

Improving Trans-ionospheric Geolocation of High Frequency Signals Using Parallel Processing and Assimilative Ionospheric Models

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ABSTRACT

All electromagnetic radiation undergoes refraction as it propagates through the atmosphere. Tropospheric refraction is largely governed by interaction of the radiation with bounded electrons; ionospheric refraction is primarily governed by free electron interactions. The latter phenomenon is important for propagation and refraction of High Frequency (HF) through Extremely High Frequency (EHF) signals. The degree to which HF to EHF signals are bent is dependent upon the integrated refractive effect of the ionosphere: a result of the signal's angle of incidence with the boundaries between adjacent ionospheric regions, the magnitude of change in electron density between two regions, as well as the frequency of the signal. In the case of HF signals, the ionosphere may bend the signal so much that it is directed back down towards the Earth, making over-the-horizon HF radio communication possible. Ionospheric refraction is a major challenge for space-based geolocation applications, where the ionosphere is typically the biggest contributor to geolocation error. Accurate geolocation requires an algorithm that accurately reflects the physical process of a signal transiting the ionosphere, and an accurate specification of the ionosphere at the time of the signal transit. Currently implemented solutions are limited by both the algorithm chosen to perform the ray trace and by the accuracy of the ionospheric data used in the calculations. This paper describes a technique for adapting a ray tracing algorithm to run on a General-Purpose Graphics Processing Unit (GPGPU or GPU) and using a physics-based model specifying the ionosphere at the time of signal transit. This technique allows simultaneous geolocation of significantly more signals than an equivalently priced Central Processing Unit (CPU) based system. Additionally, because this technique makes use of the most widely accepted numeric algorithm for ionospheric ray tracing, and a timely physics-based model of the ionosphere instead of the typically used climatologically derived one, we assert that this technique improves geolocation accuracy.

1. INTRODUCTION

There is clearly a desire for greater accuracy in geolocating the source of transionospheric signals. Accurately geolocating the transmitter of a transionospheric signal via use of a Single Site Location (SSL) receiver has broad

application. Accurate geolocation requires high precision ray tracing through the atmosphere and ionosphere. The physics and models required to perform these types of high precision ray traces have been around since the 1950s. Often times, to geolocate large numbers of transionospheric signals (e.g., television/radio broadcasting, radars and "short-wave" communications), system designers use a generalized approximation for ray tracing, along with a climatological representation of the ionosphere. The challenge being addressed by this paper is how to accurately geolocate the great volume of signals emanating from Earth in an efficient and timely manner.

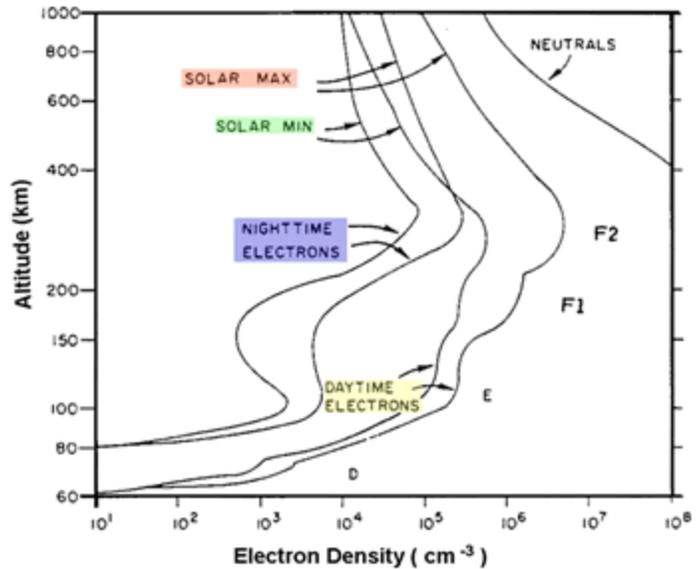


Fig. 1. Ionospheric profiles [1]

The following description of the ionosphere is derived from those of the Naval Postgraduate School [1] and Massachusetts Institute of Technology Haystack Observatory [2].

