Comparision of Different Algoritms of Orbit Determination During One Peneration a Radar

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The primary approaches used for orbit determination on the basis of a single pass through a radar are recursive (Kalman filter) and joint (least squares). If the stochastic characteristics of the errors are not completely known or the measurement errors are time correlated these techniques do not provide a guaranteed evaluation of the errors of the generated estimates. This is a significant limitation. This paper presents a comparative analysis (based on computer simulation) for the procedures based on the Guarantee method and traditional recurrent and joint processing techniques.

1. INTRODUCTION

The problem of orbit determination based on one pass through the radar field of view is not a new one. Extensive research in this area has been carried out in the USA and Russia since late 50ies when these countries started the development of BMD and Early Warning systems. In Russia these investigations got additional stimulation in early 60ies after the decision to create the Space Surveillance System whose primary task is the maintenance of the satellite catalog. These problems were a focus of research interest up to middle 1970's at which time the appropriate techniques and software had been implemented for all radars. Then for more than 20 years no new research papers arrived on this subject. This produced an impression that all the problems of track determination based on one pass had been solved and there was no area for further research.

In late 1990's the interest in this problem arose again in relation to the following. The American specialists estimate the number of orbital objects with size greater than 1-2 cm as 100 000. Collision of operational spacecraft with any of these objects may have catastrophic results. Thus, for prevention of hazardous approaches and collisions with valuable spacecraft the existing satellite catalog should be extended at least ten times. This is a very difficult scientific and engineering task. One of the issues is the development of data fusion procedures and the software capable of maintaining such a huge catalog close to real time. The number of daily processed measurements (of all types, radar and optical) for such a system may constitute millions, thus exceeding the existing amount of measurements more than ten times. Since it is known that when we have ten times more satellites and measurements the computer effort required for correlation of measurements will be two orders of magnitude greater. This can create problems for processing data close to real time even for modern computers. Preliminary "compression" of data for one pass through the field of view of a sensor can significantly reduce the requirements to computers. This compression will take place in case all the single measurements of the sensor are replaced by the orbit determined on their basis. The single measurement here means the radar parameters (range, azimuth, elevation, and in some cases range rate) measured by a single pulse.

Two types of techniques have been traditionally used for processing single measurements recurrent and joint processing. Recurrent procedures convenient for real time processing usually are based on Kalman's filtering and its modifications [1]. Less convenient joint processing techniques basically use least squares [2] or least modules [3] methods. When the errors of single measurements are time correlated and for all cases when the statistical characteristics of single measurements are not known completely these techniques do not provide the guaranteed evaluation of the errors of generated estimates. This limitation can be avoided when we use the method with the guarantee approach when the guarantee ranges of orbital parameters are obtained on the basis of guarantee ranges of the parameters of single measurements [4]. The guarantee approach has one more remarkable feature. With certain limitations on the distribution of the errors of the measurements and a large enough number of these measurements this approach leads to more accurate estimates than the traditional techniques mentioned above.

The general procedure based on the guarantee approach for several limitations is a linear programming procedure with the amount of computation significantly greater than the least squares procedure. This resulted in a lack of interest in such procedures in 60-70's. However, now the situation is different. The capacity of the computers is many and many times greater. Modern sensors have small and rather stable errors. At the same time the new task of accurate estimation of collision risk for important space vehicles requires higher accuracy for the determination of the position of the space vehicle. Thus, we have reasons to look again at this promising method.

The fundamentals for the algorithm based on the techniques mentioned above has been presented previously [5]. The subject of this work is the comparative analysis of the accuracy provided by different methods using mathematical simulation. We assume that the radar measures the local spherical coordinates of a satellite (range d, azimuth α and elevation angle β) within the zone limited in range (not more than 3000-7000 km) and elevation angle (not more than 40°-60°) with errors of single measurements of the order of tens of meters in range and several angular minutes for angular coordinates.

2. TRADITIONAL METHODS

The recurrent algorithm of orbit determination by radar measurements uses the previous estimate of the orbital parameters $\hat{\mathbf{c}}_{k-1}$ and the calculated correlation matrix of errors $\mathbf{P}_{k-1}$, referred to the time t_{k-1}, and the current scalar measurement u_k, acquired at the time $t_k \geq t_{k-1}$, to calculate the updated estimate $\hat{\mathbf{c}}_k$ and calculated correlation matrix of errors $\mathbf{P}_k$:

$$\hat{\mathbf{c}}_k = \hat{\mathbf{c}}_{k|k-1} + w_k \mathbf{P}_k \mathbf{h}_k (u_k - \mathrm{h}_k(\hat{\mathbf{c}}_{k|k-1})) \tag{1}$$
$$\tilde{w}_k = w_k/(1 + w_k \mathbf{h}_k' \mathbf{P}_{k|k-1} \mathbf{h}_k) \quad \mathbf{P}_k = \mathbf{P}_{k|k-1} - \tilde{w}_k (\mathbf{P}_{k|k-1}\mathbf{h}_k)(\mathbf{P}_{k|k-1}\mathbf{h}_k)'$$

