbital debris population with moderately negative power-law exponents (e.g.,  $\beta = -1.6$ ) [6], Fig. 5 shows that the ratio of the observed and parent differential distributions is unity if only if an albedo of 0.13 is chosen. Use of the canonical 0.1 albedo results in a 24% upward–

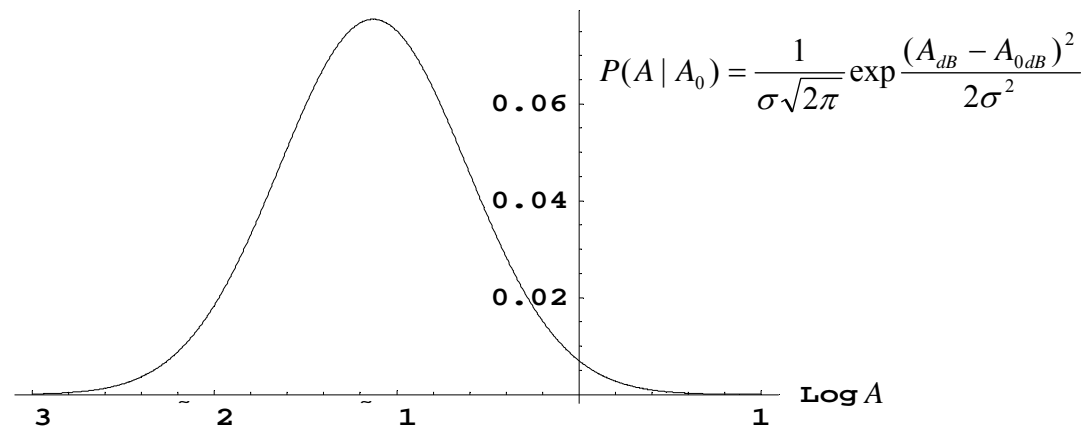


Fig. 2. The albedo distribution (i.e., the probability of finding  $A$  given  $A_0$ ) of CT debris. It has been empirically determined to be Log-Normal with a median albedo  $A_0$  of 0.076 ( $\text{Log} \langle A_0 \rangle = -1.12$ ) and log-width ( $\text{Log} \sigma$ ) of  $-0.5149$