The ionosphere is the region of the atmosphere between 50 and 2000km above the Earth. It is typically divided into several layers referred to, in increasing altitude, as the D, E, F1 and F2 layers (see Fig. 1). Each of these layers has different characteristics based on chemical composition, density of free electrons at any given time and how widely the electron density varies over the day/night cycle. In general the ionosphere is characterized by higher molecular concentrations in the lower altitudes, and lower molecular concentration in the upper altitudes. The sun's energy excites the molecules, causing them to separate into negatively charged electrons and positively charged ions. Collectively these electrons and ions are referred to as charged particles. The population density of all particles in the D-layer is higher than the E and F region. Recombination rates are higher because there are more opportunities for collisions (higher collision rate). That's why the D region disappears at night, as shown in Fig. 1, as the photoelectric energy source producing ionizing radiation is removed and the high recombination rates quickly deplete the ion density. This results in a diurnal oscillation of the D layer, with this layer being the most prevalent during the sunlit hours. Similarly, the E layer varies widely between day and night. Daily variation in the F layers is much lower due to the rarity of interactions between charged particles as altitude increases.

Why do we need to understand the structure, characteristics and composition of the ionosphere? Understanding these helps us to understand the medium through which a signal must travel. These characteristics help to describe the distribution of charged particles in the ionosphere. The charged particles interact with the electromagnetic High Frequency (HF) through Extremely High Frequency (EHF) radio waves causing them to bend as they pass through the atmosphere. In order to accurately geolocate the transmitter of a transionospheric signal we integrate the refractive effect that each layer of the ionosphere has upon the emitted signal as it is back-tracked to its point of origin. This process for geolocating a signal emitter is referred to as transionospheric SSL throughout the remainder of this paper.

2. CURRENT STATE OF TECHNOLOGY

Current Central Processing Unit (CPU) based solutions are handicapped by the exponential increase in resources/cost when scaling to geolocate a large volume of signals. This constraint is generally overcome through the implementation of a less rigorous and subsequently less accurate ray tracing algorithm. In an attempt to reduce the computational cost of geolocating a signals source some systems are designed to account for the ionospheric impact on a signal at relatively few discrete locations (e.g., between the D, E, F1 and F2 layers, at a single point in the ionosphere, or even ignore ionospheric impact altogether). The dichotomy that has established itself in these systems is accuracy being traded for computational speed: increasing either accuracy or speed negatively impacts the other, and both are desired.

In addition to a less rigorous ray tracing algorithm, most current geolocation efforts use a climatologically based model of the ionosphere as input data for the ray tracing scheme employed. The International Reference Ionosphere (IRI) has been recognized as the international standard model for the ionosphere [3]. "It provides monthly averages in the non-auroral ionosphere for magnetically quiet conditions." [4] Until very recently this was the most accurate representation of the state of the ionosphere that could be provided. However, advances in both data collection and ionospheric modeling will soon mean we can specify the state of the ionosphere with greater precision and higher resolution. Already there are models that outperform IRI under certain ionospheric conditions, and rapid progress is continuing in this field.

An approach to geolocating a signal's source based on the kinds of approximations described above is not as accurate at correcting ionospheric impacts on a transionospheric signal as one based on a near real-time profile of the ionosphere combined with a ray tracing algorithm that properly integrates the effect this profile has upon the signal.

3. OUR TECHNIQUE

To counter the above mentioned handicaps in processing, we have tailored the most widely accepted numerical algorithm for ionospheric ray tracing, the Jones-Stephenson model, for execution on a General-Purpose Graphics Processing Unit (GPGPU or GPU). This system is designed to take advantage of the potentially thousands of GPGPU threads, as opposed to the relatively few CPU cores of current systems implemented for transionospheric SSL. By taking advantage of the increased processing power of the GPGPU we are able to make use of a more accurate and computationally costly adaptive time stepping based algorithm for integrating the refractive effect of the ionosphere on the emitted signal, while yielding an order of magnitude improvement in throughput compared to a CPU based system.