where vectors 6×1 and matrices 6×6 are denoted by bold (small and capital respectively) letters, $'$ – transposition sign; $u_k = u(t_k)$ scalar parameter of the measurement for the time

t_k (range, azimuth or elevation angle); $\mathbf{c}_k = \mathbf{c}(t_k)$ orbital parameters for the time t_k – state vector $(x_k, y_k, z_k, \dot{x}_k, \dot{y}_k, \dot{z}_k)$ in local rectangular coordinates $x, y, z,$ related to the spherical ones d, α, β by relationships $x = -d \sin\alpha \cos\beta$ $y = d \sin\beta$ $z = d \cos\alpha \cos\beta$; $\mathrm{h}_k(\mathbf{c}_k)$ and $\mathbf{h}_k$ – relationships and the vector $6{\times}1$ of their partial derivatives with respect to $\mathbf{c}_k$; $\mathbf{c}_k = \mathbf{f}_k(\mathbf{c}_{k-1})$ functional operator of propagating orbital parameters from the time t_{k-1} to the time t_k; $\mathbf{F}_k(\mathbf{c}_{k-1})$ is the matrix operator of propagating the variations $\delta\mathbf{c}$ of orbital parameters $\mathbf{c}$ from the time t_{k-1} to the time t_k; $\mathbf{P}_{k|k-1}$ – operator of propagating the correlation matrix $\mathbf{P}_{k-1}$ from the time t_{k-1} to the time t_k, $w_k = 1/\sigma_{u_k}^2$ – weight characteristic of the measurement u_k.

For the propagation of the orbital parameters the differential equations of motion in the local rectangular coordinate frame x, y, z are solved using numerical Runge-Kutta methods of the 4th order. Only the second zonal harmonic of the Earth gravitation potential is taken into account. Propagation of the variations of these parameters is performed under the assumption of a linear in time change of the coordinates x, y, z. The matrix $\mathbf{F}_k$ in this case have dimensions $6{\times}6$ with the following not zero elements: $f_{i,i}{=}1$ for $i = 1, 2, ..., 6,$ $f_{i,i+3}{=}\tau_k$ for $i = 1, 2, 3,$ where $\tau_k{=}t_k{-}t_{k-1}$. Propagation of the correlation matrix $\mathbf{P}_{k-1}$ is performed by the formula $\mathbf{P}_{k|k-1} = \mathbf{F}_k{\cdot}\mathbf{P}_{k-1}{\cdot}\mathbf{F}_k{+}\mathbf{\Gamma}_k$, where in the matrix $\mathbf{\Gamma}_k$ only the diagonal elements corresponding to the velocity components of the vector $\mathbf{c}$ are not zero [6]. First using (1) we update the previous estimate by $u_k{=}d_k$, then the obtained estimate is updated by $u_k{=}\alpha_k$ and then the obtained estimate is updated by $u_k{=}\beta_k$.

Regarding the least squares method we search for $\min\limits_{\mathbf{c}} \Psi(\mathbf{c}) = \Psi(\mathbf{c}_{min})$ of the function $\Psi(\mathbf{c})$ of the shape

$$\Psi(\mathbf{c}) = \sum_{k=1}^{n} \left(\frac{1}{\sigma_{d_k}^2}(d_k - d_k(\mathbf{c}))^2 + \frac{1}{\sigma_{\alpha_k}^2}(\alpha_k - \alpha_k(\mathbf{c}))^2 + \frac{1}{\sigma_{\beta_k}^2}(\beta_k - \beta_k(\mathbf{c}))^2 \right), \tag{2}$$

where $\mathbf{x}_k{=}(d_k, \alpha_k, \beta_k)$ – k-th measurement $(k = 1, 2, ..., n)$; t_k – time reference of the k-th measurement; $\sigma_{d_k}, \sigma_{\alpha_k}, \sigma_{\beta_k}$ – RMS of the errors of d_k, α_k, β_k; $\mathbf{h}_k(\mathbf{c}) = (d_k(\mathbf{c}), \alpha_k(\mathbf{c}), \beta_k(\mathbf{c}))$ – the values of the parameters of the k-th measurement, calculated by the vector $\mathbf{c} = \mathbf{c}(\bar{t})$ of the orbital parameters referred to the time $\bar{t}$.