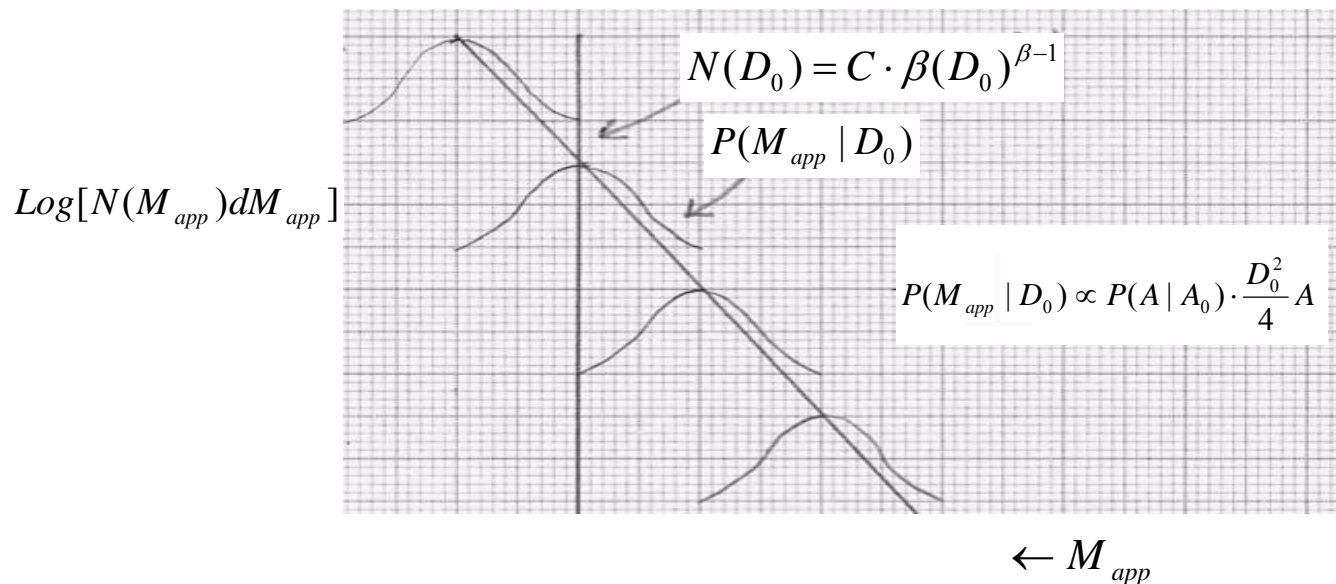


Fig. 3. Gaussian-blurring underlying the observed brightness distribution. To first order, the size distribution  $N(D_0)$  of the orbital debris population is well represented by a simple power-law (sloped line with exponent  $\beta = -1.6$  [6]). If albedo were constant, the transformation between brightness and size would be trivial. When coupled to a log-normal albedo distribution however, the resultant brightness distribution is blurred such that a unique transformational albedo must be chosen to recover the original power-law size distribution.