This system also uses ionospheric specification data produced hourly by the Global Assimilative Ionospheric Model (GAIM) instead of the current reference ionosphere model. GAIM is a Gauss-Markov Kalman Filter (GMKF or GM) based model, which assimilates near real time measurements of the ionosphere taken globally. "GAIM's purpose is to assimilate multiple types of ionospheric measurements including ground and space-based GPS, ionosonde profiles, UV airglow radiances, in situ electron and neutral densities, plasma drifts, neutral winds, neutral densities, or other types of measurements creating the equivalent of Numerical Weather Prediction (NWP) models. The assimilation of ionospheric measurements into mature first-principles ionospheric models will produce physically consistent accurate ionospheric analysis, as well as the important ionospheric drivers, therefore enabling the generation of more accurate ionospheric weather forecasts." [5] The model, as run at the Air Force Weather Agency (AFWA), produces hourly analysis and forecast data of the ionosphere. The Air Force Research Laboratory (AFRL) has performed validations of the GAIM model. Furthermore, "AFRL has confirmed (albeit with some ongoing issues) [see, for example, McNamara et al., 2010] that the GAIM-GM model is a very successful data assimilation model that provides generally reliable real-time specifications of the ionosphere." [6,7] By using GAIM data we anticipate seeing significant differences in ray tracing results under conditions where the ionosphere deviates significantly from the reference ionosphere. GMKF will soon be replaced by a much higher resolution model that is fully physics based. The full physics model is intended to address the issues of the GAIM-GMKF model. In order to use the full physics model data, the required ray tracing computational resources will significantly increase, perhaps by 500%. Again, employing a GPGPU to allow massive parallel processing is an effective counter to the expected increase in computational cost of transionospheric SSL, thereby enabling the system to take advantage of this improved ionospheric specification and forecast information.

Technical performance measures were established prior to the implementation of our techniques in order to clearly define the criteria for declaring them successful. First we must show a meaningful increase in accuracy. This is difficult to measure since this study is simulation-based, and field testing is beyond the scope of this project. Therefore our claim of increased system accuracy must be based on increased accuracy of each of the constituent parts of transionospheric SSL (i.e., the algorithm and the ionospheric model), resulting in a more accurate geolocation of a signal's origin. The degree of the difference between the different techniques for geolocation would constitute a quantitative measure for the increase in accuracy. For the purpose of establishing a performance measure it was determined that if the great circle distance between this technique and others was on the order of tens of meters a meaningful increase in accuracy had been achieved. Secondly we must show a 10 times increase in

transionospheric SSL throughput (number of rays processed) when the Jones-Stephenson 3D ray tracing algorithm is tailored to run on a GPGPU compared to the CPU.

4. ASSUMPTIONS

Some initial assumptions about a transmitting signal were made for the simplicity of getting a working prototype of the transionospheric SSL system.

As previously stated the system relies on simulations and improvements in the constituent parts as the method for validating the assertions of increased accuracy. Accuracy is assumed to increase because of the replacement of the climatologically derived representation of the ionosphere with a near real-time representation of it and the use of the Jones-Stephenson ray tracing model as opposed to one that is more approximated and less rigorous. It is acknowledged that empirical evidence is needed to bolster this assertion. This, however, was beyond the scope of this project.

Given the calculated signal path we are able to provide a geolocation for any given altitude/signal path intercept. We assume, however, that all transmitters are emitting from the surface of a perfectly spherical Earth. Recovering a transmitter's altitude would not be possible without measuring multiple rays from a transmitter simultaneously. This does not impact the validity of the study since the two primary focuses are the volume of ray tracings performed and comparison of ray tracing results to the most widely accepted algorithm for ionospheric ray tracing. Since the algorithms being compared make the same assumption, a direct comparison of speed of execution and accuracy of results can be made.

The third assumption eliminates the possibility of a signal making multiple hops to arrive at its ultimate location in the simulations. The scope of this project is transionosphere ray tracing, and it is felt that once a signal hits the ground it then becomes a ground to ground ray tracing problem. While the Jones-Stephenson model can account for ground to ground, we do not believe analysis of ground-to-ground signal propagation adds value to this particular study.

It is assumed that because the IRI model is climatologically derived, it misses some fine scale irregularities in the ionosphere. GAIM is assumed to captures much more of these irregularities because it is assimilative and physics based.

