Localized Density/Drag Prediction for Improved Onboard Orbit Propagation

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Abstract

The objectives of localized density modeling is presented and compared to existing global models currently in use. Localized models are primarily concerned with the density profiles observed by a specific spacecraft in a slow varying orbit regime. The CHAMP density and derived temperature data are presented and discussed as relating to localized modeling and truth values. Temperature perturbations due to geomagnetic storms are discussed and the basis of a new model that applies local information is discussed. Output based models are developed based on the autoregressive filters, that use previous density measurements to make predictions of future density values. Future work of combining simplified local density models with perturbation corrections is also discussed.

1. Introduction

Since the development of Luigi G. Jacchia's first density model in 1970 (J70) [1], atmospheric density modeling has steadily focused on large monolithic codes that provide global density coverage [2][3]. The most recent instantiation of the global density model is the Jacchia-Bowman 2008 (JB2008) model developed by Bruce Bowman of the Air Force Space Command [4]. As the models have evolved and improved, their complexity has grown as well. Where the J70 model required 2 indices and various time averages to determine density, the JB08 model requires 5 indices to determine density [4]. Due to computational complexity, the number of real-time inputs required, and limited forecasting abilities, these models are not well suited for onboard satellite orbit propagation.

In contrast to the global models, this paper proposes the development of a density prediction tool that is only concerned with the trajectory of a specific satellite. Since the orbital parameters of most low Earth orbiting satellites remain relatively constant in the short term, there is also minimal variation in the density profile observed by the satellite. Limiting the density model to a smaller orbit regime will also increase the ability to forecast the density along that orbital track. As a first step, this paper evaluates the feasibility of using a localized density prediction algorithm to generate the density profile that will be seen by satellite, allowing for high-accuracy orbit propagation with minimal or no input from the ground.

The algorithm evaluated in this paper is a simple autoregressive model that, given previously measured density values, provides predictions on the upcoming density profile. This first approach requires zero information about the satellite's current orbit, but does require an onboard method for determining the current, local density. Though this aspect of the onboard system is not analyzed here, it is envisioned that this current, local density (or equivalently drag acceleration) would be calculated through onboard processing of GPS or accelerometer data.

Using the trajectory of the CHAMP satellite as a test case, various samples of CHAMP density data in the "past" (i.e. before a chosen epoch time) will be input to the model, and the model will in turn predict "future" density values (i.e. beyond the chosen epoch time). The effectiveness of the local density model will be assessed by comparing its predicted density values to "true" densities in the CHAMP database at those times. Results describe the prediction accuracy of the filter and length of time over which accuracy is maintained.

2. CHAMP Density and Temperature Data

Launched on July 15, 2000, the CHAllenging Minisatellite Payload (CHAMP) has provided a wealth of data from its high accuracy onboard accelerometers. Of particular interest to the astrodynamics community is the atmospheric neutral density values derived from the drag accelerations [5][6]. The derived density for May 20, 2001 (day 140) are shown in Fig. 1a. Fig. 1b shows the derived density for the first 4 hours of the same day along with the densities predicted by the Jacchia-Bowman 2006 (JB2006) [2] and the High Accuracy Satellite Drag Model (HASDM) [3] global models. Note that while the JB2006 and HASDM models are fairly consistent with each other, they both over-predict the density for the time shown. For this analysis the CHAMP derived density is considered to be truth.