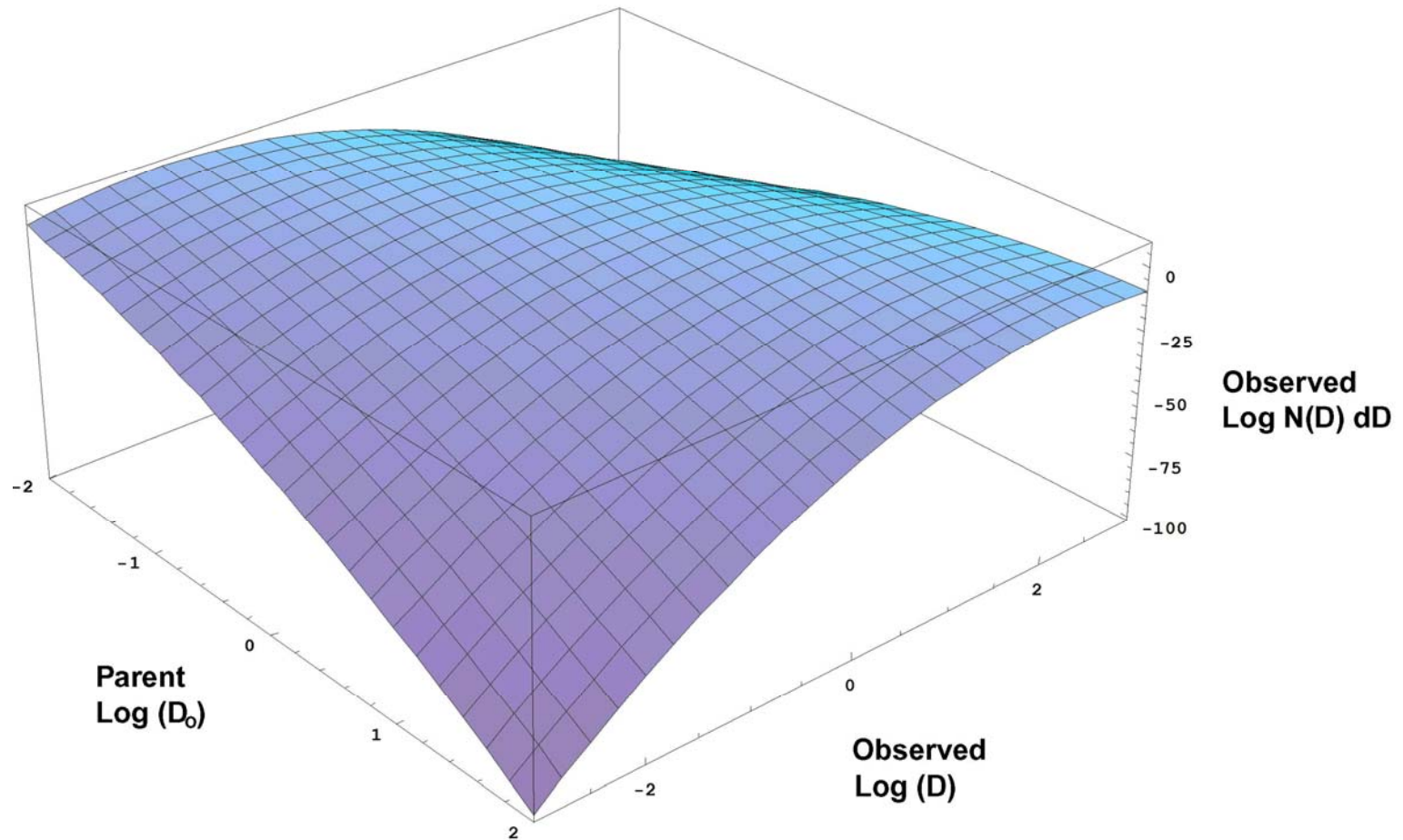


Fig. 4. Probability Distribution of Observed Diameters. This figure demonstrates the spread of apparent object size ( $D$ ) relative to the actual object size ( $D_0$ ) when the albedo distribution is considered - for each parent diameter there is a distribution of observed diameters. The full convolution of the log-normal and power-law functions ( i.e., the number-size distribution we actually observe) is the integrated area under a slice at constant diameter  $D$ . If the reference albedo is not chosen carefully, the derived distribution of object size will be disproportionately influenced by objects whose true size ( $D_0$ ) is smaller (or larger) than the diameter of interest. For moderately negative power laws this results in a positive or negative bias in the calculated number distribution. By judicious choice of a reference albedo the parent distribution can be recovered. This figure was generated with the 0.13 bias-free albedo.

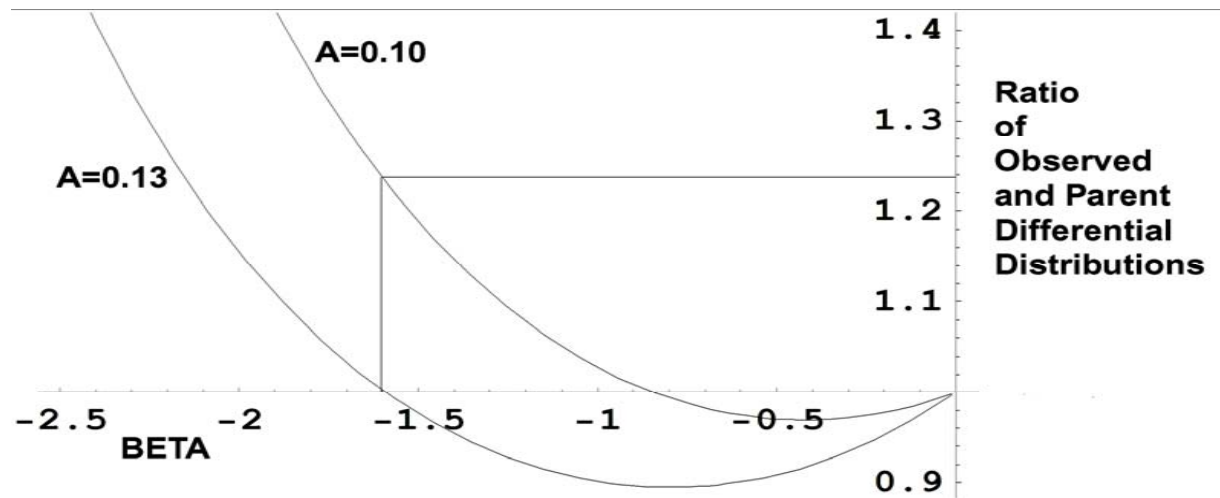


Fig. 5. Ratio of observed differential power-law size distribution to the parent power-law distribution. Selection of a 0.10 albedo yields a ratio of 1.24 at a power-law slope of  $\beta = -1.6$ . – i.e, the population is overestimated by 24% - this is “small object” bias. Selection of a 0.13 albedo conversely yields a bias-free ratio (unity). The parent number distribution is recovered with judicious choice of albedo. Other power-law exponents require alternate albedos.

