Space-Based Sensor Coverage of Earth Orbits

Islam I. Hussein

Yue Wang

Worcester Polytechnic Institute

ABSTRACT

In this paper we propose a space-based system for the surveillance of Earth-orbits. The proposed design will have the capability of completely covering spatial tubes (whose widths depends on the sensor ranges) spanning a range of Earth orbits. While the literature on ground-based surveillance is large, the problem we study in this paper (space-based surveillance of Earth orbits) is rarely studied in the past.

A surveillance satellite placed in orbit about Earth will only be able to detect objects of interest within its sensors' footprints. Objects of interest on the same orbit but outside the sensors' footprints will not be detected due to a difference in angular positions between the satellite and objects of interest. Hence, for that satellite to scan an entire orbit and ensure the detection, with probability one, of any objects on that orbit, it will have to apply control forces to march along the orbit. Hence, in this paper, we develop a simple (and cheap) orbital maneuver to effect the marching of the satellite along the orbit. The maneuver depends on the sensor range and nominal orbit size. This basic design is then extended to multiple satellite systems.

Once the basic design is introduced, we investigate the interdependence of three basic design variables: number of satellites used, fuel usage, and time to mission completion. The proposed design starts with two extreme cases: a system design that uses a single satellite but maximum fuel usage and time to mission completion; and a system design with (an analytically computable) maximum number of satellites but with zero fuel usage and time to mission completion. Each of the cases has the ability to detect, with probability one, any objects of interest in orbit within a given range of space orbits. With J_2 effect taken into account, the proposed system design will guarantee complete coverage, not only of a tube in space, but of an entire shell containing a wide range of orbits with varying inclination angles.

Detailed numerical examples are given for the coverage of Low Earth Orbits and Geostationary Orbit. This paper lays the basis for future work where the authors will consider questions of resource allocation for cases with limited resources (especially severe constraints on the number of satellites used in the mission), and a formal formulation of a multi-objective optimal design problem and solutions where the design parameters are: number of satellites used in the system, fuel-usage and time to mission completion.

1. INTRODUCTION

Coverage control has been of much interest recently due to its versatility in many aerial, terrestrial, and underwater applications such as surveillance, search and rescue/retrieval. See [1], [2] and references therein for a complete overview of the literature.

Research on coverage control includes three major categories: locational optimization [3, 4, 5], sensor redeployment [6], and effective coverage control. The first two classes neglect both the sensor mobility and the computational efficiency, while effective coverage control provides a remedy for these issues. Recent research on effective coverage control (see [7, 8, 9]) has developed deterministic control strategies using vehicles with limited sensory capabilities over large-scale domains. In the stochastic setting, the author in [10] uses the Kalman filter for estimating a spatially-decoupled, time-independent field and optimally guiding the vehicles to move in directions that improve the field estimate. In [11, 12], a novel dynamic "awareness" model is introduced and applied to the effective coverage control of large-scale domains with intermittent communications, and the decision making for search and tracking of multiple targets.

The literature on space-based *ground* surveillance is large. Recently, several Low Earth Orbit (LEO), Medium Earth Orbit (MEO), and Geostationary(GEO) satellite constellations have been proposed (see, for example, [13, 14, 15, 16]). The problem we study in this paper is to effectively cover *space* and detect spatial targets from space-based observation satellites. Every point within the circular planar orbits is guaranteed to be covered using our satellite system design and control solution, thus ensuring the detection of all possible targets.

2. PROBLEM SETUP

General Settings. The focus of this paper is on space-based surveillance of circular Earth orbits. This problem is a subset of a more general class of problems where the Earth orbits of interest are elliptic. We will first ignore the effects of the oblateness of Earth but, later in the paper, we will use J_2 effects to show how the proposed constellation design will be able to monitor an entire shell around Earth. We will investigate a simple design of an N-satellite constellation for Earth orbit surveillance. In the analysis, we will start with the assumption that we have only one satellite available for the mission, and then work our way up to a general statement about N-satellite systems.

