Ballistic Coefficient Prediction for Resident Space Objects

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Abstract

Recent improvements in atmospheric density modeling now provide more confidence in spacecraft ballistic coefficient (BC) estimations, which were previously corrupted by large errors in density. Without attitude knowledge, forecasting the BC for accurate future state and uncertainty predictions remains elusive. Our objective is to improve the predictive capability for ballistic coefficients for Resident Space Objects (RSOs), thus improving the existing drag models and associated accuracy of the U.S. Space Object Catalog. To achieve this goal we implemented a two-pronged strategy that includes elements of time series analysis and physics based simulations.

Two empirical time series prediction methods were applied and tested on simulated and measured BC time series data: a multi-tone harmonic model and an autoregressive (AR) model. Both the multi-tone harmonic model and the AR model were subjected to multiple levels of optimizations resulting in highly optimized final models that were tuned specifically with the 205 BC time series data provided by the Air Force. Two versions of the AR model were developed based on the model prediction methodology. The second version of the AR model performed approximately as well as the optimized multi-tone model. The proposed algorithms automatically select the fitting function and duration of the fit span tuned for the best prediction performance based on past known data. The results demonstrated the ability to robustly and automatically fit past BC data in order to predict forward for 1-10 days with improved accuracy. These improvements were mapped to position error predictions for typical satellites, demonstrating the utility of such predictions for conjunction analysis and other catalog applications.

An archive of simulated BC data is generated using custom 6DOF high fidelity simulations for RSOs using plate models for shapes. The simulator includes force and torque perturbations due to the nonspherical Earth, third-body perturbations, SRP, and atmospheric drag. The simulated BC profiles demonstrate significant variation over short time spans (due primarily to varying frontal areas), providing motivation to improve future BC estimation strategies. The 6DOF modeling is intended to provide a physics-based BC data set to complement the BC data set provided by the AF. The improved performance of the time series prediction algorithms applied to the physics-based simulated data suggests that the actual estimated BCs include other signatures.

Introduction

Atmospheric drag is the major source of error in the orbit determination and prediction for RSOs in low Earth orbits. In general, the drag force experienced by a satellite varies primarily due to changes in the ballistic coefficient (BC) and atmospheric density. When the attitude profile of an RSO is known, drag modeling is difficult primarily due to the inherent temporal and spatial uncertainties in atmospheric density. The new Air Force Space Command (AFSPC) density model coupled with current better solar predictions is addressing many of the deficiencies in drag modeling [1, 2, 3, 7, 8]. However, for RSOs exhibiting large frontal area variations, changes in the BC can be a major source of orbit prediction error. Real time ballistic coefficients are currently computed from the daily differential orbit corrections obtained on all RSOs. AFSPC would like to predict BC changes over the period of a week from the epoch time. Corrections to thermospheric density from current models are now accurate enough to allow reasonably good determination of these BCs. This ability to separate density and BC values paves the way for significantly improving RSO orbit prediction.

In this project, we pursued a two-pronged strategy that included elements of time series analysis and physics-based six degree of freedom (6DOF) simulations. State-of-the-art empirical time series prediction methods were applied on BC time series and tested using both simulated data and real data provided by the Air Force. Also, an archive of simulated BC data was generated using custom 6DOF high fidelity simulations.

For the empirical time series prediction algorithms, a variety of approaches were considered and two prediction models showed the most promising performance: a multi-tone harmonic model and an Auto-Regressive (AR) model. Both the multi-tone harmonic model and the AR model were subjected to multiple levels of optimizations resulting in highly optimized final models that were tuned specifically with the 205 BC time series provided by the USAF. Both algorithms exhibited similar levels of performance. The chosen algorithms automatically select the fitting function and duration of the fit span to obtain the best prediction performance based on past data. The results obtained using these algorithms demonstrated that both algorithms have the ability to automatically fit real BC data and to predict future BCs for 1-10 days with improved accuracy over the current constant model approach. The constant model approach assumes that the BC remains constant at the most recently estimated value over the entire prediction interval. For 7-day predictions, our best model provided relative root mean squared (RMS) errors of approximately 9%, compared to approximately 15% for the constant model. These improved predictions could be tremendously valuable in improving conjunction analysis and other catalog applications. While the performance of the algorithms is promising, there is still room for improvement from this initial proof-of-concept demonstration.