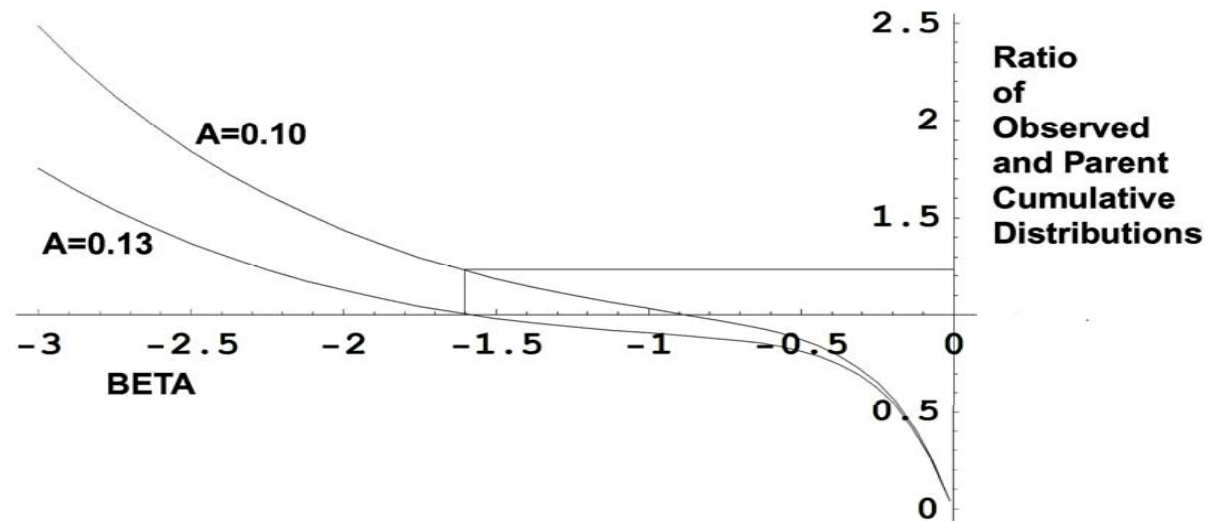


Fig. 6. Ratio of observed cumulative power-law size distribution to the parent power-law distribution. Selection of a 0.10 albedo yields a ratio of 1.22 for a parent power-law with a  $\beta = -1.6$  slope - thus the population is overestimated by 22% - this is “small object” bias. Selection of a 0.13 albedo conversely yields a bias-free ratio (unity). The parent number distribution is recovered with judicious choice of albedo. Other power-law exponents require alternate albedos.

bias in the differential and a 22% upward bias in the cumulative (Fig. 6) distributions. The observed cumulative distribution similarly matches the parent only if a 0.13 transformational albedo is employed.

## 5. APPLICATION OF A GLOBAL ALBEDO TO AN OPTICAL DATASET

One of the primary objectives of NASA-LMT optical data acquisition was the determination of orbital debris flux as function of altitude. Initially, the derived flux appeared to contradict other data sources primarily the Haystack Auxiliary radar observations[7]. LMT implied fluxes in the 10 cm regime were particularly problematic– indicating fluxes 30-40% higher than generally accepted. The disagreement is illustrated in Fig. 7a – generated by transforming year 2000 LMT data with a 0.10 albedo. This discrepancy prompted an in-depth analysis of the role of sensitivity roll-off and its coupling to observations of populations whose numbers expand with decreasing size. The log-normal behavior of orbital debris albedos was subsequently ascertained and the methodology discussed herein was developed. The imposition of a transformational albedo (which can vary with power-law exponent) is now a rigorously defensible action obviating the more subjective nature of past analyses [8] which sometimes relied heavily of visually obtained photometric values[9]. Fig. 7b was generated via the application of a 0.13 albedo to the year 2000 LMT data. The albedo-based bias is now eliminated and the disparity is small. This is not to say that all systematic bias (or random error) is eliminated, only that the choice of albedo is now both analytically and empirically justified.

