Collision Preduction for LEO Sattellites. Analysis of Characteristics

Viacheslav F. Fateev
Doctor of Science (technical sciences), Professor, Russia,
"Vympel" Corporation, President
Sergey A. Sukhanov
Doctor of Science (technical sciences), Professor, Russia,
"Vympel" Corporation, Designer General)
Yuri V. Burtsev
Ph.D (technical science), Russia, Space Forces, Head of Division)
Zakhary N. Khutorovsky
Doctor of Science (technical sciences), Russia, "Vympel" Corporation,
Head of Division (Orbital Mechanics and Catalogue Maintenance)
Sergey Yu. Kamensky
Russia, "Vympel" Corporation, Chief Designer
Victor A. Stepaniants
Ph.D. (physics and mathematics), Russia, "Vympel" Corporation, Leading Researcher
Alexander S. Samotokhin
Ph.D. (physics and mathematics), Russia, "Vympel" Corporation,
Head of Division (Software Development)
Vladimir M. Agapov
Russia, "Vympel" Corporation, Head of Division (Space Situation Analysis)

The satisfactory compliance of the calculated errors of determination of the relative positions of approaching satellites for the time of the closest approach with the real errors is the indispensable condition for reliable decision on the possibility of collision. The technical measures undertaken for the catalog maintenance procedures to attain this compliance are described in the paper. The case of the catastrophic collision between the Kosmos-2251 and Iridium-33 spacecrafts that occurred on 10 February 2009 is used to illustrate the potential capabilities of Russian Space Surveillance System to predict this event. The materials of the paper demonstrate that for the 10 days interval prior to the time of collision the average level of the real errors in the radius vector direction for the time of collision is about 30 meters and for the other directions less than 500 meters with satisfactory compliance of the calculated errors with the real ones. For the probability of collision we have determined the increase from $3{\cdot}10^{-5}$ in the beginning of the interval to $2{\cdot}10^{-4}$ in its end just before the collision. It is demonstrated as well that for the case of collision warnings generated for two spacecraft a day prior to the collision the frequency of false alarm will be about 0.016 (once in 2 months).

1. INTRODUCTION

The basic source of the observation data on the space objects are the sensors of the Space Surveillance System (SSS). This system operates in Russia for more than 40 years. The system has been developed with the focus on the special tasks of missile attack warning, ballistic missile and air defense. These tasks determined the directions for development and the technical characteristics of the SSS.

The development of this system in Russia is successful. The independent satellite catalog is maintained [1], which basic characteristics in the LEO region were close to the characteristics of the US catalog until the beginning of the 90th years of the previous century [2,3]. This result was achieved under the conditions significantly more favorable for the US possessing the network of sensors more accurate and more efficiently located across the globe [4,5]. Russia possesses extensive experience of solving various tasks related to space observations. One of these tasks is the problem of evaluating the hazard related to the orbital flight of the spacecraft.

The hazard to the spacecraft launched to the Earth satellite orbit is posed by a great amount of orbiting space objects generated by more than four thousand launches and more than hundred of break-ups that occurred in the course of the last 50 years. Though more than 20 thousand of these objects sized more than 10 cm have already finished their orbital life by reentry there are about 15 thousand objects with sizes of meter or ten centimeters and hundreds of thousands of centimeter sized objects still orbiting the Earth and posing real collision hazard to the launched spacecraft. Such a collision can damage and disfunction the operational spacecraft and for the manned missions the threat is fatal. More of that, any collision in space generates more hazard due to the fragments of collision that increase the general level of future hazard.

The guarantee protection of the operational spacecraft from collisions with other space objects can be provided by the maintenance of the catalog of all the objects with sizes about 1 cm and reliable (with probability close to unity) prediction of possible collisions. Both these tasks are very difficult and the solutions are hardly to be expected (especially for the second problem) in the predictable future. Thus, are the attempts to find some solution worthwhile ? We think that they are, since having not enough capabilities of reliable prediction of collision we still can determine its probability and in case this probability is high enough we can try to reduce it for example performing avoidance maneuver. Performing like this we will undertake many unnecessary measures, since for the great majority of cases no collision happens, however, in case when collision actualizes we will avoid the danger.

For calculation the collision predictions for cataloged objects Russian SSS uses the "direct" method presented in 1993 at the First European Conference on Space Debris and published in the Proceedings of the Conference [6]. This method uses the permanently updated element sets of the cataloged objects for determination of the future time intervals of dangerous approaches for any pair of cataloged satellites along with evaluation of the geometric characteristics of these approaches and the probability of collision. For calculation of this probability the general formula is used apparently first derived in [6]. This formula presents the collision probability P_c as function of relative position and velocity for the time of the approach of two satellites for minimum distance, covariance matrices of the errors of determination of their position for this time and the sizes of the approaching satellites.

In this work we derive from the general formula for P_c a rather accurate approximate relationship for collision probability. The presented analysis shows that this relationship explicitly demonstrates the influence of all the parameters on the collision probability. As follows from the expression for P_c the satisfactory compliance of the calculated errors of determination of the relative positions of approaching satellites for the time of the closest approach with the real errors is the indispensable condition for reliable decision on the possibility of collision. We describe the technical measures undertaken by Russian SSS in the area of maintenance of orbital catalog using radar and optical measurements which improves the character of this compliance.