The 6DOF simulations included force and torque perturbations due to the non-spherical Earth, third-body perturbations, SRP, gravity gradient, and atmospheric drag. The simulated BC profiles demonstrated significant variation over short time spans, providing motivation to improve BC prediction strategies. The 6DOF modeling provided a physics-based BC data set to complement the BC data set provided by the Air Force. Our forecasting algorithms were tested using the physics-based simulated BC data and highly accurate prediction capabilities were demonstrated. The improved performance of the time series prediction algorithms applied to the physics-based simulated data suggests that actual estimated BCs include effects not accounted for in the 6DOF simulations. These results provide further motivation for improvements in the estimation of BCs. Accordingly, in future work, we plan to leverage our 6DOF simulations to investigate potential improvements to the time series prediction models and the direct estimation of the BC series.

BC Time Series Analysis

The BC of an object in space determines its orbital decay behavior, especially in low Earth orbit (LEO), and hence is of prime importance during drag force computation. If atmospheric forces (and all other dominant forces) can be suitably modeled, the BC, as shown in Eq. 1, may be estimated.

$$BC = \frac{m}{C_D \cdot A} \tag{1}$$

While the mass, m, drag coefficient, C_D, and area, A, remain inseparable, the main variation of BC is due to changes in frontal area. For a non-spherical tumbling object, this variation can be rapid with large amplitudes. The current space catalog tracking process assumes fixed BC values over long durations and, therefore, does not effectively consider or model these potentially rapid BC variations. This has led to the growing need to model and predict the ballistic coefficient more effectively.

One of the proposed solutions is to estimate future values of the BC using time series prediction techniques. These techniques do not use complicated dynamics models, and, instead, model and predict the BC data by fitting previous data to high-order basis functions (such as Fourier series or sum-of-sine series). The time series data are smoothed using a trend analysis in order to remove random noise. The smoothed data are then used to estimate the coefficients of the fitting model. In the current work, we considered three types of fitting models: non-linear Matlab based models, the multi-tone harmonic model and autoregressive (AR) model. The performance of the non-linear Matlab based model was found to be inferior to the other two models and hence was not considered in later part of this work.

The terminology adopted for the the BC time series analysis is defined in the next section. Next, a brief overview of the multi-tone harmonics approach is presented, followed by a brief overview of the AR approach. Finally, the evaluation of both methods using the 205 BC time histories provided by the Air Force is presented. The time series data originated from actual orbit estimations for 205 different satellites over a time spans on the order of hundreds of days.

BC Time Series Terminology

Considering that we are analyzing multiple BC data files with varying time spans and observation frequencies, we defined the time series terminology given in Fig. 1.

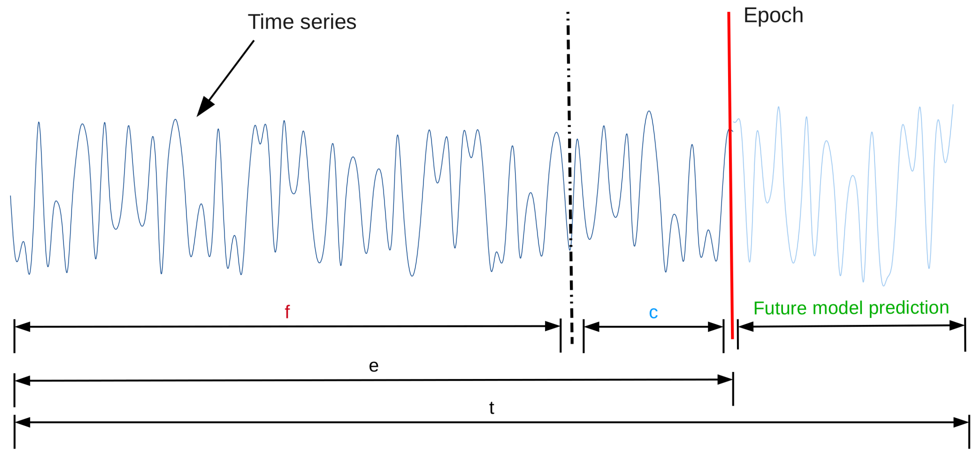

Fig. 1: Time Series Terminology

In Fig. 1, t is the total span of the input BC time series. Epoch span, e, was defined as 85% of the total span of the input BC time series. In the current study, we assumed e was our known data span and tried to predict BC values in the region beyond e. Eighty-five percent was chosen arbitrarily for this study. The calibration region, c, is the region over which various model parameter optimizations are performed. The fit span, f, defines the span of the fitted model. The fit span satisfies the following condition:

$$c \leq f < (e - c) \tag{2}$$

For a consistent measure of the prediction performance of the final model, the amplitude relative error is defined in in Eq. 3.

$$\text{amplitude relative error} = \frac{\text{(true value - predicted value)}}{\text{amplitude of the BC series over the epoch span}} \tag{3}$$

The true value of the BC is known in simulation (when the epoch span ends prior to the end of the time series history), but would be unknown when applying the algorithm in practice.

The amplitude relative error served as our metric throughout the model generation/optimization process. Using the amplitude relative error instead of the standard relative error during model generation gives more weight to long term and well-defined trends, which, in turn, helps reduce over-fitting of the BC time series data. Standard relative error was used for quantifying prediction models' capabilities in the final performance evaluation section of this report. Prior to fitting, the BC time series were prepared as described in the next section.

Preparing BC Data

Parsing

First, the 205 data files containing BC time series were parsed and imported into Matlab. The relevant data was then automatically extracted, splined, smoothed, and then passed to the fitting algorithm. The data files were also stored in a Fortran readable binary, in order to be usable with a Fortran-based prediction algorithm in the future.

Splining

The BC data loaded into Matlab was irregularly spaced with varying time intervals. This irregularly-spaced data was interpolated using splines and sampled at equal intervals, leading to a homogeneous BC time series with 1568 elements for the multi-tone model and 5000 elements for the autoregressive (AR) model. Regularly spaced data helps to maintain homogeneity and facilitates convergence of the frequency detection algorithm, as described later.

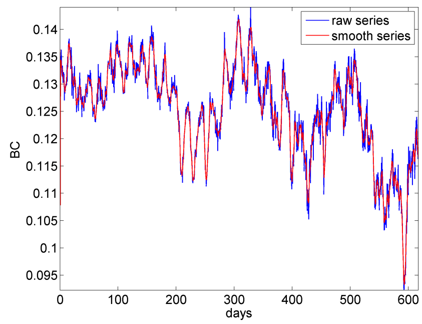

Fig. 2: Smoothed BC Time Series

Smoothing

The next step was to perform a time series decomposition analysis on the homogeneous BC data and break it into cyclic, seasonal, and random noise components. The cyclic and seasonal terms represented the long term trends in the BC time series and could be accurately modeled by the fitting function once random noise was removed. All the 205 BC time series were smoothed before being fitted. Fig. 2 shows a representative raw (in blue) and smoothed BC time series (in red).

The smoothed time series fit is superior to the fit of the un-smoothed time series in terms of mean RMSE. The smoothed BC series were used in the selected model fitting algorithms.

Multi-Tone Harmonic Model

Matlab's built-in non-linear fitting models were originally considered for fitting BC time series. Even though preliminary results were encouraging, it was quickly realized that the available Matlab models were lacking the fidelity required to capture the relevant frequencies in the BC time series. Further, custom non-linear Matlab models showed limited success and suffered from sensitivity to the initial guess of model coefficients. To overcome these problems, a multi-tone harmonic series based model was adopted which is defined in Eq. 4.

$$y(x) = m + \sum_{i=1}^{O} a_i cos(w_i x) + b_i sin(w_i x) \tag{4}$$

The BC value at time index x is y, m is the bias, i is the harmonic number, a and b are the scaling coefficients for the multi-tome harmonic, O represents the order of the harmonic series and w represents the frequency corresponding to the harmonic number.