Finally the effects of absorption on the wave due to collisions within the plasma are ignored. Because we are primarily concerned with the refraction of the ordinary wave without absorption it is not necessary to consider these collisions. Here again, since the algorithms being compared make the same assumption a direct comparison of speed and accuracy can be made.

5. JONES STEPHENSON RAY TRACING PROGRAM

Our first major endeavor in completing this project was to develop an operational version of the Jones Stephenson Ray Tracing Program (JSRP). This code, originally developed in FORTRAN, was written by R. Michael Jones and Judith J. Stephenson [8, 9]. It implements the Haselgrove equations for ray tracing:

$$\begin{aligned}\frac{r'}{c} &= \frac{1}{\mu^2} \left[v_r - \mu \frac{\partial \mu}{\partial v_r} \right] \\ \frac{\theta'}{c} &= \frac{1}{\mu^2 r} \left[v_\theta - \mu \frac{\partial \mu}{\partial v_\theta} \right] \\ \frac{\varphi'}{c} &= \frac{1}{\mu^2 r \sin \theta} \left[v_\varphi - \mu \frac{\partial \mu}{\partial v_\varphi} \right] \\ \frac{v_r'}{c} &= \frac{1}{\mu} \frac{\partial \mu}{\partial r} + v_\theta \frac{\theta}{c} + \sin \theta v_\varphi \frac{\varphi'}{c}\end{aligned}$$

$$\frac{v_{\theta}'}{c} = \frac{1}{r} \left[\frac{1}{\mu} \frac{\partial \mu}{\partial \theta} - v_{\theta} \frac{r'}{c} + r \cos \theta v_{\theta} \frac{\varphi'}{c} \right]$$

$$\frac{v_{\varphi}'}{c} = \frac{1}{r \sin \theta} \left[\frac{1}{\mu} \frac{\partial \mu}{\partial \varphi} - \sin \theta v_{\varphi} \frac{r'}{c} - r \cos \theta v_{\varphi} \frac{\theta'}{c} \right]$$

Where

r, θ, φ are coordinates of a point in a spherical coordinate system with the origin being Earth's center
 c the velocity of light in a vacuum.

μ is the refractive index of the atmospheric medium at the point of intersection with the ray

v_r, v_{θ} and v_{φ} are components of the wave normal direction in the r, θ, φ directions, normalized so that
 $v_r^2 + v_{\theta}^2 + v_{\varphi}^2 = \text{Re}\{\mu^2\}$.

These equations can be used to describe changes in the trajectory of HF through EHF signals when traversing the ionosphere. This algorithm requires knowledge of several characteristics of the received signal in order to calculate its most probable propagation path from origin to the SSL receiver: the angle of incident with the receiver, the SSL receiver's position, and transmission details, such as frequency. The Hasselgrove equations require knowledge of the refractive index of the ionosphere. This can be determined from the Appleton-Hartree formula (ignoring collisions):

$$\mu^2 = 1 - \frac{2X(1-X)}{2(1-X) - Y_T^2 \pm \sqrt{Y_T^4 + 4(1-X)^2 Y_L^2}}$$

Where

$$X = \frac{f_N^2}{f^2}$$

$$Y_T = \frac{f_H}{f} \sin \psi$$

$$Y_L = \frac{f_H}{f} \cos \psi$$

f_N is the plasma frequency

f_H is the electron gyrofrequency

f is the wave frequency

ψ is the angle between the wave normal direction and the earth's magnetic field.

As mentioned in our initial assumptions, for the sake of simplicity we chose to ignore absorption so we did not implement a plasma collision model, but did implement a dipole model of the Earth's magnetic field for mapping ordinary and extraordinary waves.

For transionospheric SSL, the JSRP was modified to model the potential path(s) back to the sender given the receiving antenna location, signal frequency, and the signal azimuth and elevation angles with respect to the antenna. Since the system's objective is to recover a signal's propagation path with a known receiver location, propagation is performed from the receiver to the signal origin; as such, several effects on the emitted signal were inverted in the algorithm to account for the "backward" propagation through the ionosphere. Another modification to JSRP was required since the model was designed to propagate signals in or below the ionosphere. In the transionospheric SSL model, checks are performed to verify whether or not a wave is above the ionosphere and the direction the wave is heading. This simply enables the system to distinguish between a ray escaping into space or headed towards the earth.

