Track-to-Track Association Using Bhattacharyya Divergence

Islam I. Hussein, Christopher W. T. Roscoe and Matthew P. Wilkins
Applied Defense Solutions, Columbia, Maryland

Paul W. Schumacher, Jr.
Air Force Research Laboratory, Kihei, Hawaii

ABSTRACT

There are three primary types of association problems of interest in space surveillance: the classical observation-to-track association (OTTA) problem, the track-to-track association (TTTA) problem, and the observation-to-observation association (OTOA) problem. In this paper, we build on recent work to further investigate the use of information theoretic criteria to solve the TTTA problem, in which we have multiple uncorrelated tracks (UCTs) to be tested for association against a given set of tracks given at a different (usually previous) time instance. Both the tracks and the UCTs are uncertain and are probabilistically described using multivariate normal distributions. This allows for a closed-form solution, based on the unscented transform and on the Bhattacharyya information divergence for Gaussian distributions. We will establish a connection to the covariance-based track association (CBTA) technique, and compare the performance of the two methods in extensive parametric Monte Carlo simulations over several orbital regimes.

1. INTRODUCTION

In general, there are three types of data association problems in space situational awareness (SSA). The first is the classical observation-to-track association (OTTA) problem, where an analyst seeks to associate each observation with a unique track (or none, i.e., a clutter-generated observation) given an observation with some known measurement statistics and a set of existing candidate uncertain resident space object (RSO) tracks. The second association problem is where we are given a set of observations at different time instances and wish to determine which of these observations were generated by the same RSO. This is the observation-to-observation association (OTOA) problem. The authors developed information theoretic solutions for this problem and were reported in Ref. [1,2]. In that work, we developed information theoretic criteria to solve the problem. Several criteria were proposed including, mutual information and information divergence. It was generally found that mutual information outperforms information divergence in solving the OTOA problem. The third and final association problem is where we have multiple tracks or UCTs at different time instances and wish to determine whether any of the tracks belong to the same RSO (and which were clutter generated). This is the track-to-track association (TTTA) problem. This problem is the subject of this paper.

We build on our recent work [1,2] to investigate the use of information criteria, specifically the Bhattacharyya information divergence, to solve the TTTA problem in SSA, where we seek to associate a given set of tracks at one point in time with another set of tracks or UCTs at a different time instance. The use of information theoretic criteria were also proposed in the space tracking community to solve the same problem -see for example, Ref. [3]. Both sets of tracks are uncertain and are probabilistically described using multivariate normal distributions. The normality assumption allows for the use of the unscented transform [4] and a closed-form solution that is based on the Bhattacharyya information divergence. To date, the main solution approach to the TTTA problem is the covariance-based track association algorithm (see Ref. [5] and references therein). As the name implies, it is a covariance-based approach to track association. The CBTA criterion for association can be linked to information theoretic metrics, especially the Bhattacharyya information divergence. In this paper, we will discuss these relationships and conduct a performance comparison analysis between the proposed information divergence solution and CBTA. We will demonstrate the main result and performance comparison with CBTA in a parametric Monte-Carlo simulation in the following orbital regimes: low earth orbit (LEO), medium earth orbit (MEO), Geostationary transfer orbit (GTO) and geosynchronous orbit (GEO).

The rest of the paper is organized as follows. First, we formulate the TTTA problem. Next, we define and discuss the information divergence, followed by a discussion on how it can be used to solve the TTTA problem. We will then briefly describe the CBTA solution and discuss its relationship to the Bhattacharyya divergence. We will then provide simulation results and performance comparison between CBTA and the Bhattacharyya criterion. We conclude the paper with a summary of the main results of the paper and a brief description of future research directions.

2. THE TRACK-TO-TRACK ASSOCIATION PROBLEM AND ASSUMPTIONS

The scenario we consider in the present analysis is one where we are given n_T uncertain tracks $\boldsymbol{x}_i$, $i = 1, \ldots, n_T$, at time instances t_i. All n_T tracks can be propagated (using an appropriate uncertainty propagation technique) to a common TTTA analysis time t^*, say the time a collection of n_U UCTs were captured by a sensor. The overall goal of TTTA is to associate each one of the n_U observed uncorrelated tracks to one or none (in case of a new RSO birth, for example) of the n_T given tracks prior to the observation time t^*.