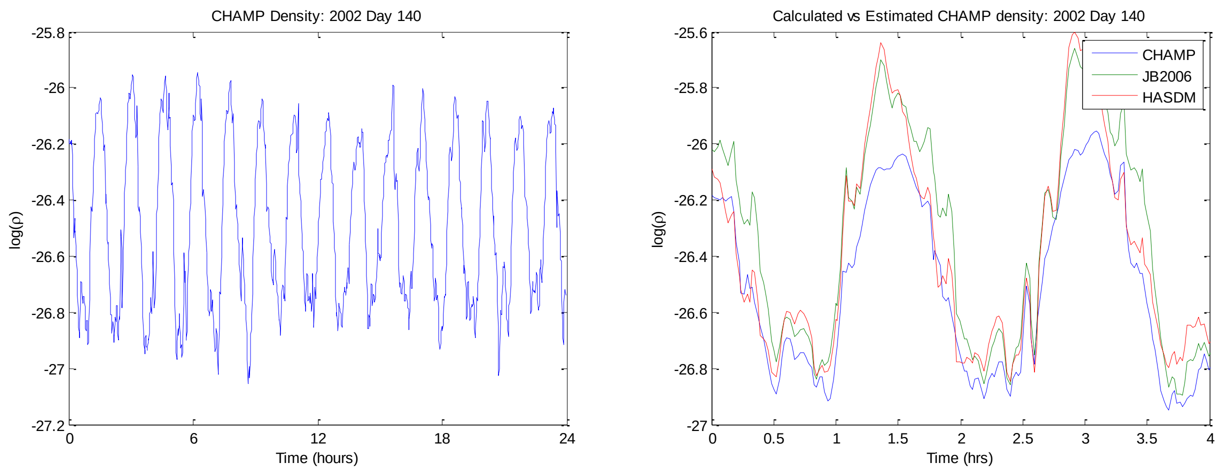

Fig. 1. (a) CHAMP derived neutral density (logarithmic scale) for 2001 Day 140. (b) CHAMP density data compared to the JB2006 and HASDM predicted densities.

Another set of data derived from the CHAMP data is the atmospheric temperature variations due to geomagnetic perturbations. For the J70 model and many of its derivative models this is the ΔT associated with the geomagnetic indices *ap* or *kp*, and is the primary means of addressing geomagnetic storms and their related density fluctuations [7]. This data is formed by evaluating the J70 model for all of the satellite position and known indices, minus the ap value. The algorithm then determines the ΔT value (typically a function of *ap*) that would result in the CHAMP observed density. The ΔT values, as a function of the 'observed' *ap* indices, for years 2001 through 2005 are shown in Fig. 2. Note that the data has a large spread for each value of ap. Similarly derived data was used to determine the new DST based geomagnetic perturbation model employed in the JB2008 model [8].

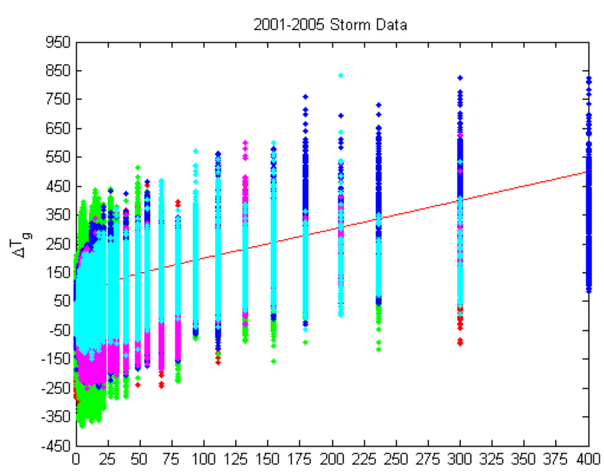

Fig. 2. J70 geomagnetic temperature perturbations derived from the CHAMP density data. The red line is the ΔT predicted by Jacchi's static model.

3. Geomagnetic Storm Perturbations

In an effort to evaluate the applicability of localized density modeling, an effort was first made to better understand the density perturbations due solely to geomagnetic storms using the CHAMP derived ΔT data. This work was also being performed as an *ap* based backup to the new DST models in the JB2008 global density model. In his paper entitled "New Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles," Jacchia outlined his empirical relation between the *Kp* and *ap* indices and ΔT to be the following [7]:

$$\Delta T_g = 28° Kp + 0.03° e^{Kp} \quad (1)$$

$$\Delta T_g = 1.0° ap + 100°[1 - e^{-0.08 ap}] \quad (2)$$

Jacchia's model is global in formulation and does not account for atmospheric variations resulting from latitudinal, temporal, or seasonal variations. The models also possess an inherent average time lag between the variations in geomagnetic index and those in temperature of 6.7 hours.