Coordinate Definitions. For an N-satellite constellation, each satellite will be indexed by $i = 1, 2, \ldots, N$. Naturally, we will adopt polar coordinates for the definition of the position of the satellite in space. Let $T_i = 2\pi\sqrt{\frac{r_i^3}{\mu}}$ be the orbit period and $n_i = \frac{2\pi}{T_i}$ be the orbit angular velocity of satellite i, where r_i denotes the orbital radius of satellite i. Let f_i denote the true anomaly (also equal to the mean anomaly M_i and eccentric anomaly E_i in the case of circular orbits).

Simple Sensor Model. Each satellite is assumed to be equipped with some sensor of interest whose range is R_i in all directions. For simplicity, assume that $R_i = R$ for all satellites. This implies that each satellite has a sensing range of $r_i - R < r < r_i + R$ radially and approximately an angular range of $f_i - \frac{R}{r_i} < f < f_i + \frac{R}{r_i}$ in-track. For a satellite to completely characterize its neighborhood (determined by the sensor range R), the sensor requires a monitoring time of τ_s. The time τ_s is the minimum amount of time required for a satellite to determine with some high probability whether any objects of interest exist in its neighborhood or not.

Orbit Maneuvers. The basic surveillance strategy is that a satellite will monitor a spherical volume whose radius is given by R and which is centered at the location of the satellite on the orbit. After the passage of an amount of time τ_s, this spherical volume would have been satisfactorily monitored and any existing object in that volume would have been completely characterized. After τ_s amount of time the presence of the satellite at that location returns no new useful information and it has to execute a maneuver to change its location to a new one where it can cater new and useful surveillance information.

The specific maneuver of interest in this paper is an in-track phase shift to a new location in the orbit such that after the maneuver the satellite covers a previously uncovered spherical volume. This in-track phase shift will be achieved by an impulsive maneuver that places the satellite on an elliptic such that after one orbital rotation, when the satellite returns to its original reference circular orbit, the satellite would have achieved a net angular phase shift. After that maneuver, the satellite monitors the new region for an amount of τ_s after which it executes another impulsive maneuver that sends it on an elliptic orbit, where after a complete orbit rotation the satellite returns to the original orbit shifted by another angular amount to cover a new region on the reference trajectory. This is repeated until the entire orbit (or portions of it for multiple satellite constellations as will be seen later in the paper) is covered. *A basic assumption in this work is the satellites of interest in space are incapable of executing maneuvers of their own.*

The key question, then, is: How should the elliptic orbit be designed in order for the satellite to return to the reference circular orbit, but shifted from its original angular location by an amount equal to $\frac{R}{r_i}$? That is, if the satellite finishes the monitoring of a spherical region around the reference orbit at a true anomaly angle of f_i, how should the elliptic orbit be chosen such that it returns to the reference orbit at a true anomaly of $f_i \pm \frac{R}{r_i}$.

In this paper, the basic maneuver of interest is an orbit size change where a satellite transfers from a circular orbit of radius of r_i^- to an elliptic orbit with semimajor axis of $a = \frac{r_i^- + r_i^+}{2}$, where r_i^+ is the apoapsis radius. We need to change the velocity from that of the circular orbit to that of the elliptic orbit at periapsis by applying an impulsive tangential force. This sends the satellite on an elliptic orbit. When the satellite returns (after one orbital rotation) to the reference circular orbit, the satellite applies a second impulsive force to transfer from the elliptic orbit to the circular orbit. For transfers between the circular orbits of sizes r_i^- and the ellipse with semimajor axis of a, the total change in (tangential) velocity during the whole process is given by

$$\begin{aligned} \Delta v_i &= 2|v_c - v_p| \\ v_c &= \sqrt{\frac{\mu}{r_i^-}}, \\ v_p &= \sqrt{\frac{\mu}{a}\left(\frac{1+e}{1-e}\right)}, \end{aligned} \tag{1}$$