What potential capabilities for collision prediction do Russian space surveillance facilities and the Space Surveillance Center (SSC), where the catalog is maintained, have ? The catastrophic collision between the non-operational spacecraft Kosmos-2251 and the operational spacecraft Iridium-33 that happened on 10 February 2009 at 16.46 UTC over the territory of Russia provided the information for the ansver to this question. The work presents the results. The analysis presents the errors of determination of relative position of the spacecraft for the time of collision as function of time for all (per revolution) updates of their orbits by radar measurements, starting from 10 days before the collision and up to the moment of collision. It is shown that for this 10 days interval the calculated RMS of the errors are in satisfactory compliance with their real values. For the same time interval the values of collision probability are calculated. The error (the frequency of "false alarm") of making decision on the collision possibility is evaluated for different time intervals before the collision. Finally we review the techniques used for collision prediction and analyze their efficiency.

2. COLLISION PROBABILITY

The basic characteristic that should be used for making decision on the possibility of future collision of two satellites is the calculated value of the probability of this event. The probability of collision P_c of two satellites for their dangerous approach is determined by the formula [6]

$$P_c = k\cdot \exp(-0.5\cdot k_{rr}) \tag{1}$$

$k{=}S\cdot v_r\cdot\left(4\pi^2\cdot k_{vv}\cdot\det \mathbf{K}_1\cdot\det \mathbf{K}_2\cdot\det\left(\mathbf{K}_1^{-1}{+}\mathbf{K}_2^{-1}\right)\right)^{-0.5}$, $k_{rr}{=}\delta\mathbf{r}\cdot(\mathbf{K}_1{+}\mathbf{K}_2)^{-1}\cdot\delta\mathbf{r}'$, $k_{vv}{=}\delta\mathbf{v}\cdot(\mathbf{K}_1{+}\mathbf{K}_2)^{-1}\cdot\delta\mathbf{v}'$; $\delta\mathbf{r}$, $\delta\mathbf{v}$ – vectors of relative positions and velocities of the objects for the time $t_{\min}$ of their approach to minimum distance; $\mathbf{K}_1$, $\mathbf{K}_2$ – covariance matrices of the errors of determination of the positions of both objects for the time $t_{\min}$; v_r – the absolute value of the relative velocity of approach; $\det \mathbf{A}$ – determinant of matrix $\mathbf{A}$; $^{-1}$, $'$ – the signs of inversion and transposition of a matrix; S – the collision cross section [6], the value of which depends on the shape and size of the approaching objects, for objects of spherical shape with diameters d_1 and d_2 $S = \pi\cdot(d_1{+}d_2)^2/4$.

The formula (1) is derived under the following assumptions [6]:

1. Within the time interval of possible collision the motion of two approaching objects is straight-line.
2. The velocities of both objects are known with small relative errors.
3. The errors of position determination for one of two approaching objects are significantly greater than the size their collision cross section.

For the operation of Russian SSS these assumptions are always valid. We can see from (1), that the probability of collision depends on the sizes of the approaching satellites, covariance matrices of the errors of position determination for the time $t_{\min}$ and the orientation of the vector of relative velocity. The formula (1) is significantly simplified if we assume that the RMS of the errors of position determination for each object are equal in all directions and are equal to σ_1 and σ_2 respectively. In this case we have

$$P_c = (S/2\pi(\sigma_1^2{+}\sigma_2^2))\cdot \exp\left(-\delta r_{min}^2/2(\sigma_1^2{+}\sigma_2^2)\right) \tag{2}$$

where δr_{min} – the estimate of the minimum distance between two satellites. The coefficient before the exponent in (2) is the ratio of the collision cross-section to the two-fold area of the dispersion circle for the errors of determination of relative position of two approaching satellites in the transversal plane. The exponent index is the ratio of the square of the estimate of the minimum distance between approaching objects and the double covariance of the error of this estimate.

However, actually the assumption of the equality of the errors of position determination in all the directions is not in place. The RMS of the errors significantly differ for different directions. The errors along the track normally are much greater then the errors in perpendicular directions. For the near-circular orbit (eccentricity less than 0.1) this means that $\sigma_n \gg \sigma_r$, $\sigma_n \gg \sigma_b$, where $\mathbf{r}, \mathbf{n}, \mathbf{b}$ – directions or radius vector, normal to the radius vector in the orbital plane and the normal to the orbital plane. Assume that one of two approaching satellites resides in near-circular orbit. The distribution of the approaching objects of the environment with respect to the elevation angle of approach direction in orbital coordinate frame is concentrated in the small vicinity of the zero value (see e.q., [7], pp. 48-51). In this case not very difficult transformations of (1) lead to the following rather accurate for practical needs approximate relationship, given here for explicitness, for the approximation of the approaching objects by equivalent spheres with diameters d_1 and d_2

$$P_c \approx \tilde{k}\cdot \exp(-A) \tag{3}$$

$\tilde{k}{=}(d_1{+}d_2)^2/(\sigma_r\cdot(11\sigma_n\sin^2\alpha_c{+}16\sigma_b\cos^2\alpha_c))$, $A{=}\delta r^2/4\sigma_r^2{+}\delta n^2/2\sigma_n^2{+}\delta b^2/(2\sigma_b^2\cos^2\alpha_c{+}2\sigma_n^2\sin^2\alpha_c)$; δr, δn, δb – projections of the vector $\delta\mathbf{r}$ of relative positions of the objects for the time $t_{\min}$ to the directions $\mathbf{r}$, $\mathbf{n}$, $\mathbf{b}$ of the orbital coordinate frame of the satellite residing in near circular orbit; α_c – the angle between the vector $\mathbf{v}_1$ and $\mathbf{v}_2$ of the velocity of satellites for the time $t_{\min}$; σ_r, σ_n, σ_b – certain values within the ranges $(\sigma_{r1}, \sigma_{r2})$, $(\sigma_{n1}, \sigma_{n2})$, $(\sigma_{b1}, \sigma_{b2})$ of the parameters of both objects for the time $t_{\min}$.