6. IMPLEMENTATION OF THE REFERENCE IONOSPHERE

Another major modification to the JSRP is the implementation of the GAIM model to define the electron density functions. Implementation of electron density functions within the JSRP requires the following measures be defined:

$$X = \frac{\partial X}{\partial r} \frac{\partial X}{\partial \theta} \frac{\partial X}{\partial \phi}$$

Where

X is the normalized electron density

$\frac{\partial X}{\partial r}$, $\frac{\partial X}{\partial \theta}$, and $\frac{\partial X}{\partial \phi}$ are the gradients of the normalized electron density in spherical coordinates (r, θ, ϕ) .

The normalized electron density X is related to standard electron density through the equation:

$$X = \frac{80.5E10^{-6} N}{f^2}$$

Where

f is the wave frequency in MHz

N is the electron density in cm^{-3}

Once X is calculated we are able to find the value of its gradients by finding the numerical partial derivative of the wave at its present position. It is vital that both X and its gradients $(\frac{\partial X}{\partial r}, \frac{\partial X}{\partial \theta}, \frac{\partial X}{\partial \phi})$ be continuous functions of position; otherwise the program will run slowly or give erroneous results. To ensure continuous functions we implemented a three dimensional linear interpolation on the electron densities. This ensures a smooth transition between layers in the reference ionosphere. The three dimensional interpolations also provide a realistic electron density value for arbitrary positions in the ionosphere compared to the relatively coarse gridded data that exists in the GAIM model. It is important to note that the GAIM model produces a 3-dimensional electron density distribution at user-specified times: these data are produced at discrete positions, in latitude, longitude, altitude coordinates. For the purpose of our comparisons, IRI data was developed at the same discrete positions available in the GAIM data set. The aforementioned linear interpolation between discrete points in the atmospheric model therefore applies the same to both data sets, and “resolution” of the two atmospheric models is the same for our studies.

As mentioned earlier, current models use a reference ionosphere based upon climatological data. We are theoretically able to obtain improved results with the use of a near real-time reference ionosphere.