Without loss of generality we will assume that the n_T tracks are all defined at the same time instant $t_i = t_*$, $i = 1, \ldots, n_T$. We will assume that the n_T tracks are Gaussian distributed. At time t_*, each track's state $\boldsymbol{x}_i$ will have mean $\boldsymbol{\mu}_i$ and covariance $\boldsymbol{\Sigma}_i$, $i = 1, \ldots, n_T$. At time t^*, the UCTs are assumed to have states $\boldsymbol{y}_j$, which are assumed Gaussian and are given by means $\boldsymbol{\eta}_j$ and covariances $\boldsymbol{\Omega}_j$, $j = 1, \ldots, n_U$. We will assume that $n_T = n_U$, where each of the n_U UCTs corresponds to one of the n_T tracks. In other words, we are assuming that there are no clutter or new births. These cases are currently being studied.

Each of the n_T tracks at time t_* will be propagated to the time t^*. Because of the Gaussianity assumption, we will use the unscented transform to propagate the uncertainty. The result will be that the propagated state $\boldsymbol{x}_i$ will have a propagated mean $\tilde{\boldsymbol{\mu}}_i$ and covariance $\tilde{\boldsymbol{\Sigma}}_i$, $i = 1, \ldots, n_T$. The Gaussianity assumption imposes performance limitations on the proposed technique. However, future research will focus on the use of particle techniques and solutions to solve the TTTA (as well as OTOA and OTTA problems, generally) problem.

3. INFORMATION DIVERGENCE AND THE TTTA PROBLEM

3.1 Information Divergence

In information theory, information divergence measures the similarity between two probability distributions. Generally speaking, information divergence is not a metric. While there are many definitions of information divergence [6–8], we will use the Bhattacharyya divergence. The Bhattacharyya divergence between two pdfs $p(\boldsymbol{x})$ and $q(\boldsymbol{x})$ is given by:

$$D_B(p||q) = -\log(B_C(p||q)), \tag{1}$$

where B_C is the Bhattacharyya coefficient and is given by

$$B_C(p||q) = \int \sqrt{p(\boldsymbol{x})q(\boldsymbol{x})}\mathrm{d}\boldsymbol{x} \tag{2}$$

If both p and q are Gaussian, then one can compute $D_B(p||q)$ in closed form and is given by:

$$D_B(p||q) = \frac{1}{8}\left(\boldsymbol{\mu}_p - \boldsymbol{\mu}_q\right) \cdot \boldsymbol{\Sigma}^{-1} \cdot \left(\boldsymbol{\mu}_p - \boldsymbol{\mu}_q\right) + \frac{1}{2}\log\left(\frac{\|\boldsymbol{\Sigma}\|}{\sqrt{\|\boldsymbol{\Sigma}_p\|\,\|\boldsymbol{\Sigma}_q\|}}\right), \tag{3}$$

where $\boldsymbol{\mu}_p$ and $\boldsymbol{\Sigma}_p$ (resp., $\boldsymbol{\mu}_q$ and $\boldsymbol{\Sigma}_q$) are the mean and covariance of the pdf p (resp., q), and where $\boldsymbol{\Sigma} = \frac{\boldsymbol{\Sigma}_p + \boldsymbol{\Sigma}_q}{2}$.

The Bhattacharyya divergence is related to the Hellinger distance D_H, where:

$$D_H(p||q) = \sqrt{1 - B_C}. \tag{4}$$

Unlike the Bhattacharyya divergence, Hellinger distance is, in fact, a proper metric because it satisfies the non-negativity, symmetry and triangle inequality properties. Bhattacharyya divergence, on the other hand, does not satisfy the triangle inequality and is hence not a metric. We do not need the triangle inequality in the proposed TTTA solution and, hence, we will directly use Bhattacharyya divergence to solve the TTTA problem as will be described in the next section.