After multiple evaluations and iterations modeling the data an more localized modeling approach was developed to represent the observed data. To do this a new modeling parameter, θ, was developed to indicate both the latitude of the spacecraft as well as whether its current location was on the day or night sides of the earth and in the summer or winter hemisphere. Due to the limitations in the data no binning was performed about the local solar time (LST) or day of year (DOY). The parameter θ is depicted in Fig. 3. Fig. 3a shows the earth, projected as a circle, divided into four quadrants representing the unions of day/night and summer/winter. Fig. 3b depicts the same projected earth with the equivalent θ angle (which is simply a circle with angle gradations from 0 to 360). A satellite in the summer hemisphere, on the night side of the earth, and located at approximately 60 degrees latitude (whether positive or negative) would have a θ value of 120 degrees. A satellite in the winter hemisphere, on the day side of the earth, and located at approximately ±60 degrees would have a θ value of 300 degrees.

Since the objective of the model being developed was an *ap* based model, the data was first sorted according to its corresponding *ap* value, and then binned according to its calculated value for θ. The results are shown in Fig. 4. Fig. 4a depicts the ΔT data in a polar plot with each *ap* set normalized to its maximum value. Fig. 4b depicts the ΔT data vs θ for each discrete value of *ap* (including *ap* = 0).

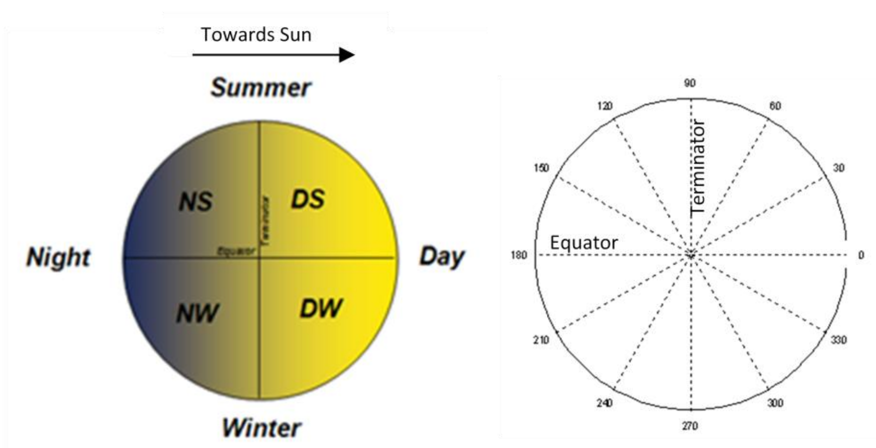

Fig. 3. (a) Earth divided into day/night/summer/winter quadrants. (b) Model parameter θ depicting appropriate quadrant and latitude.