7. GEOLOCATION PERFORMANCE

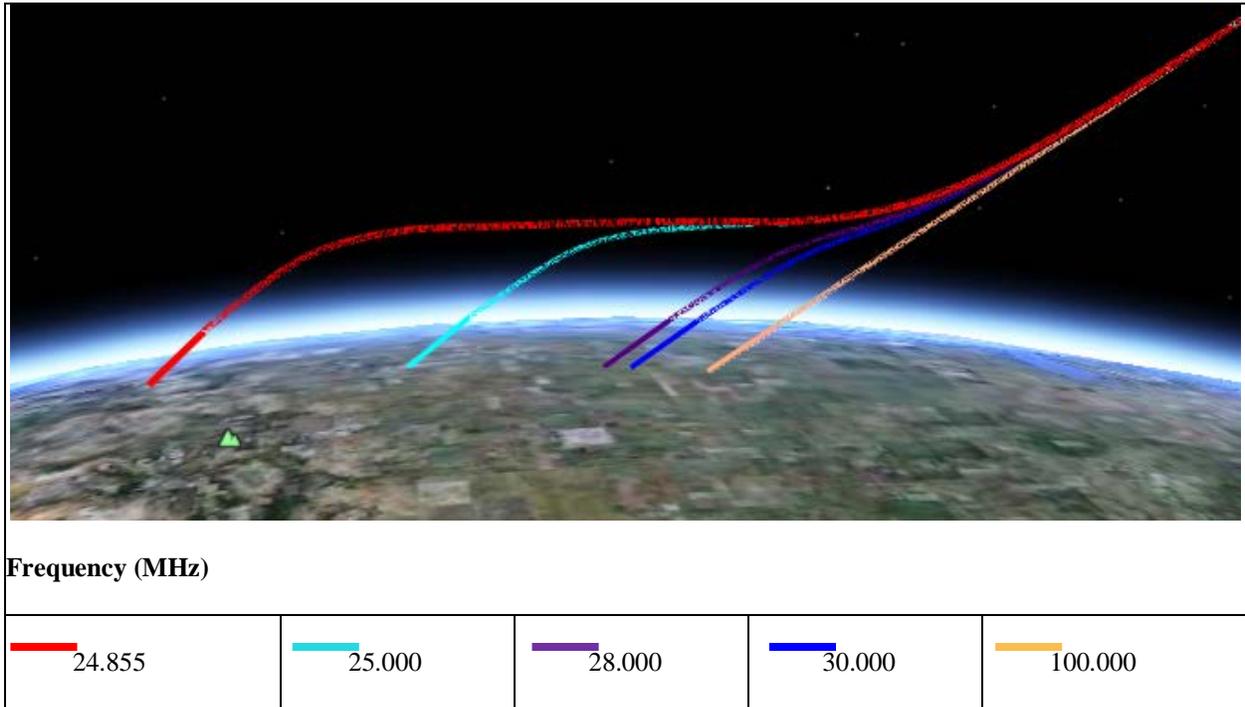


Fig. 2. Varying rf frequency has significant impacts on the refraction of trans-ionospheric radiofrequency signals.

Accurate geolocation of radiofrequency signals from a space-based platform requires an accurate representation of signal refraction through the ionosphere. To understand the dependence of frequency on ionospheric refraction, we pose an experiment with a fixed signal receiver, located at 18.90° N, 56.5° W, and altitude of 18000 km, and a fixed angle of incidence. By varying the frequency of the signal we are able to affect final calculated geolocation of the emitter, as shown in Fig. 2. One can see in this figure that, as expected, the highest frequency signals, at 100MHz, are much less affected by the ionosphere than the lower frequency signals. For our test case, the 100MHz refracted signal arrives on the earth's surface 8 cm away from the location that would be predicted using a direct-path measurement. For the lower frequencies in our experiment, the HF signals are highly affected by the ionosphere, with the 25 MHz signal arriving over 560 km away from the location predicted using direct-path measurement.

The 24.855 MHz wave shows a natural phenomenon known as tunneling in the ionosphere. Here a wave travels perpendicular to the surface of the earth in a region bounded by higher electron density on either side. Finally the signal reaches a region where the electron density is low enough for the wave to refract enough to exit the bounding area and reaches the ground. In this case the final location is over 1000 km from the straight line path.

To better understand the sensitivity of the results to the underlying ionospheric data, we compared transionospheric SSL using data from the climatologically-derived IRI to that obtained using the GAIM model. The date of November 6th 2001 was chosen for the test runs due to high solar activity, during which we would expect a much higher Total Electron Count (TEC) and the large resulting deviation from a climatologically-averaged atmosphere. The results are shown in Fig. 3.