The main idea of the Multi-tone Harmonic Model approach is to calculate w_i using an iterative frequency estimation algorithm and then obtain the model coefficients a, b and m via a least squares fit. This approach completely avoided the problem of guessing initial values for the model coefficients while still capturing higher order frequencies in the BC data. For the current study, the Quinn-Fernandes [5] algorithm was used for estimating various harmonic frequencies present in the BC time series data. The Quinn-Fernandes algorithm is an iterative, statistically optimal algorithm which may be interpreted as finding the maximum of a smoothed periodogram *.

After computing the model parameters ($w, m, a,$ and b) for each $i = 1 \ldots i_{max}$, the BC predictions were computed by using Eq. 4 for a future value of time index x. An upper limit on O was initially set to 200 and then optimized for each time series.

Model Generation

The model fitting algorithm can be divided into two stages: the calibration region optimization stage and the final model fitting stage. The smoothing of the BC time series was performed using Matlab's "smoothts" function. This processing of the input data helped suppress the random noise present in the data. It also improved the accuracy of the fitting process and increased robustness of the frequency estimation algorithm.

* HTTP://escape.library.uq.edu.au/eserv.php?pid=UQ:10626&dsIDc̄omparison-t.pdf

For each BC time series, stage 1 starts by decomposing the series into three parts. The prediction region, p, is fixed at 5 days. Values of 1, 2, 3, and 5 days were considered for the calibration region, c. For each c value, a two level optimization was carried out by adjusting the fit span, f, and the harmonic order, O. The purpose of this optimization was to minimize the amplitude relative error in the calibration region. An optimized value of f was found to be beneficial for improving the predictive capability of the fitted model over long intervals. The combination of f and O that gave the minimum amplitude relative error in the calibration region was then selected. Next the model was refit using the selected combination of f and O, including the calibration region in the optimized fit span ($f_{new} = f + c$). We then calculated the maximum amplitude relative error in the prediction region, p, for a 5 day prediction interval. The c value corresponding to the minimum error over the 5 day prediction region was finally selected and stored.

After the calibration region was selected for the 205 data files, the stage 2 algorithm decomposed each time series into three parts, the fit span, f, the calibration region, c, and the future prediction region. If this algorithm were used in real time to predict BCs, the future prediction region would correspond to a future period of time for which BC data was not yet available. As in stage 1, values of f and O were selected to minimize the amplitude relative error in the calibration region. The final model was then created by adding the calibration region to the selected fit span and then refitting over the new fit span ($f_{new} = f + c$). This process was carried out for each of the 205 BC time series data files. This final model is our working model and is used to predict the future BC values in the future prediction region.

Autoregressive Model

The autoregressive model (AR) is a linear prediction model which attempts to predict future outputs of a system based on the previous outputs. AR models are frequently used in analysis and forecasting of time series data in the field of econometrics. These models assume that the underlying series is approximately stationary, meaning that the mean remains relatively constant over the time span (a condition satisfied by BC data). The general form of an AR model for an input $y(t)$ is given in Eq. 5.

$$y(t) = \sum_{t=1}^{q} a_i y(t-1) + e(t) \tag{5}$$

The autoregressive coefficients are a_i and q is the order of the AR model. The noise term, e, is assumed to be Gaussian white noise. We used Matlab's built-in "ar" function with the default settings, to compute the AR model coefficients for a given q value. The order q depends on the input series and is generally identified using an autocorrelation analysis. Autocorrelation is the cross-correlation of a signal or a series with itself with some lag parameter. It may be described as the similarity between various observations as a function of the lag in the independent variable (i.e. time, in a BC prediction problem). This mathematical tool helps to find repeating patterns buried under noise, or in identifying the harmonic frequencies in a signal.

Model generation

Typically, before autoregressive model estimation, an autocorrelation analysis on the time series is performed. There was significant cross correlation between various values of the BC time series, so the order, q, could not be determined by directly inspecting the autocorrelation plot. To overcome this problem, we performed an optimization of q during the estimation of the AR model. The order q was allowed to vary between 1 and 35 during the optimization phase. As with the multi-tone harmonic model, we also performed an optimization of the fit span, f, which resulted in a two level optimization strategy for generating the AR model.

Prediction

The future predictions using the AR model employed MATLAB's built-in "predict" function. We implemented a linear AR prediction model which uses the latest predicted data along with the past data to make future predictions. Our approach is just one of various possible approaches to AR prediction and it may be valuable to look into other possible approaches for future work.