where v_c is the nominal circular tangential velocities of the satellite in the circular orbit with radius r_i^-, v_p is the tangential velocity of the satellite if it were at periapsis on an ellipse, and e is the eccentricity of the ellipse. Δv_i is the total amount of velocity change to effect an in-track phase shift using an elliptic orbit. The only unknown here is r_i^+ which will be appropriately selected to achieve the desired angular phase shift. Note that if $r_i^+ < r_i^-$, the satellite's radial location decreases causing an overall forward phase shift, while if $r_i^+ > r_i^-$, the satellite's radial location increases causing an overall backward phase shift. We will assume, without any loss of generality, that $r_i^+ > r_i^-$.

3. SURVEILLANCE OF AN EARTH ORBIT

Let us assume that the range of orbits to be surveyed is given by $r_l < r < r_u$, where r_l is the lowest orbit of interest and r_u is the highest orbit of interest. A K-stage surveillance problem is one that can be split into K orbits, where $K = \lceil \frac{r_u - r_l}{2R} \rceil$, which is the number of satellites that can *radially* (i.e., where $\phi_i = \phi_j$ for all $i, j = 1, \ldots, K$) cover all orbits in the range $r_l < r < r_u$. Having such an arrangement will not guarantee detection of objects in that orbital range since objects of interest may be out of phase with the satellites even if they are not radially aligned. We will return to this general case later. For now, we will consider the case where $K = 1$, specifically with $r_u = r_l + 2R$.

3.1. Single Orbit Stage Case ($K = 1$)

For the $K = 1$ case, there are two extreme solutions one may consider. The first is one that involves a maximum number of satellites with zero fuel consumption. The second is one which involves the minimum number of satellites ($N = 1$) but with the maximum use of fuel. The first solution represents a minimum (in fact a *zero*) fuel, maximum number of satellites solution and the second represents a maximum fuel, minimum number of satellite solution.

Zero-Fuel Solution. The zero fuel solution requires the distribution of $N_{\max}$ satellites on an orbit of size $r_1 = \frac{r_l + r_u}{2} = r_l + R$ evenly in the radial direction. Hence, we will need $N_{\max} = \lceil \pi \frac{r_1}{R} \rceil$ satellites where the phase shift between one satellite and the next one is given by $\Delta\phi = 2\frac{R}{r_1}$. One will not need any more than that number of satellites to cover the entire orbit with a coverage width of R. To show that such a design will require an unpractically large number of satellites, consider a low Earth orbit with $r_1 = 7,200$ km orbit (roughly an altitude of 800 km) and a sensor range of 100 km. Then the number of satellites for a zero fuel surveillance constellation is given by

$$N_{\max} = \lceil \pi \frac{7200}{100} \rceil = 227 \, \text{satellites}.$$

We will now consider the other extreme solution, one where we use exactly one satellite but with a large amount of fuel expended in the process.

Single Satellite Solution. Assume now that we have a single satellite capable only of performing orbital maneuvers as mentioned in Section 2. As mentioned above, and initializing the satellite such that its original orbit radius is $r_1 = \frac{r_l + r_u}{2} = r_l + R$ with phase $\phi_0 = 0$, the strategy is to

1. Dwell on the circular orbit for a time of τ_s to monitor a spherical region centered at the orbital location of the satellite with a radius of R.
2. Perform an orbital maneuver to an elliptic orbit of semimajor axis $a = \frac{r_1 + r_t}{2}$. The apoapsis radius r_t is chosen such that when the satellite returns to r_1 the satellite phase angle is given by $\phi_1 = \phi_0 + 2\frac{R}{r_1} = 2\frac{R}{r_1}$.
3. Dwell for a time τ_s to cover the second sector corresponding to ϕ_1.

This process is the repeated such that after each maneuver we get a change of $2\frac{R}{r_1}$. Hence, we need

$$\frac{\pi r_1}{R} \text{ transfers}$$

to cover the entire orbit for all phases.