The formula (3) rather simply and explicitly illustrate the dependence of the collision probability on the four basic parameters of the approaching satellites:

1. Average sizes d_1 and d_2.
2. Predicted positions $\mathbf{r}_1$ and $\mathbf{r}_2$ for the time $t_{\min}$.

3. The estimates σ_r, σ_n, σ_b of the errors of predicted positions.
4. The angle α_c between velocity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$.

The coefficient k depends on the ratio between the sizes of the approaching satellites and the errors of determination of their positions for the time $t_{\min}$. The exponential index A depends on the ratio between the difference of positions of two satellites for the time $t_{\min}$ and the calculated value of the errors of this difference (in case of actual collision these are the real and calculated values of determination of the distance between objects for the time of collision).

From (3) follows that the collision probability considered as function of the angle α_c, reaches its minimum (maximum) value for the "lateral" ("frontal") approach (respectively $\alpha_c = \pi/2$ and $\alpha_c = 0$). The formulas (1), (3) lead as well to the very important conclusion that only satisfactory compliance of calculated errors of position determination for approaching satellites with the real ones can result in right decision regarding the possibility of their collision. Really let us assume that the collision must happen. Then, as follows from (3), if the compliance of the calculated errors with the real ones is in place the probability of collision is in the average close to $0.4{\cdot}k$. Significant over- or under-estimation of the calculated errors significantly reduces this value. Thus, for example the overestimation of the calculated errors by 10 times result in the collision probability value obtained by (3) $\approx$100 times smaller and in case of underestimation by 10 times will practically turn to zero. The catalog maintenance procedures used by Russian SSS include special measures undertaken for reduction of incompliance between the calculated and real errors of orbit determination and prediction. These measures are described in the next section.

3. COMPLIANCE OF CALCULATED AND REAL ERRORS

The approaches used for maintenance of the catalog on the basis of radar measurements and designed to reach the better compliance of the calculated errors of orbit determination and prediction with the real ones and the results based on real data are described in detail in [8], [9]. Further we will just mention them. The sources of non-compliance of the calculated and real errors of orbit determination and prediction are the following inaccuracies of the used statistical description of the the measurement errors and the model of the satellites motion prediction:

1. The distribution of the normal component of the errors of measurements is known in general, but however, includes unknown parameters: RMS and bias,
2. For the abnormal measurements we know only the maximum values.
3. The unavoidable errors of prediction are determined by the inaccuracies of the used model of the atmosphere and the unknown variations of area-to-mass ratio (AMR), determined by the difference of the real shape of the object from the spherical one.

3.1 Parametrical uncertainty

For dealing with parametric uncertainty we use the **adaptive Bayes** approach [10], according to which the unknown parameters $\mathbf{b}$ of the errors of measurements and prediction are estimated along with the parameters $\mathbf{a} = (\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_k)^*$ of the orbits of k observed satellites by minimization with respect to parameters $\mathbf{a}$ and $\mathbf{b}$ of the functional

$$\Phi\,(\mathbf{X},\, \mathbf{a},\, \mathbf{b}) = \sum_{l=1}^{k} \Phi_l\,(\mathbf{a}_l), \tag{4}$$

where $\mathbf{X}$ – the set of all the measurements of all the tracked objects; $\Phi_l\,(\mathbf{a}_l)$ – the functional of the least squares method (LSM), based on the measurements of the l-th object. The weights of the measurements in this functional accounts of the unavoidable errors of prediction [8, 9].

For solving this problem we use the dynamical variant of the simple relaxation method of minimization of non-linear functional where the iterations of minimization for different parameters are replaced by iterations in time, i.e.

$$\hat{\mathbf{a}}^{(n)} = \arg\min_{\mathbf{a}} \Phi(\mathbf{X}_0,\, \mathbf{X}_1, ..., \mathbf{X}_n,\, \mathbf{a}, \hat{\mathbf{b}}_{n-1}) \tag{5}$$

$$\hat{\mathbf{b}}_n = \arg\min_{\mathbf{b}} \Phi(\mathbf{X}_0,\, \mathbf{X}_1, ..., \mathbf{X}_n,\, \hat{\mathbf{a}}^{(n)}, \mathbf{b}), \tag{6}$$

* Each set of the parameters $\mathbf{a}_i$ includes 6 orbital parameters and AMR.

where $\mathbf{X}_n$ – the set of measurements acquired during time interval τ_n; $\tau_0, \tau_1, ..., \tau_n, ...$ – are consequent not intersecting time intervals.

The algorithm performs recurrent in time processing. For the most laborious part (5) the independent processing of the measurements of different objects by LSM using the previously obtained estimate $\hat{\mathbf{b}}_{n-1}$ of the vector of parameters $\mathbf{b}$ is organized. The joint processing of the measurements of different objects is performed by significantly more simple algorithm based on (6). The calculated estimates $\hat{\mathbf{b}}_n$ of parameters $\mathbf{b}$ are used by the next step of processing of the new portion of measurements $\mathbf{X}_{n+1}$. The vector $\mathbf{b}$ of the "interfering" parameters has the shape: $\mathbf{b} = (\mathbf{m},\ \vec{\sigma},\ \vec{\alpha})$, where $\mathbf{m}{=}(\mathbf{m}_1,\ \mathbf{m}_2, ..., \mathbf{m}_p)$, $\vec{\sigma}{=}(\vec{\sigma}_1,\ \vec{\sigma}_2, ..., \vec{\sigma}_p)$ – the biases and the RMS of the errors of different sensors, p – the number of the sensors; $\vec{\alpha}{=}(\alpha_1,\ \alpha_2, ..., \alpha_q)$ – RMS of the relative errors of determination of AMR for the objects with significant atmospheric drag, q – the number of these objects. The parameter α of each atmospheric object is included into the weights of the measurements in the LSM functional and its estimate is performed along with the estimate of orbital parameters $\mathbf{a}$. Using iterations [8, 9], we select the value of α, providing the best compliance of the real and calculated errors of prediction within the interval of measurements used to estimate the parameters $\mathbf{a}$.