Time Series Fitting Models Performance Evaluation

The predictive capability of the selected time series fitting models (both the multi-tone and AR) was evaluated over prediction intervals of approximately 1, 3, 5, 7, 9 and 11 days. For each of these predictions, the RMS relative errors and max relative errors were computed and plotted. Upon investigation it was found that the multi-tone model is not very sensitive to the smoothing parameter, hence, for simplicity, a moving average value of 2 was used in Matlab's "smoothts" function to smooth the BC time series for all 205 models. Fig.s 3 and 4 show the RMS and max prediction errors over the various prediction intervals. The worst 5 outlying cases have been removed from these plots as the model failed to fit due to spikes and excess noise in the raw BC data. As expected, the multi-tone models performed very well for 1 and 3 day prediction, and the RMS errors for a 11 day prediction were found to be less than 12%. The AR model is comparable in performance to the multi-tone harmonic model over the whole range of prediction intervals.

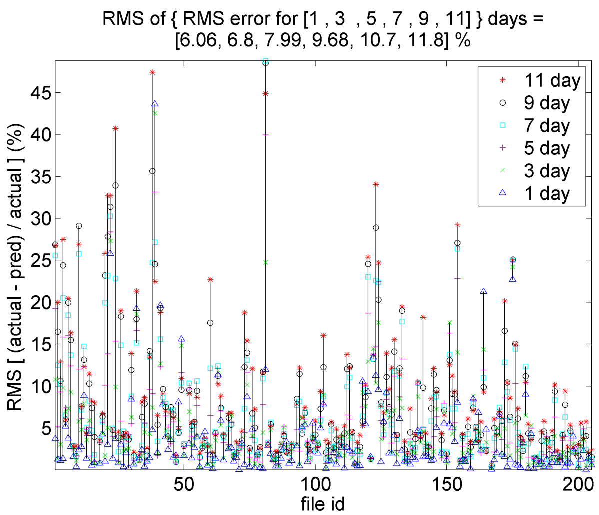

Fig. 3: Multi-tone Model RMS Relative Error

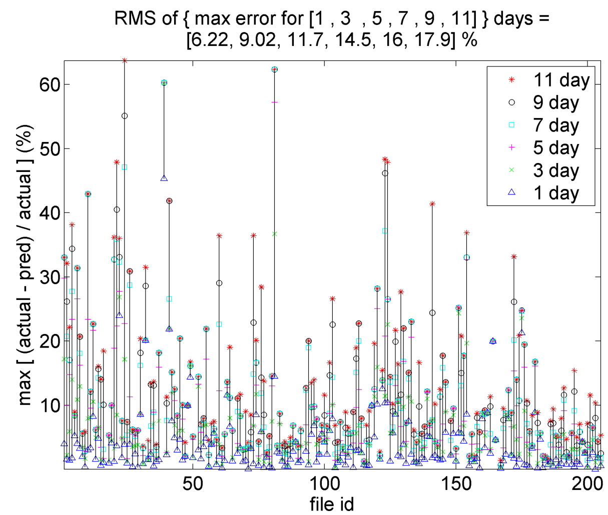

Fig. 4: Multi-tone Model Maximum Relative Error

Example fits using the multi-tone model for two representative cases are given in Figs 5 and 6. Note that many of the files have highly erratic evolutions of the BC that likely either represent BC estimation errors or maneuvers, such as seen in the Fig. 6, where the BC hovers near zero and even dips below zero in some cases. This unexpected behavior in the Air Force-provided BC data is an important motivation to use physics-based simulations to improve the forecasting and also the estimation of the BC values.