Hence, we seek to choose r_t such that after a single complete elliptic orbit maneuver the satellite returns with a phase shift of exactly $2\frac{R}{r_1}$. First note that transferring from r_1 to r_t and back takes a total time of

$$T_t = 2\pi\sqrt{\frac{(r_1 + r_t)^3}{8\mu}}. \tag{2}$$

During that time, a reference satellite on the original reference circular orbit would have changed its true anomaly by

$$\Delta f = 2n\pi\sqrt{\frac{(r_1 + r_t)^3}{8\mu}},$$

where n is the angular rate of the reference orbit. We Desire that $\Delta f = 2\pi + \Delta\phi$, where $\Delta\phi = 2\frac{R}{r_1}$ is the desired phase shift. Solving for r_t one obtains

$$r_t = r_1\left(2\left(1 + \frac{R}{\pi r_1}\right)^{\frac{2}{3}} - 1\right). \tag{3}$$

From this one can compute the amount of velocity change per orbit transfer and back:

$$\Delta v \quad = \quad 2\left|\sqrt{\frac{\mu}{r_1}} - \sqrt{\frac{2\mu}{r_1 + r_t}\left(\frac{1+e}{1-e}\right)}\right|, \tag{4}$$

as well as the total energy involved in the entire surveillance process:

$$\Delta v_{\text{tot}} = \pi\frac{r_1}{R}\Delta v. \tag{5}$$

Lets consider the example given above with $R = 100$ km and $r_1 = 7,200$ km. This gives a transit orbit size of $r_t = 7,242$ km (i.e., a change of 42 km in orbit size per transfer). This gives a total velocity change of 4938.5 m/s, which is relatively high.

Non-Zero-fuel, Multiple Satellite Solution. Lets say we now have $1 < N < N_{\max}$ satellites. One can devise a solution analogous to that suggested in the previous paragraph for a single satellite. We begin by distributing the satellites evenly on the orbit. That is, if the first satellite has a phase of $\phi_1 = 0$, then the second satellite will be placed at $\phi_2 = \frac{2\pi}{N}$, the third at $\phi_3 = \frac{4\pi}{N}$, and so on. Thus, satellite i starts with a phase of

$$\phi_i = \frac{2(i-1)\pi}{N},\ i = 1, \ldots, N.$$

In this scenario, each satellite has to advance its phase angle by a total of $\frac{2\pi}{N} - \frac{2R}{r_1}$ radians only instead of a complete phase shift of 2π. As in the previous $N = 1$ setting, each satellite will transfer from a circular orbit to a transit elliptic orbit and back to the original circular orbit such that the overall phase shift after this transfer is given by $\frac{2R}{r_1}$. Hence, the size of the transit orbit is identical to that computed above in Eq. (3) with exactly the same amount of velocity change as in Eq. (4). In the present case, however, instead of each satellite having to perform $\frac{\pi r_1}{R}$ transfers it now has to do only:

$$\frac{\pi r_1}{NR} \text{ transfers.}$$

Hence, we see that the total energy expended by each satellite under this design is given by

$$\Delta v_{\mathrm{s/c}} = \frac{\pi r_1}{NR} \Delta v, \tag{6}$$

where Δv is given by Eq. (4). Hence, each satellite will have its required energy be reduced (from that required for a single satellite system) to cover its portion of the orbit that spans a radial angular amount of $\frac{2\pi}{N}$.

For the same example used above, each satellite only requires a total velocity change of 49.385 m/s if one employs a total of 100 satellites distributed with a phase shift of 0.0628 radians, or 3.6 degrees. This corresponds to an arc length of 452.4 km. However, note that the total velocity change for the overall system is still given by the expression in Eq. (5). This is because we have to multiply the quantity in Eq. (6) by N to determine the total energy required by the entire surveillance system to cover the orbit. Hence, we see that *the main benefit one obtains by using more than a single satellite is that the energy requirement* ***per satellite*** *can be significantly reduced.*

Time to Mission Completion. The second benefit in using multiple satellites as opposed to a single satellite is that the time to mission completion is reduced as a function of N. For $N = N_{\max}$, the time to mission completion $T_{N=N_{\max}}$ is zero. That is because we have located satellites evenly on the orbit such that each point on the orbit is within the sensory range of some satellite.