3.2 Non-parametrical uncertainty

The example of non-parametric uncertainty is provided by abnormal errors of the measurements. No account of these errors can result is distortion of orbital parameters and thus lead to underestimation in the calculated errors their real values. In the process of orbit determination on the basis of measurements in the course of satellite tracking we perform the selection and removing from the process the "alien" measurements and the abnormal components of "own" measurements. For this case we use the **robust** approach [10]. The following algorithm is implemented.

The components of the new (just correlated and which did not participate in the updating of the orbital parameters) measurements with great errors can significantly distort the orbit during the minimization process and thus can hamper the selection process. That is why they are selected prior to minimization of the LSM functional $\Phi(\mathbf{a})$ by comparing with high threshold ($\approx$100) of the squares of the normalized residuals, obtained by the correlation process (residuals prior to minimization of $\Phi(\mathbf{a})$ or input residuals). This is the rough selection. Further more accurate selection of the abnormal components of the measurements is performed by multi-pass minimization of $\Phi(\mathbf{a})$ with selection at each pass the abnormal components of all the measurements using the normalized residuals of their measured and estimated values. Here we use the residuals after minimization of $\Phi(\mathbf{a})$ or output residuals. At the first pass of minimization all the old (participated previously in orbit updating) measurements are used with the weights obtained by the previous update, i.e. the weights of the components of the measurements which were considered abnormal are set to zero. We set to zero as well the weights of the components of the new measurements which had not passed the mentioned above rough selection prior to minimization. The other components of new measurements participate in the first pass of minimization with their weights calculated under the assumption that they are not abnormal. Before each next pass all the components of all the measurements (the old as well as the new ones) are checked for abnormality by comparing with small threshold ($\approx$10) of the square of the normalized residual (of the respective component in $\Phi(\mathbf{a})$) in the point of the minimum obtained by the previous pass. The weights of thus detected abnormal components are set to zero and the previous not abnormal components incorrectly excluded from the process are returned to processing with restored weights. If the result of performed minimization reveals that all the weights are selected correctly, the passes are finished. The described algorithm is rather laborious. The deciding feature for its choice as the research demonstrated [10], was its higher efficiency compared with other robust procedures.

3.3 The calculated errors and their prediction

By "calculated errors" of orbit determination we mean the covariance matrix $\mathbf{K}_{\mathbf{a}(t)}$ of dimensions $7{\times}7$ of the errors of determination of $\mathbf{a}(t)$ parameters of the updated orbit, obtained by LSM for the measurements, where t – the time of the most "new" measurement which participated in obtaining $\mathbf{a}(t)$. $\mathbf{K}_{\mathbf{a}(t)}$ is calculated using Gauss-Newton formula

$$\mathbf{K}_{\mathbf{a}(t)} = (\mathbf{E}'{\cdot}\mathbf{K}_{\mathbf{x}}^{-1}{\cdot}\mathbf{E})^{-1} \qquad \mathbf{E} = \frac{\partial \mathbf{x}(\mathbf{a}(t))}{\partial \mathbf{a}(t)} \tag{7}$$

where $\mathbf{x}(\mathbf{a})$ – functional relationship of parameters of the measurements and the orbital parameters; $\mathbf{E}$ – matrix of the partial derivatives of the function $\mathbf{x}(\mathbf{a})$; $\mathbf{K}_{\mathbf{x}}$ – correlation matrix of the errors of the measurements, which diagonal elements are the result of the adaptive and robust procedures described above. In the calculated using (7) matrix $\mathbf{K}_{\mathbf{a}\,(t)}=\{r_{ij}\sigma_i\sigma_j\}$ $i,j=1,2,...,7$ the RMS of the errors σ have lower limit constants that values are determined by the errors of the motion prediction procedure used by the orbit determination algorithm. The values of these errors for all 7 orbital parameters can be obtained for every satellite from the orbital archive. The most simple is the procedure for AMR since in all prediction procedures currently used in Russian SSS it is assumed that AMR does not change within the interval of prediction.

If the orbit determined on the basis of measurements has the reference time t is propagated to a certain time t_{pr} in the future $(t_{pr}>t)$, then the prediction of the calculated errors is performed using the formula

$$\mathbf{K}_{\mathbf{a}\,(t_{pr})} = \mathbf{H}\cdot\mathbf{K}_{\mathbf{a}\,(t)}\cdot\mathbf{H}' \qquad \mathbf{H} = \frac{\partial\mathbf{a}\,(t_{pr})}{\partial\mathbf{a}\,(t)} \tag{8}$$

where $\mathbf{H}$ – is the matrix of partial derivatives of the functional relationship $\mathbf{a}\,(t_{pr})$ with respect to $\mathbf{a}\,(t)$, describing the used model of motion.

4. COLLISION OF KOSMOS-2251 AND IRIDIUM-33

On February 10 of the year 2009 at 16.56.00 UTC over the territory of Russian Federation the catastrophic collision of two spacecrafts occurred: not operational Russian spacecraft Kosmos-2251 (international designator 1993-036A) and operational prior to the collision US spacecraft Iridium-33 (international designator 1995-053C). The collision resulted in a great amount of fragments, by 01.08.2009 Russian and US space surveillance systems track more than 1300. The further tables 1, 2, 3 present the orbital parameters of collided satellites for the time of the last radar measurement*, acquired prior to collision†, characteristics of observations and measurement errors‡, characteristics of collision§.