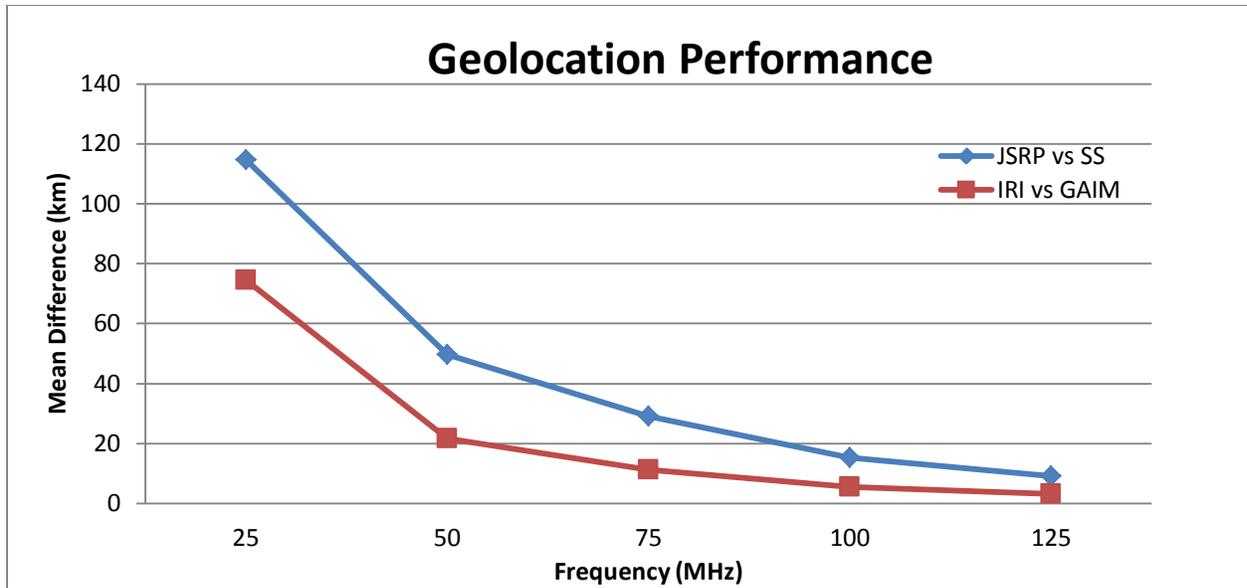


Fig. 3. Comparison of geolocation performance using November 6, 2001 ionospheric data. The lower line, in red, shows the average difference in predicted emitter location when the ray tracing algorithm is run with GAIM data versus IRI data. The upper line, plotted in blue, shows the average difference between geolocation using a Single Shell (SS) algorithm and the ray tracing algorithm (both run on the same GAIM data).

A variety of incident angles and receiver locations were tested, however the same altitude was used in all backwards ray tracings. A “mean difference” metric is plotted in this figure, defined as follows. Difference measures are obtained by varying only the atmospheric model (IRI versus GAIM) at a given frequency, incident angle and receiver location; a collection of difference metrics is then obtained by varying incident angle and receiver location. The collection of difference metrics is then averaged to obtain the mean difference. A large “mean difference” is indicative of a large variation between test cases, atmospheric models in this test. As is evident in Fig. 3, the results obtained using the IRI data are vastly different than those using GAIM. We assume this is because the IRI model represents the ionosphere as more or less homogenous over large regions, and often misses some of the fine scale irregularities since it is climatological in nature. GAIM, on the other hand shows much more of such irregularities since it is assimilative and physics based.

Additionally we compared the JSRP algorithm with a single shell algorithm (both using the GAIM data). The single shell algorithm is mainly used when modeling GPS signals that are of much higher frequencies than those tested here. It attempts to account for the deflection of the signal throughout the entire ionosphere at a single point that represents the greatest index of refraction climatologically. As can be seen in Fig. 3, the SS algorithm gives rather poor geolocation results at the tested frequencies.

8. SPEED OF GEOLOCATION

Computations were performed using an Nvidia Tesla C2070 with CUDA 4.0 GPGPU, and an Intel Xeon 2.4 GHz as our reference CPU. The use of the GPGPU enabled the transionospheric SSL to perform magnitudes better than the CPU implementation of the JSRP in certain circumstances. It is important to note that the ionospheric model used has no impact on the speed of execution. This is expected, since the data sets from each of the two ionospheric models were developed with the same 3-dimensional grid spacing. Fig. 4 shows the amount of time in seconds to calculate a varying number of transionospheric paths using 1 km integration resolution. Integration resolution is the maximum propagation distance a ray can travel before recalculating the Haselgrove equations.

Initially it is faster to calculate a batch of rays on the CPU. The primary reason for this is that the core clock of the GPGPU is 1150MHz compared to the CPUs 2.4GHz. In addition there is an overhead cost associated with the starting up of the GPGPU, as well as the time to allocate and transfer all needed data to and from it. It takes roughly 2.65 seconds to startup, transfer, and return from the transionospheric SSL GPGPU version algorithm. After this