On the other end of the spectrum, if we use $N = 1$ satellites, the time to mission completion is given by the sum of all orbit transfer times and the total dwell time at each new phase angle. The time taken to transfer to the transit orbit and back is given by T_t in Eq. (2). One has to add the dwell time τ_s to T_t to obtain the time it takes to sense and effect an overall phase shift. Let $T_{\text{single}} = \tau_s + T_t$ denote the time taken from the beginning of one sensing phase to the beginning of the following one. For $N = 1$, the total time taken to complete the surveillance problem is then given by

$$T_{N=1} = \frac{\pi r_1}{R} T_{\text{single}}. \tag{7}$$

For example, assume that $\tau_s = 60$ seconds. Using the same orbital and sensor parameters as above, we find that the total mission time is 16 days, 3 hours and 29 minutes.

However, for a generic number of satellites N between 1 and $N_{\max}$ we see that the total surveillance time is given by

$$T_N = \frac{\pi r_1}{NR} T_{\text{single}}. \tag{8}$$

For the above example, but with $N = 100$, this means that we have reduced the total surveillance time from 16 days, 3 hours and 29 minutes by a factor of 100, or for a total of $T_{N=100} = 3.87$ hours, or 3 hours and 52.5 minutes.

Table 1: Summary of results for $K = 1$ (LEO)

Number of Satellites	Mission Time
$N = 1$	16d:3h:29m
$1 < N = 100 < N_{\max}$	3h:53m
$N = N_{\max} = 227$	0

Number of Satellites	Energy Usage per sat.
$N = 1$	4,938.5 m/s
$1 < N = 100 < N_{\max}$	49.385 m/s
$N = N_{\max} = 227$	0

Number of Satellites	Total System Energy Usage
$N = 1$	4,938.5 m/s
$1 < N = 100 < N_{\max}$	4,938.5 m/s
$N = N_{\max} = 227$	0

Summary. The results for a single orbit stage are summarized in Table 1. We note that as the number of satellites increase the mission time decreases (in fact inversely proportional to N as shown in Figure 1) as well as the fuel usage per satellite. Note, however, that the total system fuel usage is the same for all $N < N_{\max}$, but drops to zero for the case with $N = N_{\max}$.

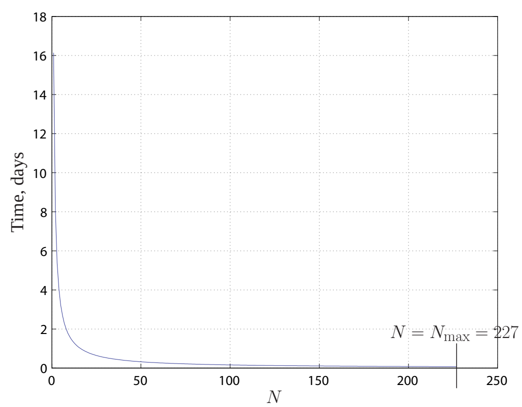

Figure 1: Time to mission completion versus number of satellites in surveillance constellation.

GEO Case with $K = 1$. The results for the $K = 1$ case for a $35,800$km Geostationary orbit (with $r_1 = 42,200$km) are summarized in Table 2.

Surveying a Shell Around Earth. Looking at the three-dimensional picture, the solution proposed above, regardless of N, will survey a tube centered about an orbit of nominal radius equal to $r_l + R$ for some inclination $\imath$. In the above, we have not considered J_2 effects. While J_2 will generally affect the trajectory. The most important effect J_2 will have on the surveillance system is that the nominal (inclined) orbital plane will precess relative to inertial space and the constellation will scan across the globe at a constant rate, effectively returning to its initial orbit after one nodal period. The precession rate of the orbit plane is given by

$$\dot{\Omega} = -\frac{3}{2}\frac{R_o^2 J_2}{r_1^2}\sqrt{\frac{\mu}{r_1^3}}\cos\imath, \tag{9}$$

where $R_o = 6378.14$ km is the Earth's radius, $J_2 = 0.00108263$ is the second zonal harmonic of the Earth, $\mu = 3.986005 \times 10^5\ \mathrm{km}^3/\mathrm{s}^2$ is the Earth's gravitational constant, and $\imath$ is the inclination. The precession