Table 1. Orbital parameters of collided spacecrafts

Name	Date (d m y)	Time (h m s)	i (°)	Ω (°)	h_a (km)	h_p (km)	AMR (m^2/kg)
Iridium-33	10.02.2009	16.46.56	86.4	121.3	796	756	0.016
Kosmos-2251	10.02.2009	16.46.45	74.0	17.3	794	767	0.001

Table 2. Observations and errors of measurements

Name	d (m)	n_{rad}	n_{rev}	n_{obz}	p_{an}	$p_{an,d}$	$p_{an,\varepsilon}$	$p_{an,\gamma}$	$p_{an,\dot{d}}$	$p_{an,\dot{\varepsilon}}$	$p_{an,\dot{\gamma}}$ (%)
Iridium-33	2.6	9	12	42	20.0	0.3	2.5	5.0	2.5	4.0	16.0
Kosmos-2251	1.7	9	10	18	18.6	0.1	2.5	4.0	2.7	3.9	10.0

Table 3. Characteristics of collision

Date (d.m.y)	Time (h.m.s)	λ_{col} (°)	φ_{col} (°)	h_{col} (km)	v_{rel} (km/sec)	α_c (°)	β_c (°)
10.02.2009	16.56.00	97.9	72.5	789	11.65	102.5	0.035

We can see from these tables that both objects are very good for tracking. They are one of the most informative satellites. The orbits are close to circular. The influence of the atmospheric drag and solar radiation pressure is small. The prediction procedures used for the tracking of these satellites in the space surveillance center have the smallest methodical and real errors. Thus the unique fact of collision of Kosmos-2251 and Iridium-33 spacecrafts provides very important information for understanding of the potential capabilities of Russian SSS in preventing possible collisions.

* The measurement here means the state vector generated at the facility by smoothing the unit measurements of range, azimuth, elevation angle and range rate for certain time interval [8, 9].

† Inclination i, longitude of ascending node Ω, altitudes in apogee h_a and perigee h_p, AMR correlated with measurements.

‡ Average size d, number of different radar sensors that performed the measurements n_{rad} , the average per day number of observed revolutions n_{rev} and measurements n_{obz}, percent $p_{an,d}$, $p_{an,\varepsilon}$, $p_{an,\gamma}$, $p_{an,\dot{d}}$, $p_{an,\dot{\varepsilon}}$, $p_{an,\dot{\gamma}}$ of abnormal measurements for different parameters (range d, azimuth ε, elevation angle γ, range rate $\dot{d}$, azimuth range $\dot{\varepsilon}$, elevation angle rate $\dot{\gamma}$ [8, 9] and at least for one of them p_{an}.

§ Date and time of collision, longitude λ_{col} and latitude φ_{col} of satellite projection to the Earth surface for the time of collision, altitude h_{col} above the Earth surface for the time of collision, absolute value of relative velocity v_{rel} , the angle α_c between the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ of velocity for the time of collision, elevation angle β_c of the approach direction of Kosmos-2251 to Iridium-33 in the orbital coordinate frame of Iridium-33.

For automatic tracking of these satellites the system uses the analytical motion prediction algorithm developed in "Vympel" Corporation in the 80ies of the previous century. First this algorithm was published in [11]. In 1994 the algorithm was presented at the First US-Russian workshop on satellite observations. [12] In 2009 was published in [13]. The algorithm takes into account all significant harmonics of gravitational potential including the 8th (field 8×8) and Russian dynamic model of atmospheric density. [14] ¶ Solar radiation pressure and gravitational perturbations from the Sun and the Moon are not taken into account. Currently in the catalog maintenance process the analytical algorithm is used for tracking of ≈70% of satellites. The process of automatic tracking is close to the real time process. The space surveillance center collects the radar measurements. Using these measurements for each informational revolution the orbit is determined referred to the time of the last measurement. Orbit determination involves all the radar measurements available for the satellite for certain time interval which depends on the object (for our case the interval is 10 days). The orbit is calculated using modified LSM [8, 9]. On the basis of measurements 7 parameters are estimated: 6 orbital parameters and AMR. The AMR principally can be evaluated using measurements by the perturbations from the solar radiation pressure and atmosphere. In our case we meet a paradoxical situation when the not accounted by prediction perturbations of solar radiation pressure at least by an order of magnitude exceed the atmospheric ones included into the algorithm. Thus the value of AMR obtained by orbit determination process is a certain fictitious matching parameter. All the orbits and all the measurements were stored in the archive and after the collision were recovered. Using the orbits with timing within the interval $(t_{min}-10,\, t_{min})$ the vector $\delta\mathbf{r}$ of the distance between objects for the time t_{min} was calculated. Fig. 1 presents the time changes from $t_{min}-10$ to t_{min} of projections δr, δn, δb of the vector $\delta\mathbf{r}$ to the axes of the orbital coordinated frame of Iridium-33. We can see from Fig. 1, that for the whole 10 days interval $(t_{min}-10,\, t_{min})$ the distance between the spacecrafts for the time of collision in the directions $\mathbf{n}$ and $\mathbf{b}$ have been less than 1 km, and for $\mathbf{r}$ less than 100 m.