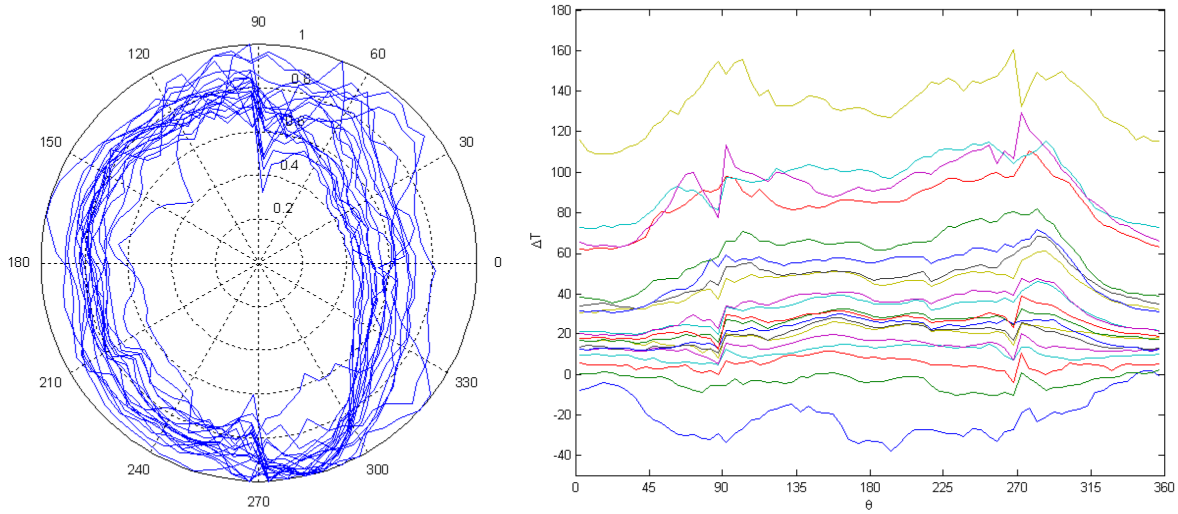

Fig. 4. (a) Polar plot of normalized ΔT data for each discrete value of *ap*. (b) ΔT data for each discrete value of *ap* vs the parameter θ.

It is not the intent of this paper to interpret the physical aspects of this data (such as the perturbations due to seasonal conditions) but to analyze the ability to apply corrections to a localized density model. An inspection of the data in Fig. 4b indicates that temperature perturbations (and equivalently density perturbations) due to magnetic storms do show, to some extent, predictable behavior about an orbit. The data for each value of *ap* can easily be modeled by a second-order fourier series. Given a base model for the local density it should be possible to apply correctors, based on observed data, to provide reasonably accurate localized density predictions. The remainder of this paper address the some of the initial, though incomplete, steps towards developing the localized base model.

4. Localized Model: Output Based Predictions

As a first step towards a base model we evaluated output based predictions, based on signal processing theory. Signal processing filters, such as the autoregressive moving average (ARMA) model, can make signal predictions based on past inputs and/or past outputs [9]. The autoregressive (AR) model makes predictions of future signal values based solely on past outputs. Using this type of model allows for density predictions based solely on past density measurements (derived through onboard accelerometers). The generic AR model is:

$$\mathrm{y_t} = c + \sum_{i=1}^{n} a_i y_{t-i} + \epsilon_t \tag{3}$$

where c is a constant, ϵ_t is white noise, a_i are the model coefficients, $\mathrm{y_{t-i}}$ are previously measured signals, and $\mathrm{y_t}$ is the predicted signal. Solving for the model coefficients can be performed using Matlab's LPC function.

This AR model predicts one step into the future. Given that the CHAMP data samples being analyzed are spaced 80 seconds apart, this represents a model that has an 80 second forward looking prediction capability. The results, shown in Fig. 5, depict that these short term predictions work reasonably well. With this type of based model there is no need to account for density perturbations beyond the basic AR model. Predictions a few minutes into the future though is perhaps a trivial solution and not particularly useful for long term predictions.

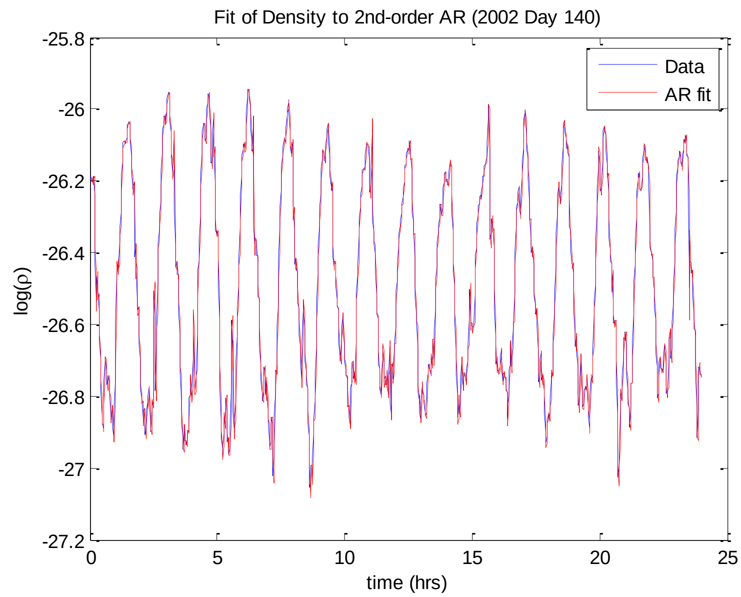

Fig. 5. Fit of the CHAMP data using a second-order AR model to predict one step into the future. Data samples are at 80 second intervals.