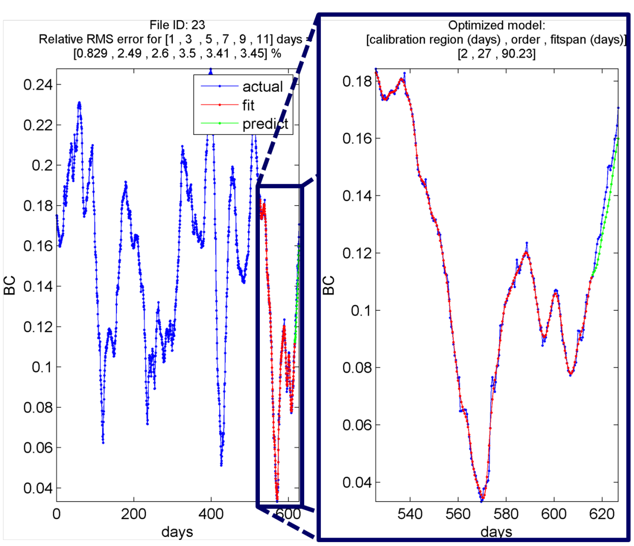

Fig. 5: BC time series fit for case with good performance

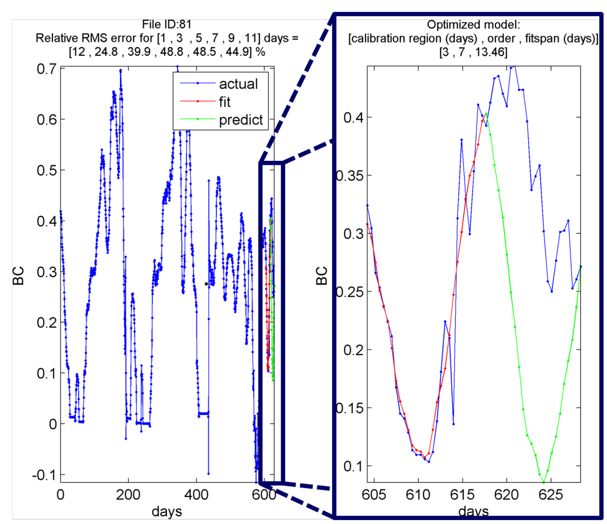

Fig. 6: BC time series fit for a case with noisy/poor input data

Fig. 7 summarizes the RMS and max relative errors for all the models considered in this report. We can see that the constant case model has super-linear error growth for the first 7 days, while the multi-tone and the AR model both show a linear progression in error. Another thing to note is that the maximum relative error for the multi-tone model is less than the RMS relative error of a constant value model for prediction intervals of 9 days or more. On average for a 7 day prediction, the multi-tone model performs better than the constant case model in 67% of 205 BC data sets.

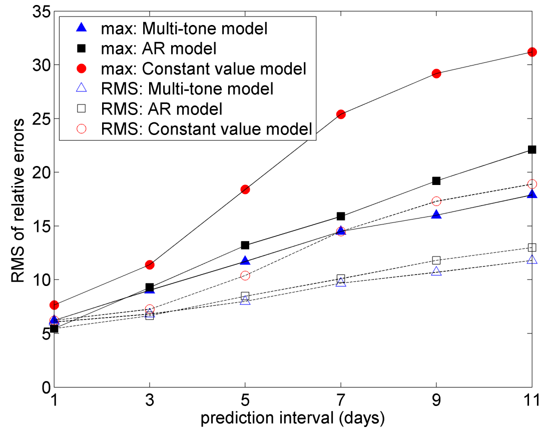

Fig. 7: Performance of the Multi-tone, AR and the Constant Value Models for all BC Data Files

Six Degree of Freedom Simulation

The goal of the 6DOF simulations was to aid in the prediction of the drag coefficient of an RSO. The behavior of the drag coefficient is determined by real, physical processes. The 6DOF simulations, therefore, aimed to model the physics as accurately as possible. It is impossible to expect that the simulation will show exactly the same behavior as the drag histories estimated from real satellite measurements. However, the simulation can help in identifying certain trends that can be expected in the real data.

The simulator models the effects on the motion of RSOs due to the non-uniformity of Earth's gravity field, third-body perturbations (using a 33x33 resolution mascon model that is easy to implement in MATLAB [6]), SRP, gravity-gradient torques and atmospheric drag. Thus, to first-order a physics-based history of the BC could be created. In order to compute SRP and drag perturbations, each RSO configuration used in the simulation was modeled as a system of flat plates. The forces on each plate were computed individually and summed to compute the total acceleration and torque on the RSO. The plate models were created using AutoCAD and then converted to MATLAB structures that could be used with the simulator. Several basic RSO shapes, as shown in Fig. 8, and 70 sets of initial conditions were used in the BC simulator.