Table 2: Summary of results for $K = 1$ (GEO)

Number of Satellites	Mission Time
$N = 1$	3yr:230d:19h
$1 < N = 100 < N_{\max}$	13d:6h:11m
$N = N_{\max} = 1326$	0
Number of Satellites	**Energy Usage per sat.**
$N = 1$	2,047.4 m/s
$1 < N = 100 < N_{\max}$	20.474 m/s
$N = N_{\max} = 1326$	0
Number of Satellites	**Total System Energy Usage**
$N = 1$	2,047.4 m/s
$1 < N = 100 < N_{\max}$	2,047.4 m/s
$N = N_{\max} = 1326$	0

period of the node is given by $\frac{2\pi}{\dot{\Omega}}$. For an 800-km altitude orbit inclined at 45 degrees to the equator, the precession period is 77 days. One problem with this design is that a target satellite on a circular orbit of the same inclination will precess along with our surveillance system and will never be detected. One way to remedy this issue is to perform an inclination maneuver after every precession period, which will affect the precession rate, and which allows for "catching up" with a target satellite.

3.2. Multiple Orbit Stage Case ($K > 1$)

We now consider the general case where $K > 1$. The sensory domain covers from r_l to r_u with radial range of $2KR$. Similar to the $K = 1$ case, there are also two extreme solutions one may first consider. The zero fuel solution and the K-satellite solution.

Zero-Fuel Solution. The zero fuel solution requires the distribution of $N_{j,\max}$ satellites evenly in the radial direction for each circular orbit j starting with the orbit $j = 1$ with $r_1 = r_l + R$ to the K^{th} orbit whose size is $r_2 = r_l + 2R(K-1) = r_u - R$. For example, using the same orbital and sensor parameters as the single orbit case and set $K = 5$. The radii of the five orbits are 7200 km, 7400 km, 7600 km, 7800 km and 8000 km. The number of satellites for a zero fuel surveillance constellation is given by

$$\begin{aligned}\sum_{j=1}^{K} N_{j,\max} &= \lceil \pi \frac{7200}{100} \rceil + \lceil \pi \frac{7400}{100} \rceil + \lceil \pi \frac{7600}{100} \rceil + \lceil \pi \frac{7800}{100} \rceil + \lceil \pi \frac{8000}{100} \rceil \\ &= 227 + 233 + 239 + 246 + 252 = 1197 \text{ satellites.}\end{aligned}$$

Clearly, this zero-fuel solution requires a very large number of satellites.

K-Satellite Solution. Assume that we have a single satellite in each orbital stage. We can initialize the satellites such that their original orbit radii are from r_1 to r_2 with, without any loss of generality, the same phases $\phi_0 = 0$. The maneuver strategy is exactly the same as that of the single stage $K = 1$ orbit case.

For the above example with five orbits, the radii of the transit circular orbits are given by Equation (3) with different value of r_1 from 7200 km to 8000 km. Therefore, $r_{t1} = 7242$ km, $r_{t2} = 7442$ km, $r_{t3} = 7642$ km, $r_{t4} = 7842$ km and $r_{t5} = 8042$ km. The velocity change of each satellite given by Equation (5) is $\Delta v_{\text{tot},1} = 4938.5$ m/s, $\Delta v_{\text{tot},2} = 4871.9$ m/s, $\Delta v_{\text{tot},3} = 4807.9$ m/s, $\Delta v_{\text{tot},4} = 4746.4$ m/s, and $\Delta v_{\text{tot},5} = 4687.1$ m/s. Thus, the overall system energy usage is $\sum_{j=1}^{K} \Delta v_{\text{tot,j}} = 4938.5 + 4871.9 + 4807.9 + 4746.4 + 4687.1 = 24052$ m/s.