3.2 Solving the TTTA Problem Using Information Divergence

As described above, the n_T given tracks at time t_* are propagated to the common observation time t^*. Because of the Gaussianity assumption made at t_*, the unscented transform (i.e., the propagation step in the unscented Kalman filter (UKF)) is used to map the uncertainty from time t_* to t^*. Let $p_g(\boldsymbol{x}_i; \tilde{\boldsymbol{\mu}}_i, \tilde{\boldsymbol{\Sigma}}_i)$ be the normally distributed probability density function for track i at time t^*, $i = 1, \ldots, n_T$. Here, $\boldsymbol{x}_i$ is the full 6-dimensional orbital state of the track i. Moreover, also described previously, the n_U (with $n_u = n_T$) UCTs are assumed Gaussian with densities $p_g(\boldsymbol{y}_j; \boldsymbol{\eta}_j, \boldsymbol{\Omega}_j)$. These UCTs are typically the outcome of an initial orbit determination (IOD) analysis given a set of sensor measurements. These measurements may be angles only, range and range-rate, etc. The specific type of measurements is irrelevant to this analysis and we assume that $p_g(\boldsymbol{y}_j; \boldsymbol{\eta}_j, \boldsymbol{\Omega}_j)$ is the resulting uncertainty for a given UCT, where $\boldsymbol{y}_j$ is the 6-dimensional state of the UCT. In the simulation section, we assume that both $\boldsymbol{x}_i$ and $\boldsymbol{y}_j$ are the full position-velocity coordinates of the tracks and UCTs. Other coordinates can be considered for describing the state of the orbit (e.g., a set of 6 classical or Equinoctial orbital elements). In the simulation section, we also demonstrate the effect of using Equinoctial orbital elements in the complete analysis and compare it against using Cartesian coordinates for propagation, both with and without higher order perturbations. Perturbations considered in the simulations are: solar radiation pressure, 8×8 EGM gravity model, drag, and Sun and Moon third body effects.

The postulate behind the use of information divergence for solving the TTTA problem is that the track and UCT that are most similar in a probabilistic sense are the ones that are most likely to be associated. So we proceed by hypothesizing that the track-UCT pair $\boldsymbol{x}_i$ and $\boldsymbol{y}_j$ are associated, implying that $\boldsymbol{x}_i = \boldsymbol{y}_j$, and then compute the divergence $D_B(p_g(\boldsymbol{x}_i; \tilde{\boldsymbol{\mu}}_i, \tilde{\boldsymbol{\Sigma}}_i)||p_g(\boldsymbol{x}_j; \boldsymbol{\eta}_j, \boldsymbol{\Omega}_j))$. This is repeated for every track $\boldsymbol{x}_i$ against every UCT $\boldsymbol{y}_j$. For a fixed track i the UCT that has the smallest divergence is the one chosen to associate the track. Note that in this approach it may be the case that more than one track i gets associated with a single UCT j. We call this a *duplicate association*. Because we are assuming that for every track i there exists a single UCT j, duplicate associations imply at least one false positive association (i.e., asserting that an association is correct when it is not). And because of the same assumption, we must then have an equal number of false negatives (i.e., asserting that an association is incorrect when it is in fact correct). Therefore, in the simulation section we will only report the true positive rate r_{TP} (defined as the rate of correct associations achieved by the algorithm among all the true associations) since it contains all the needed information about the performance of the proposed techniques, with the false positive and false negative rates both being $r_{FP} = r_{FN} = 1 - r_{TP}$. The same is not true for CBTA as we will explain in the next section.

A remark on the case of clutter or newly-born objects. The scenario considered in this paper ignores the possibility that a UCT may not correspond to any of the n_T tracks, as would arise, for example, if a new object is born or if a UCT was caused by some clutter source. One statistical approach to treating these possibilities relies on the availability of a clutter and birth statistical models. Given such models, we simply add two (or more, in the case when there are multiple clutter and birth models) new probability density functions to the list of n_T track densities and treat the new hypotheses as if they were tracks. We then test each UCT n_U against the $n_T + 2$ (or more) "tracks" in a way analogous to the procedure described above. In this case, the false positive and false negative rates do not have to be equal. Future research will address this more general TTTA problem.