The minimum of $\Psi(\mathbf{c})$ is found by iterations and for the initial approximation $\mathbf{c}_0$ we take the estimate of the parameters obtained on the basis of measurements $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n$ by recurrent procedure. It is not an easy task to find a rather simple and computationally efficient technique for the minimization of $\Psi(\mathbf{c})$, which will have a guaranteed and quick convergence. However the limitation for the elevation angle of the radar's field of view leads to the result that the major updating effect for the least squares (within the tracking time) is the reduction of the errors of the velocity components $\dot{\mathbf{u}}{=}(\dot{x}, \dot{y}, \dot{z})$ of the state vector $\mathbf{c}$. Coordinate parameters $\mathbf{u}{=}(x, y, z)$ of the vector $\mathbf{c}$ in this case are almost not updated. Thus it is expedient to perform the minimization of $\Psi(\mathbf{c}){=}\Psi(\mathbf{u}, \dot{\mathbf{u}}){=}\psi(\dot{\mathbf{u}})$ only with respect to the vector $\dot{\mathbf{u}}$. Taking into account the limits for the errors of the measurements mentioned in the introduction and the selected initial approximation the function $\mathbf{x}(\dot{\mathbf{u}})$ is close to linear. Thus the iterative process converges very quickly. In fact for all the cases determining the minimum of $\dot{\mathbf{u}}_{min}$ the function $\psi(\dot{\mathbf{u}})$ requires only one iteration. For the minimum point $\dot{\mathbf{u}}_{min}$ of the function $\psi(\dot{\mathbf{u}})$ in the case when it is reached by one iteration, Newton's formula is used $\dot{\mathbf{u}}_{min}{=}\dot{\mathbf{u}}_0{-}\left(\frac{\partial^2\psi}{\partial\dot{\mathbf{u}}^2}(\mathbf{c}_0)\right)^{-1}{\cdot}\frac{\partial\psi}{\partial\dot{\mathbf{u}}}(\mathbf{c}_0)$, where the first and the second derivatives of the function $\psi(\dot{\mathbf{u}})$ with respect to parameters $\dot{x}, \dot{y}, \dot{z}$ are calculated using a finite differences technique.

3. GUARANTEE APPROACH

The basic assumption of the guarantee approach in the non-linear problems is the low level of the errors of the measurements and the known upper limits for them. The term guarantee in this case means that the procedure provides not only the calculated values of the orbital parameters, but the maximum possible values of their errors as well. The essence of the approach is as follows. Assume for the times t_k $(t_k \leq t_{k+1};\ k = 1, 2, ...n)$ we acquire the measurements u_k of certain functions $h_k(\mathbf{c})$ of the m-dimensional vector of parameters $\mathbf{c}$ $(m<n)$, and the errors of the measurements $\delta u_k = u_k - h_k(\mathbf{c})$ have known upper limits $\delta_{k,max}$. The estimate $\bar{\mathbf{c}}_n$ of the parameters $\mathbf{c}$ and the vector $\delta\bar{\mathbf{c}}_{n,max}$ of the maximum errors of the components of this estimate provided by the guarantee approach have the following geometrical interpretation. The limits for the errors of the measurements in the m-dimensional space of parameters $\mathbf{c} = (\mathbf{c_1}, \mathbf{c_2}, ..., \mathbf{c_m})$ determine a domain $\mathbf{D}_n = \bigcap_{k=1}^{n} \{u_k - \delta_{k,max} \leq h_k(\mathbf{c}) \leq u_k + \delta_{k,max}\}$ of possible values of $\mathbf{c}$. We project this domain on the coordinate axes of the components of the vector $\mathbf{c}$ and among the projected points for each axis we find the most right $\mathbf{c}_{n,r}$ and the most left $\mathbf{c}_{n,l}$. They define the boundaries (maximum and minimum values) for the changes of each of the components of $\mathbf{c}$. The estimate $\bar{\mathbf{c}}_n$ of the orbital parameters and the maximum errors $\delta\bar{\mathbf{c}}_{n,max}$ of this estimate are determined as $\bar{\mathbf{c}}_n = 0.5 \cdot (\mathbf{c}_{n,r} + \mathbf{c}_{n,l})$ $\delta\bar{\mathbf{c}}_{n,max} = 0.5 \cdot (\mathbf{c}_{n,r} - \mathbf{c}_{n,l})$. If the measured parameters are linearly connected with the determined parameter $\mathbf{c}$, i.e. $h_k(\mathbf{c}) = \mathbf{h}'_k \cdot \mathbf{c}$, where the components $6{\times}1$ of the vector $\mathbf{h}_k$ do not depend on $\mathbf{c}$, this problem for the certain shape of the minimized function can be formulated as a standard linear programming problem. The solution is known, but the procedure is rather sophisticated and we will not describe it here.

For the evaluation of the accuracy characteristics of the parameters provided by this method it is useful to consider a model case: we measure a scalar parameter c, with $h = 1$ and $\delta_{k,max} = \delta_{max}$. In this one-dimensional model $\mathbf{D}_n = \bigcap_{k=1}^{n} \{[\,u_k - \delta_{max},\ u_k + \delta_{max}\,]\} = [\,\max_k u_k - \delta_{max},\ \min_k u_k + \delta_{max}\,]$ $\bar{c}_n = 0.5 \cdot (\max_k u_k + \min_k u_k)$ $\delta\bar{c}_{n,max} = \delta_{max} - 0.5 \cdot (\max_k u_k - \min_k u_k)$, where $[\,a,\ b\,]$ - denotes a segment with the left end a and the right end b. The estimate $\bar{c}_n$ has several interesting features, which to a certain extent are retained in the multi-dimensional case:

1. The estimate $\bar{c}_n$ does not depend on δ_{max}, that means that is not critical to the knowledge of the characteristics of the errors – in this case to the accuracy of the knowledge of δ_{max}.
2. In case we know exactly δ_{max} the value $\delta\bar{c}_{n,max}$ is a correct upper estimate for the error of the estimate $\bar{c}_n$, for any correlations of the errors of different measurements. Correctness in this case means the following. On one side the true errors of the estimate $\bar{a}_n$ for any n can not exceed $\delta\bar{a}_{n,max}$. On the other – the change of $\delta\bar{c}_{n,max}$ with the increase of n corresponds to the change of the correlation characteristics of the errors of the measurements. If the correlation interval of the errors is limited the value $\delta\bar{c}_{n,max}$ with the increase of n can be reduced infinitely. If this is not the case, i.e. the measurements includes certain systematic error, the value $\delta\bar{c}_{n,max}$ for any n will have the lower limit of this error.
3. For the uncorrelated errors of the measurements the estimate $\bar{c}_n$ regarding the accuracy is not lower and sometimes can be a significantly more accurate (!) of the estimate $\hat{c}_n = (u_1 + ... + u_n/n$ of the least squares. Thus, for example, for the frequently practical case of uniform distribution of the measurements within the interval $(-\delta_{max}, \delta_{max})$ the estimates $\hat{c}_n$ and $\bar{c}_n$ are non-biased, and the RMS of their errors are equal $\sigma_{\bar{c}_n} \simeq 1.4 \cdot \delta_{max}/n$ and $\sigma_{\hat{c}_n} \simeq 0.58 \cdot \delta_{max}/\sqrt{n}$ [7]. We can see that the estimate $\bar{a}_n$ is $\approx 0.4\sqrt{n}$ times more accurate. These features provides the advantages for the guarantee approach with respect to the least squares technique and makes this approach attractive for solving practical problems of radar operation. Especially this refers to rather precise

and stable radars where the abnormal measurements (in case they appear) are identified during the preliminary processing and do not enter the orbit determination algorithm.

Let us consider the problem in more detail. Assume the radar for the times t_k $(k = 1, 2, ..., n)$ measure the range d, azimuth α and elevation angle β in the local spherical coordinate frame, and the errors of the measurements do not exceed respectively the values $\delta_{d,max}$, $\delta_{\alpha,max}$, $\delta_{\beta,max}$. The task is to determine for the time $\bar{t}{=}0.5{\cdot}(t_1{+}t_n)$ the six-dimensional vector of orbital parameters $\mathbf{c} = (d,\, \alpha,\, \beta,\, \dot{d},\, \dot{\alpha},\, \dot{\beta})$ in this coordinate frame and its maximum errors. The following procedure is suggested. Divide all the measurements into $n/2$ groups. The first group includes the measurements performed at t_1 and $t_{0.5n}$, the second – the measurements performed at t_2 and $t_{0.5n+1}$, etc. For each k-th group $(k = 1, 2, .., 0.5{\cdot}n)$ by two position vectors $\mathbf{u}_k = (d_k,\, \alpha_k,\, \beta_k)$ and $\mathbf{u}_{0.5n+k} = (d_{0.5n+k},\, \alpha_{0.5n+k},\, \beta_{0.5n+k})$ in the local spherical coordinate frame we determine the six-dimensional vector of orbital parameters $\check{\mathbf{c}}_k = (\check{d},\, \check{\alpha},\, \check{\beta},\, \check{\dot{d}},\, \check{\dot{\alpha}},\, \check{\dot{\beta}})_k$ in the same coordinate frame for the time $\bar{t}$. Instead of the laborious operation of determination of the domain $\mathbf{D}_n$ and its projections on the coordinate axes of the phase vector we suggest to project on these axes the domain $\mathbf{D}_k$, corresponding to the k-th group, and further for each axis look for the intersections of these projections in k. The domain $\mathbf{D}_k$ is approximated by the six-dimensional parallelepiped with the center in the point $\check{\mathbf{c}}_k$ and 64 vertexes determined by the formula $\check{\mathbf{c}}_k \pm \delta_{d,max}{\cdot}\mathbf{j}_1 \pm \delta_{\alpha,max}{\cdot}\mathbf{j}_2 \pm \delta_{\beta,max}{\cdot}\mathbf{j}_3 \pm \delta_{d,max}{\cdot}\mathbf{j}_4 \pm \delta_{\alpha,max}{\cdot}\mathbf{j}_5 \pm \delta_{\beta,max}{\cdot}\mathbf{j}_6$, where $\mathbf{j}_1$, $\mathbf{j}_2$, $\mathbf{j}_3$, $\mathbf{j}_4$, $\mathbf{j}_5$, $\mathbf{j}_6$ – the lines of the matrix of partial derivatives of the functional transformation $\mathbf{c}_k{=}\mathbf{f}_k(\mathbf{u}_k,\, \mathbf{u}_{0.5n+k})$ in point $\check{\mathbf{c}}_k$. The boundary projections $\mathbf{c}_{k,l}$ and $\mathbf{c}_{k,r}$ of these 64 points of the six-dimensional phase space for each of its axes determine the boundaries of the vector interval $(\mathbf{c}_{k,l},\, \mathbf{c}_{k,r})$ of possible values of all the six orbital parameters constructed using the k-th group of measurements. Having found the common part $(\mathbf{c}_l,\, \mathbf{c}_r) = \bigcap_{k=1}^{n} (\mathbf{c}_{k,l},\, \mathbf{c}_{k,r})$ of these intervals for all groups of measurements, we find the estimate of orbital parameters and its maximum errors.