Ray Tracing Time

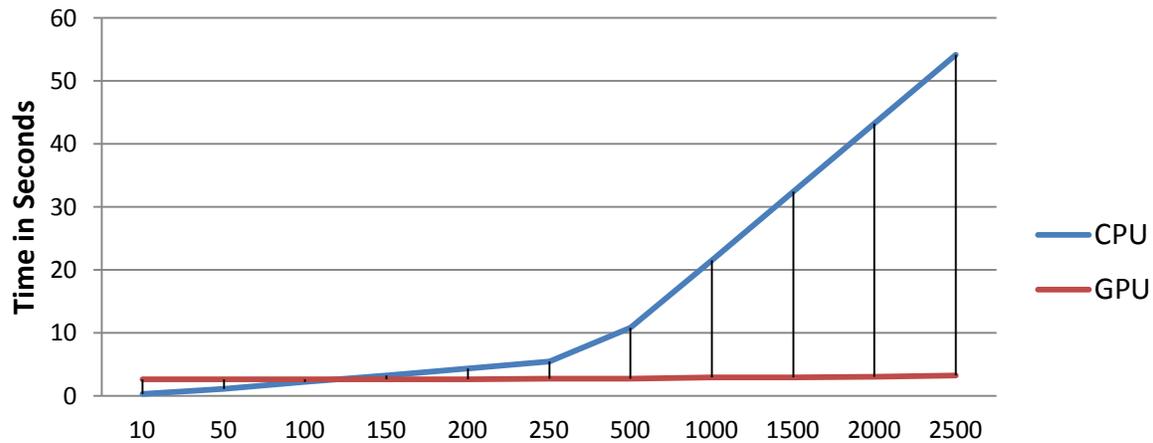


Fig. 4. Ray tracing time, comparing CPU to GPU execution time.

initial hit in computation time, calculating the propagation path of a ray on the GPGPU happens at an extremely rapid rate due to its ability to massively parallelize. The GPGPU has 448 processors per card while the CPU has 6 cores per processor. As a result, adding 500 rays to the GPGPU calculation only adds an additional 0.156 seconds while it adds 10.697 seconds to the CPU computation time.

Our technical performance goal of a 10 times increase in throughput is reached within 3 seconds. As shown in Fig. 4, it takes 3.28 seconds to calculate 150 profiles on the CPU and only 2.965 seconds to calculate 1500 profiles on the GPGPU. We are also able to get 10 times the resolution compared to the CPU. For instance, the transionospheric SSL can calculate 150 profiles on the GPGPU with 0.1 km integration resolution in 3.25 seconds compared to 3.28 seconds for 1 km resolution on the CPU.

9. DISTINCTIVE ATTRIBUTES AND ADVANTAGES

With this transionospheric SSL system we feel we have accomplished our goals.

We have successfully adapted the JSRP for execution on a GPGPU. The use of a GPGPU can significantly improve the performance of transionospheric SSL systems without resorting to trading reduced accuracy for speed. This system not only delivers an increase in computational power but can give a near real-time representation of the ionospheric effects on HF through EHF signals, the main factor contributing to error in geolocating transionospheric signals.

The system was also able to use the GAIM-GM model, or any other electron density based ionospheric model, to perform ray tracing calculations. With this system we are able to obtain higher throughput and resolution. This will help us to meet the processing demands of the much higher resolution fully physics based GAIM model.

From a computational perspective, the approach used here, leveraging GPU's and massive parallelization, could be used to deal with other ray tracing problems including signal propagation in other regions of the atmosphere, surface obstacle ray tracing of acoustic and electromagnetic signals, and underwater acoustic or electromagnetic signal ray tracing.

10. CONCLUSION

Accurately geolocating the transmitter of a transionospheric signal via use of a SSL receiver has broad application. Accurate geolocation requires high precision ray tracing through the atmosphere and ionosphere. The physics and models required to perform these types of high precision ray traces have been around since the 1950s. The challenge

being addressed by this paper was how to accurately geolocate the great volume of signals emanating from Earth in an efficient and timely manner.

In order to accurately geolocating the transmitter of a transionospheric signal we integrate the refractive effect each layer of the ionosphere has upon the emitted signal as it is back-tracked to its point of origin. By replacing the climatologically-derived reference ionosphere with a near real-time model of the ionosphere (GAIM) we are theoretically able to obtain improved results. By tailoring the most widely accepted ionospheric ray tracing algorithm, the Jones-Stephenson model, for execution on a GPGPU we are able to markedly improve the performance of ray tracing to a level that accommodates both the volumes of transionospheric signals expected and accommodate the increased resolution of the expected full physics based models of the ionosphere expected in the near future.

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