The software tools of the space surveillance center include more accrate motion prediction procedure. This algorithm due to several reasons (including computational efforts required) is used only for ≈20% of the tracked satellites. These are not geostationary objects with significant value (more than 130 minutes) of orbital period or its variation per revolution (more than 0.01 s) or orbits with high eccentricity (more than 0.1). This is the numerical method based on the Runge-Kutta technique of the 8th order [15], with account of the gravitational perturbations of the Earth, the Sun and the Moon, atmospheric drag with dynamical model of density [18], the direct solar radiation pressure and the transformations of time and coordinates performed with methodical error not exceeding 1 m. The functions presented in Fig. 1, were calculated also for the case when for orbit determination and prediction of the orbits of Kosmos-2251 and Iridium-33 the more accurate numerical prediction was used instead of analytical one. The results are presented in Fig. 2-4 (model of the Earth gravitational field 8×8, 16×16 and 36×36 respectively). The table 4 presents the resulting from Fig. 1-4 ranges of the errors (the difference between the maximum and minimum values) in meters for the different times prior to collision.

Table 4. Ranges of the errors in meters

prior to collision	anl	num 8×8	num 16×16	num 36×36
less than 10 days	2100	1100	900	650
less than 7 days	1100	650	300	300
less than 3 days	1000	350	250	250
less than 2 days	1000	350	250	250
less than 1 day	300	200	180	200

One can see from the Fig. 1-4 and table 4 that the use of the more precise propagator assists to reduce the errors of determination of the distance between satellites for the time of collision in the average two times for directions $\mathbf{n}$, $\mathbf{b}$ and three times for direction $\mathbf{r}$. For the ten days interval before the collision the errors in the direction $\mathbf{r}$ are in the average ≈30 m.

What was the character of the compliance of calculated and real errors of determination of relative coordinates of the satellites for the time of collision ? Fig. 5-7 present the plots of time variation of the RMS of the errors of position determination for both objects regarding the parameters r, n, b of each of the satellites and their

¶After 2007 replaced by atmospheric density model [18].

difference in the same parameters in the orbital coordinate frame of Iridium-33. Fig. 8-10 for each of three parameters r, n, b present together the real errors of determination of relative position of the objects for the time of collision and the calculated range $(-2\sigma, 2\sigma)$ of their variation with probability not less than 0.9. For the real errors we took the errors of determination and prediction of orbits using numerical propagator with earth gravitational field 8×8 (Fig. 2), and the RMS were calculated using (7), (8), where only the RMS of errors of determination of AMR had lower limits. The limiting constants are $0.004\,m^2/kg$ and $0.001\,m^2/kg$ for Kosmos-2251 and Iridium-33 respectively. These values are calculated according to the real time variation of the matched with measurements AMR of these objects, which is presented in Fig. 11-12. Here for Kosmos-2251 the changes of AMR are presented for the month interval and for Iridium-33 for the interval with no significant corrections of the orbit (16 days). One can see from Fig. 8-10, real and calculated errors of determination of the relative positions of two satellites for the time of their collision for the whole analized time interval $(t_{min}-10,\, t_{min})$ for all three coordinates r, n, b are in good compliance with each other. There are no cases when the real errors exceed the level of "three RMS" and almost always the real errors are less than two RMS.

Currently there is no permanent positioning of Russian radars that result in systematic and slowly shifting components or the errors of the measurements that are comparable with stochastic errors. In addition the practice shows that the RMS of stochastic errors not always comply with the "passport" values, and for some radars 10-20% of the measurements (see Table 2) include abnormal errors*. Under these conditions the obtained good compliance of the calculated and real errors is a good result. If the tracking algorithm used in Russian SSC did not have the adaptive and robust features the characteristics of the real errors and their compliance with the calculated ones would be different. This is illustrated by Fig. 13-15 and 16-18. We can see that no account of the fact of the inaccuracy of the existing statistical description of the not abnormal errors of the measurements (Fig. 13-15) lead to the increase of the real errors $\delta r, \delta n, \delta b$ and their calculated values twice and more times. If in addition we do not take into account the possibility of abnormal measurements (Fig. 16-18), the real errors increase up to 10 times with regular cases of incompliance with calculated values up to 8σ.

The probability of collision can be estimated using (1), (3). In the case of compliance of calculated and real errors the exponential multiplier in the average is equal to 0.4. The expression for the coefficient k includes RMS of the errors in the directions $\mathbf{r}, \mathbf{n}, \mathbf{b}$ for each of the satellites, presented in Fig. 5-6, and the angle α_c between the velocity vectors, given in table 3. Using these values we will have that the collision probability for the beginning of the time interval $(t_{min}-10,\, t_{min})$ was $\approx 3\cdot 10^{-5}$, and by the end of the interval just before the collision was $\approx 2\cdot 10^{-4}$. The value 10^{-4} for collision probability is a significant figure. This value is close to the daily probability of collision of any two cataloged objects [7]. We can see from (3) and table 3, that collision between Kosmos-2251 and Iridium-33 was not a simple case for prediction. The approach was in fact "lateral" and for this case there is only one precise coordinate of three possible ones – the radius-vector. If the approach was "frontal" there would have been two coordinates (radius-vector and binormal), the collision probability for the whole interval $(t_{min}-10,\, t_{min})$, calculated using (3), would have been $\approx 4\cdot 10^{-4}$.

Thus, there have been significant reasons to make a decision of possible collision of the spacecrafts on the basis of the data of Russian space surveillance facilities with possible measures for reduction of damage. However, no such decision was in place and the collision happened. Let us assume that such a decision have been made and the orbit of one of the spacecrafts (it could only be a cooperable† spacecraft Iridium-33) would have been corrected to make the collision impossible. Let us assume as well that such a decision is always made for all the operational spacecraft in these situations. We will evaluate the frequency of mistakes. The probability of collision is small. Thus in fact all the avoidance decisions for operational spacecrafts will be false. Thus the frequency of mistakes is the probability that there is a dangerous approach in space. What is the frequency of such approaches ?