4. RESULTS OF MODELING

The modeling was performed with the following initial conditions:

1. *coordinates of the radar* (latitude, longitude, altitude): $\lambda = 0$, $\varphi = 0.7$, $h = 0$;
2. *errors of the measurements*: RMS of the uncorrelated errors of the measurements of parameters $\sigma_d = 0.05km$, $\sigma_\alpha = 0.001$, $\sigma_\beta = 0.001$; there are no correlated and abnormal errors; distribution of the errors is either uniform or normal;
3. *radar field of view* regarding the elevation angle is limited by minimum of 5° and maximum of 60°.
4. *satellite trajectories.* Two orbits were selected both with an inclination of 1 radian, one with average altitude 800 km, the second one – 1500 km. For each orbit two trajectories within the field of view were selected. The first one - with "assault" aspect angle $(|\dot{\beta}|{\gg}|\dot{\alpha}|)$, the second one - with "transiting" one $(|\dot{\alpha}|{\gg}|\dot{\beta}|)$.
5. *distribution of the times of measurements.* We consider that the first measurement is aquired when the satellite enters the sector (for the considered trajectories this happens for the minimum elevation angle). Further the measurements are performed with a 5 sec. interval until the satellite leaves the sector (regarding elevation angle).
6. *the number of realizations for modeling* – $n_r = 100$.
7. comparison of the empirical (averaged over realizations) and theoretical (obtained from the Fisher's information matrix) errors of orbit determination by Kalman's filtering and least squares

respectively, is performed by each 10-th measurement, i.e. for the 50, 100, 150 etc. seconds of tracking.

The table 1 present the results of comparing the empirical errors of the estimates of parameters $d, \alpha, \beta, \dot{d}, \dot{\alpha}, \dot{\beta},$ obtained by Kalman's filtering (KF) and least squares method (LSM) between themselves and with theoretical values of these errors (ξ is the ratio of the RMS of the errors provided by KF (LSM) and theoretical RMS). The reference time for the compared values – the middle of the tracking interval $\bar{t}$. The estimate of the orbital parameters calculated by LSM is obtained just for the time $\bar{t}$, and the estimate by KF, is interpolated from t_n to $\bar{t}$. Analysis of the results of modeling leads to the following conclusions:

1. Up to 50 s tracking interval the errors of determination of velocity components by KF and LSM are close (the difference does not exceed 20%) and the empirical values are close to theoretical. When the tracking time increases for all the parameters, except for the radial velocity, the situation is virtually the same. For the radial velocity the LSM provides a more accurate estimate than the KF By the end of the tracking interval (450-750 s for the orbit of the first type) the difference in the accuracy of the estimates by LSM and KF reaches 2-3 times. The source of the effect is the non-linear character of the considered problem.

2. Parameters of the position vector are updated only by KF. Comparing with the theoretical errors of these parameters shows that for the angular components of the position vector the correspondence is satisfactory (the difference does not exceed 15% for the whole tracking uinterval). Regarding the range the situastion is different. For the tracking time more than 300 s, the theoretical errors are essentially lower, and by the end of the tracking the difference reaches 2.7 times. The source of the effect is again the non-linearity of the problem.

In the table 2 we compare the empirical RMS of the errors of the estimates of parameters $d, \alpha, \beta, \dot{d}, \dot{\alpha}, \dot{\beta},$ for the guarantee approach (GA) and the LSM. The time reference for the compared values – the middle of tracking interval $\bar{t}$. The comparing is performed for the maximum tracking interval (maximum number of measurements) for the uniform distribution of the errors of the measurements. The results of comparing GA and LSM presented in the table 11 are somewhat unexpected. For the simplest model example (the scalar measured parameter with no variations within the time interval and uniform distribution of the errors), mentioned in the description of the guarantee approach, the estimate provided by the GA for the number of measurements $n \geq 10$ is about $0.4\sqrt{n}$ times more accurate that the LSM estimate. Thus there was a hope that the GA will have advantages with respect to LSM for our six-dimensional nonlinear problem as well. However, this hope was not justified. The gains were insignificant and for some parameters the LSM estimate is essentially more accurate.

Surely, the effects valid for the one-dimensional problem may not take place for the multidimensional nonlinear case. However, this can be hardly predicted. The most likely situation is that the effects exist for the considered problem, but due to some reasons they were not revealed. We assume the following three reasons:

1. *Not equal accuracy of the measured parameters.* The error of the range in linear measure is about 2 orders of magnitude smaller than the errors of the angles. This significantly not equal accuracy of different measurements is a feature that distinguishes our problem from the model example where all the measurements have equal accuracy and may become a source of some additional effects. To avoid this, the errors of the range were increased to the level of angular errors. For these conditions we repeated the comparing of GA and LSM. The results are

presented in Table 2. We can see that the ratios $\sigma_{\text{ga}}/\sigma_{\text{lsm}}$ for different parameters became more even and the accuracy of GA and LSM estimates became closer. However, no principal change is in place – the LSM estimate remains more (up to 2 times) accurate than the GA one for the majority of the orbital parameters. Thus the unequality of the accuracy of the measurements of different parameters is not the basic reason for the situation.