5. Multi-Step Predictions

Now that we've shown the ability to predict the density one step into the future, let's suppose that I'm interested in predicting the density 50 steps into the future. Reformulating equation 3, for a second-order filter yields:

$$y_{t+50} = b\eta_t + a_1 y_{t-1} + a_2 y_{t-2} \tag{4}$$

$$y_t = b\eta_t + a_1 y_{t-51} + a_2 y_{t-52} \tag{5}$$

This indicates that data being predicted is the linear sum of data 51 and 52 time steps in the past. Since Matlab's LPC and FILTER functions operates on the idea of equally spaced data, another method must be used to generate the model coefficients. To generate the coefficients we rewrite the equation as:

$$y_t = \begin{bmatrix} y_{t-51} & y_{t-52} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + b\eta_t \tag{6}$$

This can be formulated as a least-squares problem:

$$y = Hx + e \tag{7}$$

Where $y = y_t$, $H = \begin{bmatrix} y_{t-51} \\ y_{t-52} \end{bmatrix}$, $x = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$, and $e = b\eta_t$. Solving for the coefficients x provides the following equation:

$$x = (H^T H)^{-1} H^T y \tag{8}$$

Using the CHAMP density data from May 20, 2001 (DOY 140) to predict 73 time steps into the future (approximately one orbital period at 80 second data samples) yields the following values for the filter coefficients.

$$a_1 = 0.19453$$
$$a_2 = 0.80313$$

Given one orbit worth of density data (approximately 72 values at the given data rate) we can now predict the density over the next orbit. The model is then applied to the following days worth of data, using the first 72 values to seed the model. The results for May 21, 2001 (DOY 141) are shown in Fig. 6. Note that the future predicted orbit densities are similar (though not identical) to the previously measured density values. Inspection of the results shown that this model responds to density perturbations with a one orbit delay. The model is essentially saying that without any additional information the density profile of the next orbit should look similar to the previous density profile. The advantage of this model is that we can model (at a base level) the density profile for one full orbit as opposed to a few minutes for the previous formulation. This same formulation can also be used to predict multiple orbits into the future by simply repeating the predicted orbit profile multiple times.

Fig. 7 depicts the ratios of the CHAMP data to the JB2006, HASDM, and one orbit AR models. Note that the JB2006 and HASDM models both over-predicted the density, while the AR model has a mean ratio near zero. The JB2006 and HASDM models are both operating in an open loop while the AR model uses a moving window of the 'measured' density to model future values.

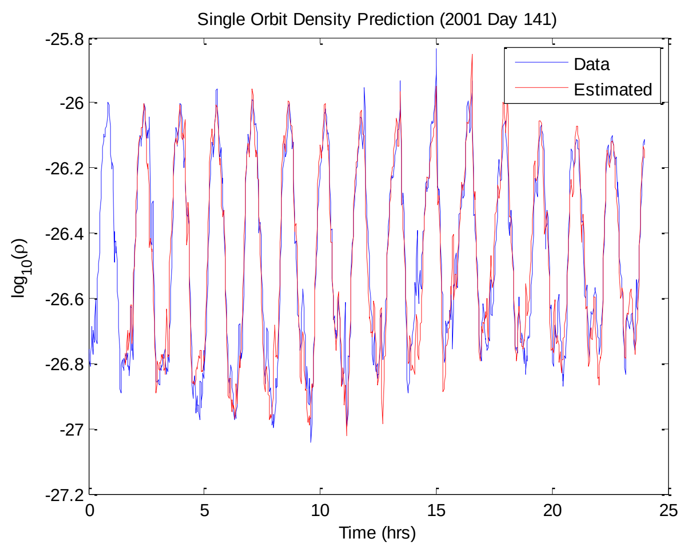

Fig. 6. Fit of the CHAMP data using a second-order AR model to predict one orbit into the future. Data samples are at 80 second intervals.