Usually some statistical model of space debris are used to answer these questions. In Reference [6] more simple technique for solving these tasks which is based on the archive of dangerous approaches (ADA),

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* The value of abnormal error in elevation angle and the respective range sometimes is comparable with the a'priori values of these parameters.

† Operational spacecraft, capable to perform orbital corrections following the commands of mission control center.

that can be maintained along with the process of catalog maintenance is suggested. This archive contains information on all the dangerous approaches among all the tracked satellites. The approach is considered dangerous either in case the distance of approach is less than the given value (normally 5-10 km) or the collision probability is greater than the certain level (normally 10^{-11}). For each dangerous approach the archive in addition to administrative characteristics of the approaching objects includes the geometrical characteristics of the approach and the collision probability. References [6, 7, 16, 17] describe a simple and universal technique for different tasks related to collision risk analysis. The technique is based on the use of ADA. In particular for evaluation of the approaches of cataloged satellites for the distances δr, δn, δb, less than the given thresholds c_r, c_n, c_b, we should calculate using ADA the number of such approaches in the course of certain time interval and divide the result by the length of the interval. Now we are interested not in all the approaches but only those where an operational spacecraft is involved. From Fig. 2 we can see that the treshold c_r, c_n, c_b and the radius r_{sph} of the equivalent in volume sphere (in fact the maximum errors for these parameters) for different values of time Δt_{col}, remaining until the collision are as follows:

Table 5. The errors for different times remaining until the collision

Δt_{col} (days)	c_r (km)	c_n (km)	c_b (km)	r_{sph} (km)
1	0.03	0.06	0.06	0.06
2	0.03	0.10	0.08	0.08
3	0.03	0.25	0.20	0.15
4	0.03	0.25	0.20	0.15
5	0.03	0.30	0.30	0.18

We could use the current ADA to obtain the desirable characteristics depending on the time Δt_{col}, remaining until the collision. However, we will not do this. For the demonstration of prediction and approximation possibilities of the technique described in [6, 7, 16, 17] we will use the data, presented there. Reference [7] presents the distribution of the minimum distance between approaching satellites on the basis of ADA archived in the space surveillance center in the course of 2.5 years from June 1992 until December 1994 which comprise the data on ≈2.5 millions of these approaches. Table 6 presents the data from [7], p.61 for the daily average number n_Δ of the pairs of objects approaching each other for the distance smaller than Δ, for different values of Δ.

Table 6. The daily average number of the pairs of satellites approaching fop the given distance (1992-94)

Δ	0.1 km	0.2 km	0.3 km	0.5 km	1 km	2 km	3 km
n_Δ	0.75	3.0	4.6	11.9	47.7	190	426

To obtain the required characteristics we should extrapolate the data of Table 6, to the different time and then transform them to the number of pairs comprising an operational spacecraft. We assume that with acceptable accuracy:

1. $n_\Delta \approx n_1 \cdot \Delta^2$, where n_1 – certain constant.
2. Altitude distributions of cataloged satellites are close for 1992-94 and 2009.
3. Altitude distributions of the cataloged and operational satellites are close.

Fifteen years ago the catalog in the LEO region comprised about 7000 satellites. Now there are 1.7 times more. According to 2. for the year 2009 all the figures in the table 6 should be multiplied by $1.7^2 \approx 2.9$. Currently there are about 250 operational spacecraft in the LEO region, i.e. 55 times smaller than the total number of cataloged objects in this region. Thus according to 3. for obtaining the frequency of approaches of cataloged satellites with operational in the year 2009 all the figures of table 6 should be multiplied by $2.9/55 \approx 0.053$. Finally, using 1. we obtain the following frequencies of approaches $\nu(r_{sph})$ of the objects of interest with the cataloged satellites for the distance less than the given threshold r_{sph} for different values of time Δt_{col}, remaining until the collision:

t_{col}	(days)	1	2	3	4	5
r_{sph}	(km)	0.060	0.080	0.15	0.15	0.18
$\nu(r_{sph})$	(1/days)	0.016	0.028	0.10	0.10	0.14

Thus for the case of making alarm of possible collision of two spacecraft a day prior to collision the frequency of false alarms will be 0.016 (once in two months).

5. CONCLUSIONS

1. The basic component of solving the task of collision prediction is attaining the compliance between the real errors of orbit determination and prediction with their RMS values calculated in the process of catalog maintenance on the basis of measurements of radar and optical sensors. Russian Space Surveillance System used for solving this task rather sophisticated adaptive and robust techniques. In this paper the example of the collision of Kosmos-2251 and Iridium-33 spacecrafts is used to illustrate the efficiency of these methods.

2. The analysis of the statistical characteristics of decisions made for collision prediction should include the deterministic approach along with traditional statistical models of space debris. For this purpose it is suggested to use the regularly maintained on the basis of actual catalog the *archive of dangerous approaches*, which in fact will accumulate complete information on collision hazard for cataloged satellites. This paper presents the evaluation of the frequency of false alarms for predictions of collision of cataloged satellites with operational spacecraft using the data of this archive.

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APPENDIX

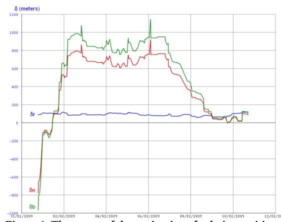

Figure 1. The errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision (analytical prediction).