2. *Non-linearity of the problem.* The relationship between the measured and determined parameters is non-linear. Thus the domain $\mathbf{D}_k$ is not a six-dimensional parallelepiped and such a replacement leads to some errors. However, these errors can be neglected when the errors of the measurements are small. For evaluation of this effect we improved the angular errors to the quality of the errors of the range and again compared the methods. The results are presented in Table 2. Comparison with case, when the errors of the range were increased to the level of angular errors, shows that the figures in both tables are very close, with the errors of the measurements reduced (in the average) by an order of magnitude, however. Thus the linearizatio n errors are probably small and can not be considered the reason for the unexpectedly low efficiency of the suggested guarantee approach algorithm.
3. *The simplifying assumptions accepted in the suggested algorithm.* That is the principal reason. The accepted simplifying assumptions lead to the observed effect. The basic one is the replacement of the real domain $\mathbf{D}$ by the intersection of six-dimensional parallelepipeds $\mathbf{D}_k$, projected to the coordinate axes. If we try to avoid this simplification we will be driven to the linear programming problem where we have to minimize a certain criterion function of estimated parameters with non-equality shape constraints for certain linear functions of these parameters. For solving this task usually a set of well known methods are used. It looks expedient to consider the possibility of using these methods for the considered problem. Now we do not see any unsurmountable difficulties for doing this. However, this is beyond the scope of this work and may become a subject for further research.

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APPENDIX: TABLES

Table 1 Normalized characteristics of the errors ξ
for parameters d, α, β, $\dot{d}$, $\dot{\alpha}$, $\dot{\beta}$ for different number of measurements n

trajectory 1al, normal distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot{d}}$ KF	$\xi_{\dot{d}}$ LSM	$\xi_{\dot{\alpha}}$ KF	$\xi_{\dot{\alpha}}$ LSM	$\xi_{\dot{\beta}}$ KF	$\xi_{\dot{\beta}}$ LSM
10	1.03	1.03	1.01	1.01	0.88	0.88	1.03	0.86	1.11	1.08	0.96	0.94
20	1.00	1.00	1.03	1.03	1.00	1.00	1.21	0.93	0.95	0.93	1.10	1.07
30	0.94	0.94	1.04	1.04	1.01	1.01	1.20	0.94	0.95	0.96	1.18	1.06
40	0.81	0.81	1.04	1.04	1.03	1.03	1.31	1.03	1.01	1.01	1.08	1.00
50	0.87	0.87	1.05	1.05	1.01	1.01	1.29	0.99	1.05	1.05	1.15	1.08
60	0.94	0.94	1.03	1.03	1.11	1.11	1.27	0.92	0.99	0.98	1.04	1.00

trajectory 1al, uniform distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot{d}}$ KF	$\xi_{\dot{d}}$ LSM	$\xi_{\dot{\alpha}}$ KF	$\xi_{\dot{\alpha}}$ LSM	$\xi_{\dot{\beta}}$ KF	$\xi_{\dot{\beta}}$ LSM
10	1.05	1.05	0.99	0.99	0.97	0.97	1.09	0.98	1.05	1.04	1.01	0.97
20	1.02	1.02	1.01	1.01	0.93	0.93	1.36	1.05	1.02	1.01	0.94	0.94
30	1.03	1.03	1.04	1.04	0.92	0.92	1.28	1.02	0.93	0.87	1.03	0.98
40	1.01	1.01	1.01	1.01	1.00	1.00	1.31	1.05	1.06	1.02	1.01	1.00
50	1.02	1.02	0.98	0.98	1.01	1.01	1.41	1.09	0.96	0.94	1.01	0.96
60	1.08	1.08	0.98	0.98	0.96	0.96	1.36	1.02	0.96	0.97	1.06	1.03

trajectory 1ac, normal distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot{d}}$ KF	$\xi_{\dot{d}}$ LSM	$\xi_{\dot{\alpha}}$ KF	$\xi_{\dot{\alpha}}$ LSM	$\xi_{\dot{\beta}}$ KF	$\xi_{\dot{\beta}}$ LSM
10	0.95	0.95	0.94	0.94	1.14	1.14	1.12	0.93	0.89	0.89	0.91	0.92
20	0.95	0.95	1.03	1.03	1.06	1.06	1.07	0.99	1.00	0.86	1.06	1.01
30	0.93	0.93	0.97	0.97	1.06	1.06	1.03	0.95	1.12	1.03	1.07	1.07
40	1.00	1.00	1.00	1.00	1.01	1.01	0.99	0.96	1.01	0.96	1.07	1.07
50	1.02	1.02	1.02	1.02	1.05	1.05	1.02	1.01	1.00	0.96	1.03	1.03
60	1.08	1.08	1.06	1.06	1.04	1.04	1.02	0.98	0.96	0.95	1.05	1.05