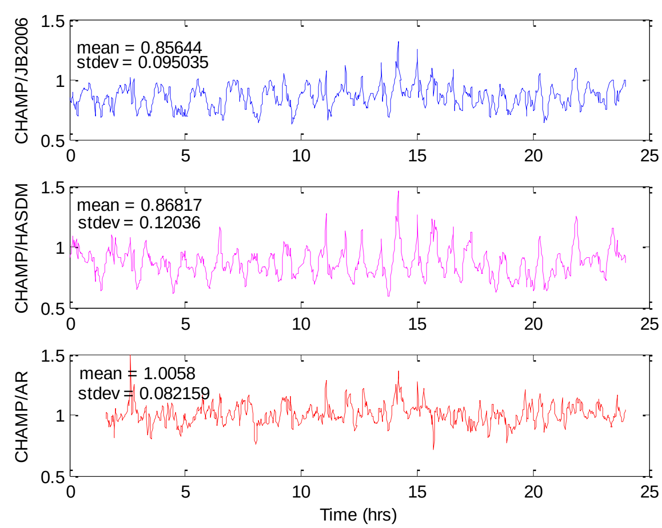

Fig. 7. Ratios of the CHAMP density data to the JB2006, HASDM, and AR model predictions.

It can be pointed out that the results shown in Fig. 6 are for fairly benign geomagnetic conditions and are based on coefficients generated on the previous days data. To determine if this model approach can apply to perturbed condition, such as during a geomagnetic storm, the model was re applied to October 3, 2001 (DOY 276). This day experienced a strong geomagnetic storm with an *ap* value of 132 ($Kp = 7$). The same coefficients are maintained for this new data set. The AR model results are shown in Fig. 8. The ratios for the JB2006, HASDM, and AR models are shown in Fig. 9.

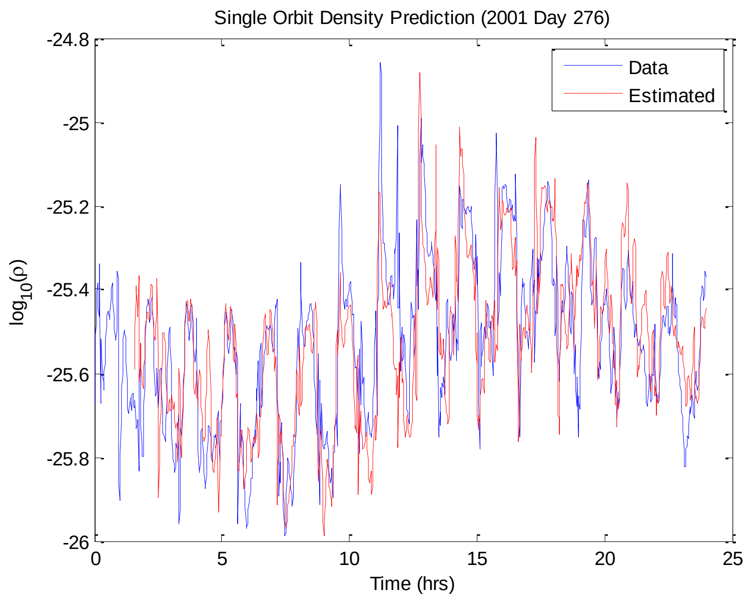

Fig. 8. Fit of the CHAMP data using a second-order AR model to predict one orbit into the future. Data samples are at 80 second intervals.