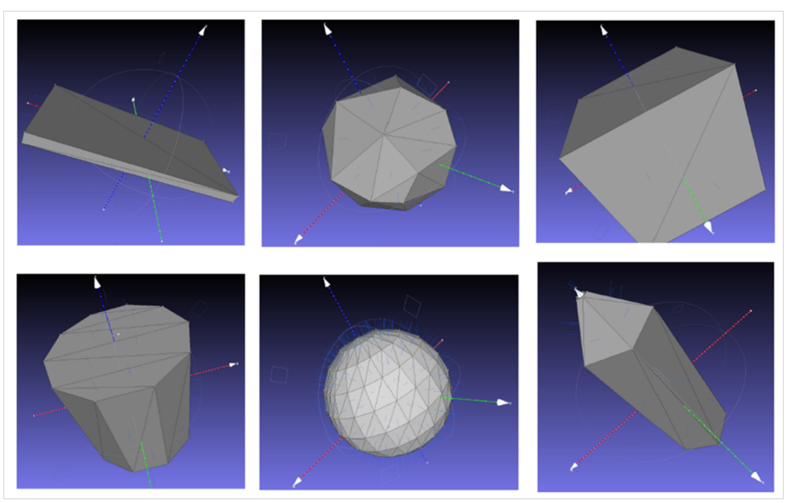

Fig. 8: Spacecraft shape models for high fidelity BC simulations

The results of the 6DOF simulation show the same kind of behavior as the actual BC data. However, due to the fast time scales of the attitude motion, this prototype 6DOF simulation can only be used to model BC over a much shorter time than the length of time covered by the real BC data. In reality, AFSPC estimates an averaged BC over several hours, whereas our simulations provide the full osculating values with frequencies as small as the period of a spinning satellite. Future work can leverage the high resolution data provided in these simulations. Simulations using each of the RSO models were run with random initial conditions in the orbit, attitude, and attitude rates. For the orbits, the following bounds are chosen, based on the fact that they are typical of orbits containing space debris [4]:

- Semi-major axis: 7000 - 7400 km
- Eccentricity: 0 - 0.06
- Inclination: 75° - 98°

For all 70 simulation runs, the initial attitude was fixed by using a uniform random distribution between 0 and 1 for all 4 quaternion elements and then normalizing them. The initial body-fixed angular velocity of the RSO determined the tumble behavior of the RSO. Out of 70 cases, 46 cases had an initial angular velocity between 0 and 10°/sec in all three axes. In the remaining 24 cases, the RSOs were given a random spin velocity of up to 57°/sec (1 rad/sec) along a random axis.

Typical results from 12-hour simulations for all the RSO configurations with various initial conditions are shown in Fig. 9.

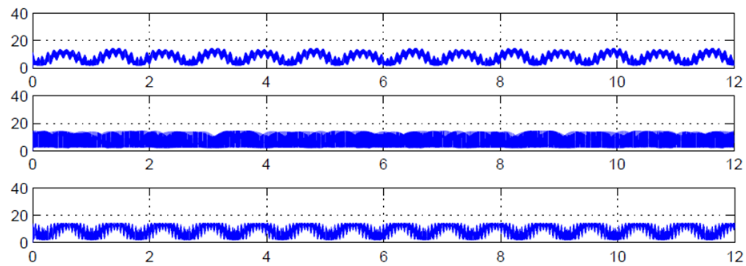

Fig. 9: Simulated BC (kg/m2) vs. time (hrs) for 3 initial conditions for flat plate RSO

To test and validate the time series prediction algorithms, an example BC series was simulated for 10 days, 5% random noise was added to the data, and the multi-tone model was applied to the simulated data to predict future BC values. The resulting fit is illustrated in Fig. 10. The results validated the optimization process for choosing model parameters, as the predictions easily capture the major frequencies in the data, even over several cycles. In addition to validating the time series prediction methods, the results also suggested that estimated BCs from the Air Force include effects not accounted for in the 6DOF simulations.