trajectory 1ac, uniform distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot{d}}$ KF	$\xi_{\dot{d}}$ LSM	$\xi_{\dot{\alpha}}$ KF	$\xi_{\dot{\alpha}}$ LSM	$\xi_{\dot{\beta}}$ KF	$\xi_{\dot{\beta}}$ LSM
10	1.05	1.05	1.23	1.23	1.04	1.04	1.18	1.02	1.05	0.96	0.96	0.93
20	1.04	1.04	1.17	1.17	1.07	1.07	1.22	1.07	1.14	1.08	1.01	1.00
30	1.06	1.06	1.06	1.06	1.04	1.04	1.18	1.04	1.17	1.09	1.02	1.01
40	1.12	1.12	1.01	1.01	1.00	1.00	1.08	1.01	1.18	1.14	1.05	1.05
50	1.05	1.05	1.03	1.03	1.02	1.02	0.95	0.85	1.02	1.00	1.03	1.02
60	1.04	1.04	1.04	1.04	1.01	1.01	0.93	0.89	1.15	1.13	1.03	1.02

trajectory 2al, normal distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot d}$ KF	$\xi_{\dot d}$ LSM	$\xi_{\dot\alpha}$ KF	$\xi_{\dot\alpha}$ LSM	$\xi_{\dot\beta}$ KF	$\xi_{\dot\beta}$ LSM
10	1.05	1.05	1.01	1.01	1.02	1.02	1.13	1.02	1.06	1.02	1.15	1.13
20	1.01	1.01	1.06	1.06	1.05	1.05	1.25	0.91	1.02	0.98	1.17	1.05
30	1.05	1.05	1.05	1.05	0.94	0.94	1.29	0.90	1.00	0.97	1.14	1.02
40	1.13	1.13	1.07	1.07	0.85	0.85	1.52	0.92	0.93	0.89	1.17	1.70
50	1.08	1.08	1.06	1.06	0.88	0.88	1.67	1.00	1.00	1.00	1.03	0.99
60	1.13	1.13	1.03	1.03	0.92	0.92	1.81	0.97	0.93	0.94	1.16	1.05
70	1.26	1.26	1.03	1.03	0.90	0.90	2.10	1.01	1.03	1.02	1.30	1.19
80	1.41	1.41	1.03	1.03	0.94	0.94	2.33	1.05	0.98	0.98	1.17	1.13
90	1.43	1.43	1.02	1.02	1.01	1.01	2.53	1.00	1.07	1.06	1.38	1.11
100	1.53	1.53	1.03	1.03	1.02	1.02	2.66	0.98	0.91	0.91	1.45	1.25

trajectory 2al, uniform distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot d}$ KF	$\xi_{\dot d}$ LSM	$\xi_{\dot\alpha}$ KF	$\xi_{\dot\alpha}$ LSM	$\xi_{\dot\beta}$ KF	$\xi_{\dot\beta}$ LSM
10	1.13	1.13	0.93	0.93	1.04	1.04	1.25	0.98	0.97	0.94	1.10	1.08
20	0.97	0.97	0.84	0.84	1.04	1.04	1.26	0.96	1.03	1.01	1.11	0.98
30	0.90	0.90	0.94	0.94	1.00	1.00	1.56	1.04	1.06	1.03	1.15	0.91
40	1.04	1.04	0.90	0.90	1.02	1.02	1.53	1.04	0.92	0.93	1.32	1.06
50	1.18	1.18	0.94	0.94	1.02	1.02	1.71	1.09	0.86	0.83	1.24	1.09
60	1.27	1.27	0.91	0.91	0.96	0.96	1.74	1.02	0.97	0.94	1.21	1.13
70	1.28	1.28	0.91	0.91	0.97	0.97	1.83	0.94	1.00	0.98	1.33	1.28
80	1.31	1.31	0.93	0.93	1.01	1.01	2.05	1.03	0.84	0.83	1.24	1.17
90	1.33	1.33	0.94	0.94	1.00	1.00	2.14	1.03	0.90	0.89	1.30	1.27
100	1.32	1.32	0.95	0.95	0.96	0.96	2.16	1.04	0.95	0.94	1.13	1.11

trajectory 2ac, normal distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot d}$ KF	$\xi_{\dot d}$ LSM	$\xi_{\dot\alpha}$ KF	$\xi_{\dot\alpha}$ LSM	$\xi_{\dot\beta}$ KF	$\xi_{\dot\beta}$ LSM
10	1.06	1.06	1.00	1.00	0.93	0.93	1.05	0.83	1.12	1.07	1.15	1.08
20	1.02	1.02	0.91	0.91	1.09	1.09	1.12	0.90	0.98	0.99	0.98	0.98
30	1.07	1.07	0.96	0.96	1.08	1.08	1.16	0.93	0.96	0.90	0.84	0.82
40	1.11	1.11	1.08	1.08	0.99	0.99	1.21	1.00	1.06	1.01	0.98	0.94
50	1.09	1.09	1.04	1.04	0.98	0.98	1.16	1.02	1.03	0.99	0.91	0.87
60	0.94	0.94	1.05	1.05	0.90	0.90	1.21	1.02	1.01	0.99	0.99	0.96
70	0.96	0.96	1.04	1.04	0.93	0.93	1.26	1.08	1.11	1.10	1.05	1.03
80	1.00	1.00	1.07	1.07	0.93	0.93	1.24	1.00	1.07	1.06	1.06	1.02
90	1.08	1.08	1.01	1.01	0.90	0.90	1.25	0.98	1.01	1.00	1.05	1.00
100	1.32	1.32	1.03	1.03	0.93	0.93	1.24	0.97	0.94	0.97	1.00	0.96
110	1.50	1.50	0.99	0.99	0.88	0.88	1.36	0.98	0.94	1.00	1.00	0.91
120	1.65	1.65	0.96	0.96	0.92	0.92	1.51	1.04	0.97	1.07	1.12	1.02
130	1.82	1.82	0.95	0.95	0.96	0.96	1.77	1.04	1.01	1.04	1.17	1.01
140	2.31	2.31	0.97	0.97	0.92	0.92	1.99	1.07	1.05	1.16	1.16	1.00
150	2.73	2.73	0.96	0.96	0.92	0.92	2.23	1.01	1.08	1.25	1.15	0.96