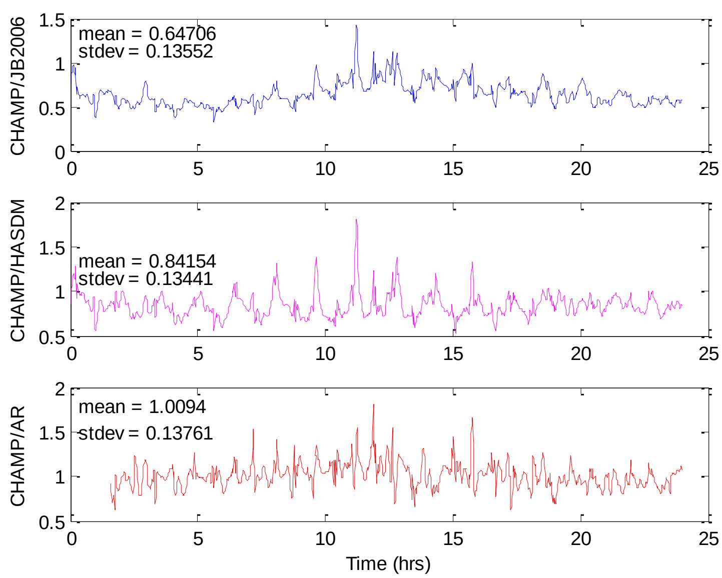

Fig. 9. Ratios of the CHAMP density data to the JB2006, HASDM, and AR model predictions.

The AR model results for the stormy DOY 276 perform similarly to those seen for the quiet DOY 141. The predicted density profile for the following orbit are similar to those measured in the previous orbit. Degraded performance is seen as a result of the significant perturbations. The mean of the predictions for DOY 141 and 276 are both nearly 1, but DOY276 has a standard deviation of 13.8% compared to 8.2% for DOY 141.

6. Conclusion

In contrast to large global density models, a new approach to onboard localized density modeling is sought. Localized density models and predictors can be used for improved onboard propagators where drag plays a significant role in A new approach to temperature perturbations resulting from geomagnetic storms was presented. Results, based on temperature variations derived from CHAMP data, show that storm perturbations do show structure and predictability and can likely be applied to base level models to account for variations in a spacecraft's local density profile. An approach for a base level density model was then shown which uses previously measured density values in an autoregressive model to predict future density models. The autoregressive model was able to accurately predict density values in the short term (or very small fractions of an orbit). The model was modified to predict density values over one orbit into the future based on the previous orbit's density. Results showed that the model predicts the future orbit density profile to look similar to the previously measured profile. The model also lagged by one orbit in responding to density perturbations for small to moderate perturbations. The model was then run against data during a strong geomagnetic storm. While the model behaved similarly to the case with small perturbations, predicting the next orbit's density profile will be similarly to the previously measured profile, the strong, short-term perturbations in density were not accurately captured. Future efforts will included improved base models for a spacecraft's local density and combining perturbation corrections to the base models. Another effort for study is switching from a local density model to a local drag model, eliminating the need to determine density values from accelerometers and allowing for orientation specific accommodations to drag.

7. References

1. Jacchia, L.G., New Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles, SAO SR 313, 1970
2. Bowman, B.R et al, The JB2006 Empirical Thermospheric Density Model, *Journal of Atmospheric and Solar-Terrestrial Physics*, Vol. 70, 774-793, 2008
3. Storz, M.F, Bowman, B.R., Branson, J.I., High Accuracy Satellite Drag Model (HASDM), AIAA/AAS Astrodynamics Specialist Conference, Monterey, CA, Aug. 2002.
4. Bowman, B.R. et al, A New Empirical Thermospheric Density Model JB2008 Using New Solar and Geomagnetic Indices, AIAA/AAS Astrodynamics Specialist Conference, Honolulu, HI, Aug. 2008
5. Bruinsma, S., Biancale, R., Total Densities Derived from Accelerometer Data, *Journal of Spacecraft and Rockets*, Vol. 40, 230-236, March-April 2003
6. McLaughlin, C.A., Hiatt, A., Lechtenberg, T., Calibrating Precision Orbit Derived Total Density, AIAA/AAS Astrodynamics Specialist Conference, Honolulu, HI, Aug. 2008.
7. Jacchia, L.G., Slowey, J., Verniani, F., Geomagnetic Perturbations and Upper-atmosphere Heating, SAO SR 218, 1966
8. Huang, C.Y. et al, Thermospheric Density Modeling During Magnetic Storms, A AIAA/AAS Astrodynamics Specialist Conference, Honolulu, HI, Aug. 2008
9. Percival, D.B. and Walden, A.T., *Spectral Analysis for Physical Applications*, Cambridge University Press, Cambridge, United Kingdom, 1993