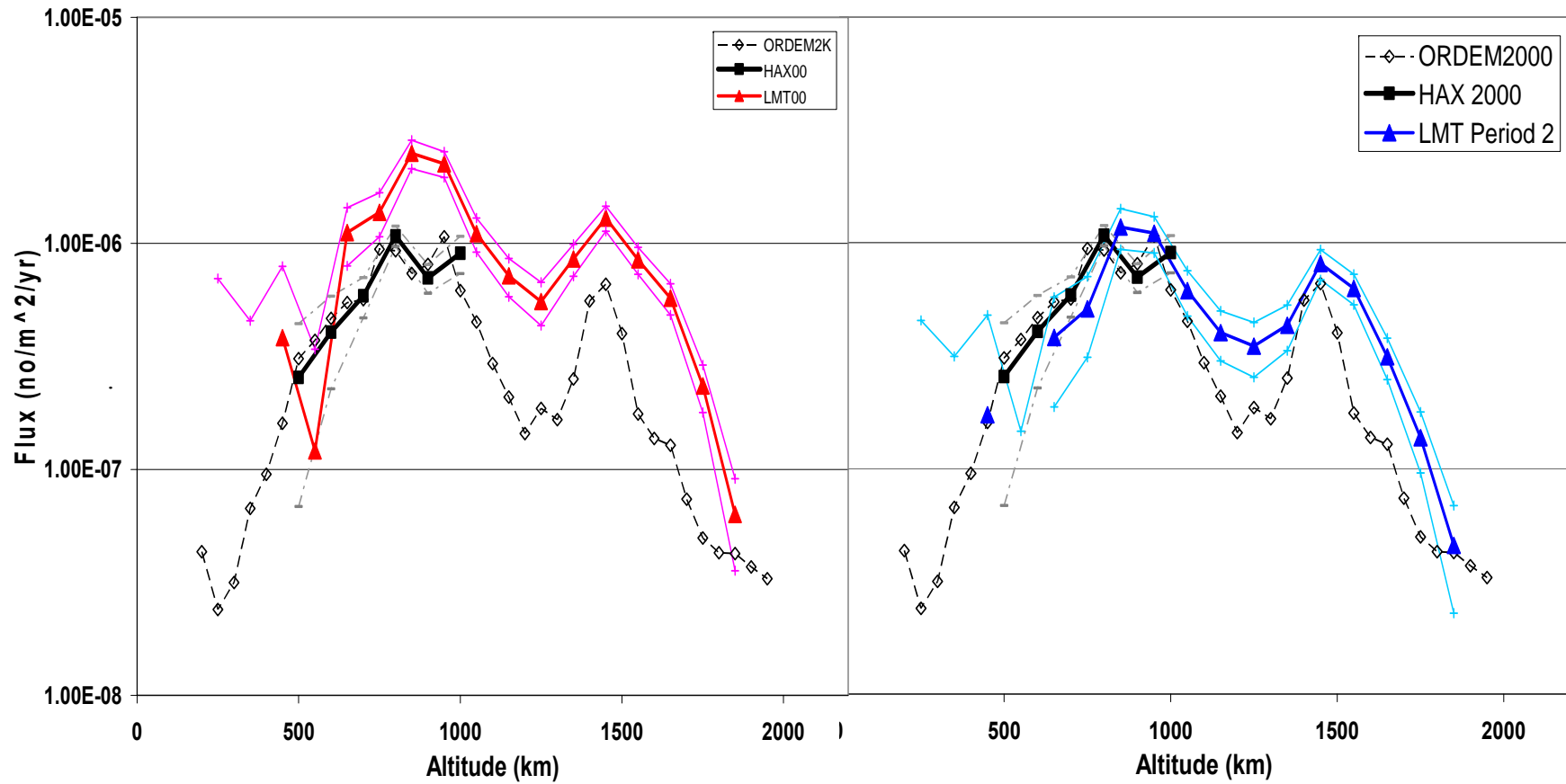
## 6. CONCLUSION

For the orbital debris population with moderately negative power-law exponents (e.g. -1.6) and a log-normal albedo distribution, a significant overestimate of the number of objects at a given size results if the canonical 0.1 albedo is employed. This disproportionately transforms intrinsically smaller objects to larger sizes resulting in a skewed and overestimated size distribution. The choice of albedo is critical – as variations of a few percent can dramatically alter derived number or flux distributions – over estimating the population at a given size if the choice is too low, underestimating if too high.

By carefully matching the transformation albedo to the power law exponent, it is possible to eliminate the constant offset of the power-law and completely recover the parent distribution. Using the power-law size distribution with  $\beta = -1.6$  and an empirically derived log-normal albedo distribution based on CT debris, it was determined that a transformational albedo of 0.13 applied to the observed brightness distribution fully recovered the intrinsic size distribution of the parent population.

Based on this more comprehensive analysis a global value of 0.13 should be applied to all orbital debris albedo-based brightness-to-size transformations regardless of data source or orbital regime.





Figs. 7a and 7b. The fluxes for the LMT[10], HAX[7], and ORDEM2000[7] (Nasa's Orbital Debris Engineering Model) for the Year 2000 dataset. Application of a 0.13 albedo (right) removed the albedo-based bias introduced by using the canonical 0.10 albedo (left).

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