Non-Zero, Multiple Satellites Solution. Following the same procedure in the single orbit case, we can come up with a design solution for any N_j between 1 and $N_{j,\max}$, where $j = 1, 2, \cdots, K$. The energy

Table 3: Summary of results for $K = 5$ (LEO)

Number of Satellites	Mission Time
$N = 5$	20d:23h:19m
$1 < N = 500 < N_{\max}$	5h:3m
$N = N_{\max} = 1197$	0
Number of Satellites	**Energy Usage per sat.**
$N = 5$	$\{$4,938.5, ..., 4,687.1$\}$ m/s
$1 < N = 500 < N_{\max}$	$\{$49.385, ..., 46.871$\}$ m/s
$N = N_{\max} = 1197$	0
Number of Satellites	**Total System Energy Usage**
$N = 5$	24,052 m/s
$1 < N = 500 < N_{\max}$	24,052 m/s
$N = N_{\max} = 1197$	0

expended by a single satellites is still given by Equation (6) with different values of r_1 and Δv_j. For example, consider a case where the number of satellites in each orbit is $N = 100$. Hence, the energy expended by each satellite is given by $\Delta v_{\mathrm{s/c},1} = 49.385$ m/s for satellites in the first orbit, $\Delta v_{\mathrm{s/c},2} = 48.719$ m/s for satellites in the second orbit, $\Delta v_{\mathrm{s/c},3} = 48.079$ m/s for satellites in the third orbit, $\Delta v_{\mathrm{s/c},4} = 47.464$ m/s for satellites in the fourth orbit, and $\Delta v_{\mathrm{s/c},5} = 46.871$ m/s for satellites in the fifth orbit. The total amount of energy consumption of the system is still 24052 m/s.

Time to Mission Completion. For the K satellites solution and $N = 1$ satellite on each orbit, the time taken to complete the surveillance problem is given by Equation (7) with different values of r_1 and T_{single}. They are respectively, 16 days, 3 hours and 29 minutes; 17 days, 6 hours and 45 minutes; 18 days, 11 hours and 8 minutes; 19 days, 16 hours and 39 minutes; and 20 days, 23 hours and 19 minutes. The time to mission completion is given by the maximum time spent on all orbits, that is, 20 days, 23 hours and 19 minutes.

However, for the non-zero fuel, multiple satellites solution, we see that the total surveillance time is given by Equation (8) with different values of r_1 and T_{single}. For the above example, with $N = 100$, the surveillance time is reduced to $T_{1,N=100} = 3.88$ hours, $T_{2,N=100} = 4.15$ hours, $T_{3,N=100} = 4.44$ hours, $T_{4,N=100} = 4.73$ hours, and $T_{5,N=100} = 5.04$ hours. Therefore, the time for mission completion is the time taken by the satellite on the 5^{th} orbit which equals to 5.04 hours.

Summary. The results for a K orbit stage are summarized in Table 3.

GEO Case for $K > 1$. The results for the $K = 5$ case for a $35,800$km Geostationary orbit are summarized in Table 4.

4. CONCLUSION

In this paper, we designed a space-based surveillance system. A basic orbit transfer between circular and elliptic orbits is introduced as a basic maneuver that allows the surveillance system completely scan an orbit. We investigate two extreme cases one with zero-fuel consumption with maximal number of satellites used in the system, and one with a single satellite but that consumes maximum number of fuel per satellite. The relations between three design variables (number of satellites used, fuel-usage and time to mission completion) are also described.