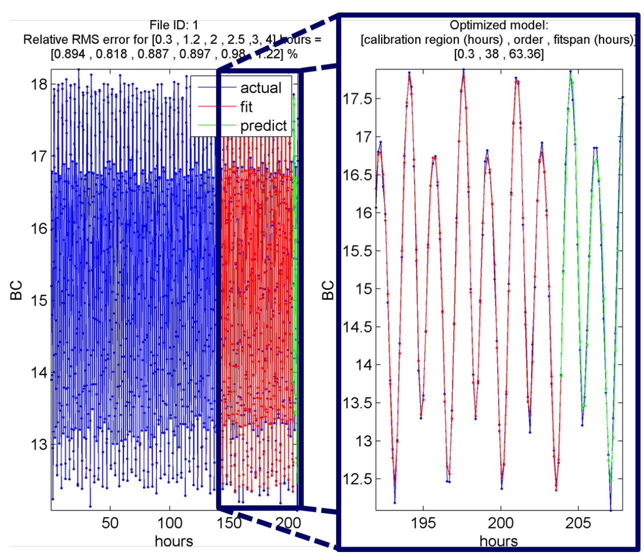

Fig. 10: Empirical time series algorithm applied to simulated BC data

Conclusions

Conclusions and Future Work in Time Series Analysis

The main focus of this project was to develop a black box BC time series prediction methodology. Three types of prediction models were considered, namely: the non-linear MATLAB based model, the multi-tone harmonic model and the autoregressive (AR) model. The non-linear MATLAB model was discarded due to inferior performance and the other two models were further developed. Both the multi-tone model and the AR model were subjected to multiple levels of optimizations resulting in highly optimized final models with improved prediction capabilities. Specifically, the multi-tone model performs well, giving 1 to 7 day prediction errors (RMS) of less than 10% and 11 day prediction errors (RMS) of approximately 12%. The AR model performs almost as well as the multi-tone model, with 1 to 7 day prediction errors (RMS) of less than 10% and 11 day prediction errors (RMS) of approximately 13%. Even though the performances of the multi-tone and AR models were satisfactory, there exists substantial room for improvement. Suggested improvements include the following:

1. Remove large amplitude and correlated noise from BC data to reduce model divergence via denoising algorithms, which can be found in signal processing literature.
2. Use known local derivative information to influence model parameters for a more accurate short term prediction model.
3. Port the algorithm to a compiled language (e.g., Fortran or C), to allow improved algorithm speed and efficiency for higher fidelity model optimization.
4. Understand and implement ARMA and ARMAX models, which improve on the AR model because they take into account the noise present in the BC data. ARMA models are particularly helpful when there are unobserved shocks present in the data. The ARMAX models incorporate exogenous terms capable of modeling some stochastic processes present in the BC data.
5. Identify model frequencies more accurately by using data with larger time spans and less noise.
6. Design a weighted hybrid model using the multi-tone and the ARMAX models, which could provide better predictive capabilities over longer prediction intervals.

Conclusions and Future Work on 6 DOF Simulations

For this project, a 6DOF simulator for RSOs was created in MATLAB. The simulator models the effect of perturbations on the motion of RSOs due to the non-spherical Earth, third-body perturbations, SRP, and atmospheric drag. Thus, a physics-based profile of the BC can be created.

Instead of assuming the RSO to be a point mass for SRP and drag perturbation computations, a flat plate model was created for each RSO configuration used in the simulation. The forces were computed for each plate individually and summed to compute the total acceleration and torque. The plate models were created using a CAD program such as AUTOCAD and then converted to MATLAB structures that could be used with the simulator.

The results of the simulations of various RSO shapes with differing initial conditions show that the physics-based BC of RSOs is smooth and periodic. The noisiness of the real data is, therefore, due to the uncertainties in the estimation process.

For future work, the MATLAB simulator will be converted to a compiled language such as Fortran. This will result in a large decrease in CPU time, which will allow for simulations over larger time scales. Also, more accurate atmospheric models should be used, since only the exponential density model was used here. For the drag computations, only the C_D has been used. However, the complete aerodynamic coefficient vector can be used during future work. The simulator will also be expanded to include effects such as full and partial plate shadowing.

Acknowledgments

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