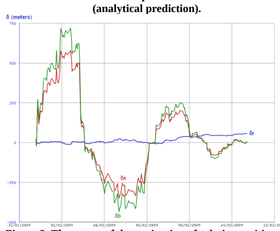

Figure 2. The errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision (numerical prediction, field 8×8).

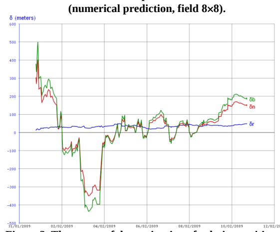

Figure 3. The errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision (numerical prediction, field 16×16).

Figure 4. The errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision (numerical prediction, field 36×36).

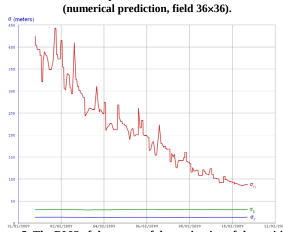

Figure 5. The RMS of the errors of determination of the position of Kosmos –2251 spacecraft for the time of collision with Iridium-33 spacecraft.

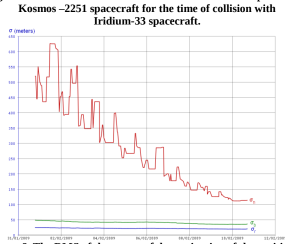

Figure 6. The RMS of the errors of determination of the position of Iridium-33 spacecraft for the time of collision with Kosmos-2251 spacecraft.

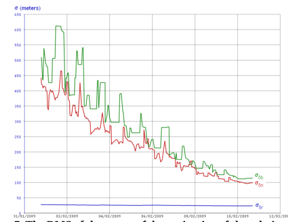

Figure 7. The RMS of the errors of determination of the relative position of Iridium-33 and Kosmos-2251 spacecrafts for the time of collision (numerical prediction, field 8×8).

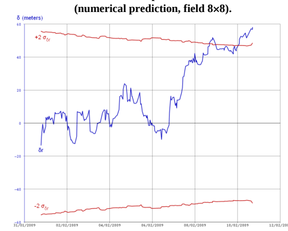

Figure 8. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σr (numerical prediction, field 8×8).

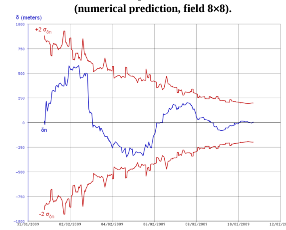

Figure 9. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σn (numerical prediction, field 8×8).

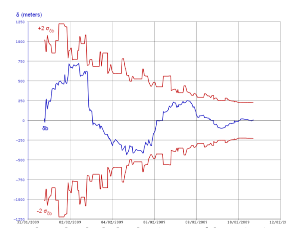

Figure 10. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σb (numerical prediction, field 8×8).

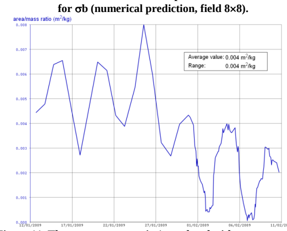

Figure 11. The area to mass ratio (correlated with measurements) as function of time for Kosmos-2251 spacecraft.

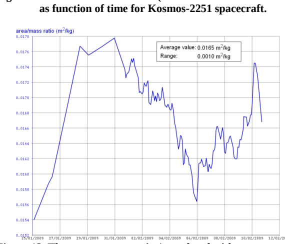

Figure 12. The area to mass ratio (correlated with measurements) as function of time for Iridium-33 spacecraft.

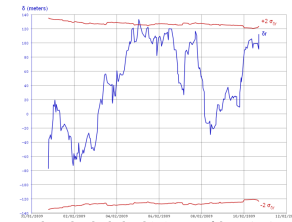

Figure 13. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σr (numerical prediction, field 8×8, no adaptation).

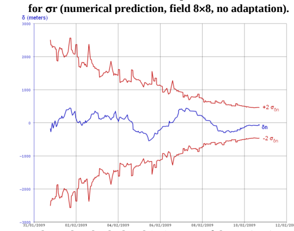

Figure 14. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σn (numerical prediction, field 8×8, no adaptation).

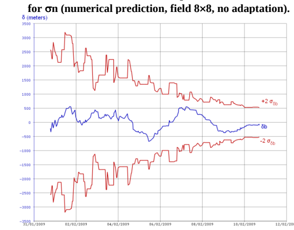

Figure 15. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σb (numerical prediction, field 8×8, no adaptation).

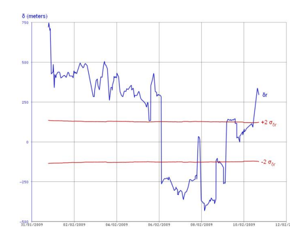

Figure 16. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σr (numerical prediction, field 8×8, no adaptation and robust estimates).

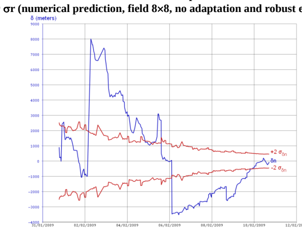

Figure 17. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σn (numerical prediction, field 8×8, no adaptation and robust estimates).

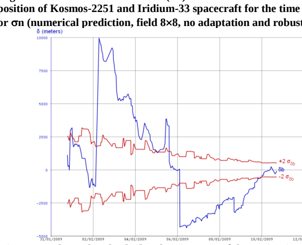

Figure 18. The real and calculated (2σ) errors of determination of relative position of Kosmos-2251 and Iridium-33 spacecraft for the time of collision for σb (numerical prediction, field 8×8, no adaptation and robust estimates).