Table 4: Summary of results for $K = 5$ (GEO)

Number of Satellites	Mission Time
$N = 5$	3yr:294d:10h
$1 < N = 500 < N_{\max}$	13d:21h:28m
$N = N_{\max} = 6694$	0
Number of Satellites	Energy Usage per sat.
$N = 5$	{2,047.4, ..., 2,028.3} m/s
$1 < N = 500 < N_{\max}$	{20.474, ..., 20.283} m/s
$N = N_{\max} = 6694$	0
Number of Satellites	Total System Energy Usage
$N = 5$	10,189 m/s
$1 < N = 500 < N_{\max}$	10,189 m/s
$N = N_{\max} = 6694$	0

References

[1] J. Baillieul and P. J. Antsaklis, "Control and Communication Challenges in Networked Real-time Systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 9 – 28, January 2007.

[2] C. G. Cassandras and W. Li, "Sensor Networks and Cooperative Control," *European Journal of Control*, vol. 11, no. 4 – 5, pp. 436 – 463, 2005.

[3] Z. Drezner, *Facility Location: A Survey of Applications and Methods*. New York , NY: Springer, 1995.

[4] A. Okabe and A. Suzuki, "Locational optimization problems solved through Voronoi diagrams," *European Journal of Operational Research*, vol. 98, no. 3, pp. 445 – 456, 1997.

[5] Q. Du, V. Faber, and M. Gunzburger, "Centroidal Voronoi tessellations: Applications and algorithms," *SIAM Review*, vol. 41, no. 4, pp. 637 – 676, 1999.

[6] J. Cortés, S. Martínez, T. Karatus, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243 – 255, 2004.

[7] I. Hussein and D. Stipanović, "Effective Coverage Control using Dynamic Sensor Networks," *2006 IEEE Conference on Decision and Control*, pp. 2747 – 2752, December 2006. [Online]. Available: http://users.wpi.edu/ ihussein/Publications/HuSt-CDC-06.pdf

[8] ——, "Effective Coverage Control for Mobile Sensor Networks with Guaranteed Collision Avoidance," *IEEE Transactions on Control Systems Technology, Special Issue on Multi-Vehicle Systems Cooperative Control with Applications*, vol. 15, no. 4, pp. 642 – 657, 2007.

[9] ——, "Effective Coverage Control using Dynamic Sensor Networks with Flocking and Guaranteed Collision Avoidance," *2007 American Control Conference*, pp. 3420 – 3425, July 2007. [Online]. Available: http://users.wpi.edu/ ihussein/Publications/HuSt-ACC-07.pdf

[10] I. I. Hussein, "A Kalman-filter based control strategy for dynamic coverage control," *Proceedings of the American Control Conference*, pp. 3271–3276, 2007.

[11] Y. Wang and I. I. Hussein, "Awareness Coverage Control Over Large Scale Domains with Intermittent Communications," *2008 American Control Conference*, 2008.

[12] Y. Wang, I. I. Hussein, and R. S. Erwin, "Awareness-Based Decision Making for Search and Tracking," *2008 American Control Conference*, 2008.

[13] M. Richharia, P. H. Hansel, P. W. Bousquet, and M. O'Donnell, "A Feasibility Study of a Mobile Communication Network using a Constellation of Low Earth Orbit Satellites," *Global Telecommunications Conference and Exhibition. 'Communications Technology for the 1990s and Beyond'. GLOBECOM '89*, vol. 2, pp. 773 – 777, Nov 1989.

[14] F. Vatalaro, G. E. Corazza, C. Caini, and C. Ferrarelli, "Analysis of LEO, MEO, and GEO Global Mobile Satellite Systems in the Presence of Interference and Fading," *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 2, pp. 291 – 300, February 1995.

[15] T. Taleb, U. Dharmaratna, N. Kato, and Y. Nemoto, "A Geographical Location Based Satellite Selection Scheme for a Novel Constellation Composed of Quasi-Geostationary Satellites," *IEEE International Conference on Communications*, vol. 3, pp. 1464 – 1468, May 2005.

[16] A. Melin, R. S. Erwin, , and V. Chellaboina, "Optimal Dynamic Assignment for Low Earth Orbit Satellite Constellations," *45th IEEE Conference on Decision and Control*, pp. 3204 – 3209, December 2006.