trajectory 2ac, uniform distribution of the errors

n	ξ_d KF	ξ_d LSM	ξ_α KF	ξ_α LSM	ξ_β KF	ξ_β LSM	$\xi_{\dot d}$ KF	$\xi_{\dot d}$ LSM	$\xi_{\dot\alpha}$ KF	$\xi_{\dot\alpha}$ LSM	$\xi_{\dot\beta}$ KF	$\xi_{\dot\beta}$ LSM
10	1.00	1.00	0.96	0.96	0.99	0.99	1.10	1.01	1.14	1.04	1.04	1.04
20	0.96	0.96	0.93	0.93	0.96	0.96	1.18	0.99	1.04	0.96	1.00	0.98
30	0.96	0.96	0.91	0.91	0.91	0.91	1.14	0.99	1.00	0.99	1.00	0.99
40	0.97	0.97	0.96	0.96	0.96	0.96	1.09	0.89	1.02	1.00	0.96	0.95
50	1.00	1.00	0.90	0.90	0.94	0.94	1.03	0.90	1.09	1.10	1.02	1.03
60	0.97	0.97	0.88	0.88	0.96	0.96	1.00	0.87	1.01	1.01	1.00	0.99
70	0.95	0.95	0.86	0.86	0.94	0.94	1.09	0.96	0.92	0.92	0.93	0.93
80	1.07	1.07	0.84	0.84	0.93	0.93	1.15	0.93	0.96	0.93	0.94	0.93
90	1.21	1.21	0.82	0.82	0.91	0.91	1.22	0.88	0.99	1.01	0.99	0.99
100	1.29	1.29	0.85	0.85	0.91	0.91	1.31	0.90	1.12	1.06	1.03	1.01
110	1.51	1.51	0.80	0.80	0.92	0.92	1.35	0.94	1.08	1.09	1.04	1.03
120	1.64	1.64	0.83	0.83	0.89	0.89	1.50	0.96	0.99	1.04	1.05	1.04
130	1.87	1.87	0.86	0.86	0.86	0.86	1.59	0.95	0.94	1.06	1.08	1.00
140	2.42	2.42	0.85	0.85	0.88	0.88	1.86	0.91	0.92	1.07	1.04	0.92
150	2.53	2.53	0.84	0.84	0.90	0.90	1.95	0.86	1.00	1.09	1.06	0.88

Table 2 The ratio $\sigma_{\rm ga}/\sigma_{\rm lsm}$ of the RMS of the errors for parameters d, α, β, $\dot d$, $\dot\alpha$, $\dot\beta$ provided by GA and LSM for all selected trajectories (naximum number of measurements, uniform distribution of the errors)

$\sigma_d = 0.05\,km, \quad \sigma_\alpha = \sigma_\beta = 0.001$

	d	α	β	$\dot d$	$\dot\alpha$	$\dot\beta$
1al	3.55	0.78	1.03	3.80	1.37	4.66
1ac	3.42	0.90	0.99	4.40	1.05	2.10
2al	3.05	1.00	0.93	5.60	1.62	1.16
2ac	2.80	1.15	1.24	5.50	0.89	3.24

$\sigma_d = 3.0\,km, \quad \sigma_\alpha = \sigma_\beta = 0.001$

	d	α	β	$\dot d$	$\dot\alpha$	$\dot\beta$
1al	1.21	0.95	1.37	1.60	2.00	1.97
1ac	1.17	1.22	0.99	1.72	1.70	1.36
2al	1.23	0.97	1.27	2.17	1.61	1.86
2ac	1.49	1.72	1.35	2.21	2.18	1.55

$\sigma_d = 0.2\,km, \quad \sigma_\alpha = \sigma_\beta = 0.0001$

	d	α	β	$\dot d$	$\dot\alpha$	$\dot\beta$
1al	1.35	0.88	1.30	1.62	1.87	1.80
1ac	1.28	1.10	1.00	1.80	1.57	1.38
2al	1.38	0.98	1.13	2.23	1.65	1.72
2ac	1.60	1.58	1.32	2.45	2.10	1.57