4. CBTA AND ITS RELATIONSHIP TO INFORMATION DIVERGENCE

4.1 CBTA

In covariance-based track association (see [5] and references therein), we proceed again by testing the association of a track i and a UCT j. We first compute the difference in the means: $\boldsymbol{\delta}_{ij} = \tilde{\boldsymbol{\mu}}_i - \boldsymbol{\eta}_j$ and the sum of the covariances $\boldsymbol{\Delta}_{ij} = \boldsymbol{\Sigma}_i + \boldsymbol{\Omega}_j$. We then compute the 6-dimensional χ^2 statistic:

$$k^2 = \boldsymbol{\delta}_{ij}^{\mathrm{T}} \boldsymbol{\Delta}_{ij}^{-1} \boldsymbol{\delta}_{ij}. \tag{5}$$

One then uses the χ distribution to test whether there is statistical significance between the track i and the UCT j. The statistical significance level is a parameter set by the analyst and that is taken to be 1% (0.01) in this paper (i.e., the track and the UCT are associated with a confidence level of 99%). This corresponds to a k value of $k \simeq 4.1$.

Unlike in the proposed information divergence solution where each track i gets associated with exactly one UCT j, a track i may not achieve any significance level with any of the UCTs in CBTA. Hence, in CBTA, the false positive rate may not be equal to the false negative rate. In our simulations, while we computed and recorded all rates (true positive, false positive and false negative) we only report the true positive rates and compare those to those of the information divergence solution.

4.2 CBTA and Information Divergence

Comparing Eq. (3) and Eq. (5), we can see that Bhattacharyya divergence and CBTA measure the difference in means (they differ only by the $1/8$ factor). Additionally, the Bhattacharyya divergence measures differences in covariance (via the log and trace terms in Eq. (3)). Because of this, one would then expect information divergence to outperform CBTA as it accounts for deviations in both mean and covariance discrepancies between tracks and UCTs. This will be demonstrated in the next section in simulation, where we compare the Bhattacharyya divergence with CBTA over multiple orbital regimes and parametrizations. We will demonstrate that this is true with and without higher order perturbations and using different coordinate systems for uncertainty propagation.

5. SIMULATION RESULTS

In this simulation, we compare the performance of the proposed Bhattacharyya divergence-based and CBTA solutions for the TTTA problem in four orbit regimes: LEO, MEO, GTO and GEO. For the first set of results, we consider $n_T = n_U = 10$ objects, with identification numbers $0, 1, \ldots, 9$, in some proximity to one other. The locations of the 10 objects are normally randomly selected at time t_* (chosen to be 30 minutes before midnight on January 1, 2004). All 10 objects have the same nominal mean value but with various standard deviations. The nominal mean values are given in orbital elements in Table 1 for the orbital cases considered, while the separation position and velocity from that mean value are varied. For position, it is varied from a value of $1.0e^{-6}$ m to 2 km (the same for all three Cartesian directions). Four values of velocity standard deviations where chosen: $3, 5, 7$ and 9 m/s (the same for all three Cartesian directions). Given the statistical nature of the initial locations of the RSOs, to measure performance we perform 1000 Monte Carlo runs per position/velocity standard deviation value. The average true positive for both methods was recorded for each position/velocity standard deviation value combination and is used to compare the performance of the information divergence and CBTA solutions. The analysis was performed three times for each of the orbital cases, each time with a different simulation duration between t_* and t^*: 1.0, 3.0, and 5.0 sidereal days.

Table 1. Parameters of the True Orbit and Measurement Model

Parameter	LEO	MEO	GTO	GEO
Semimajor Axis, km	6 991.0	26 600.0	24 567.0	42 164.0
Eccentricity	0.0	0.2	0.716286	0.0
Inclination, deg	97.9	55.0	11.0	0.0
Argument of Perigee, deg	0.0	-120.0	0.0	0.0
Right Ascension of the Ascending Node, deg	207.0	207.0	207.0	0.0
True Anomaly, deg	-30.0	20.0	169.0	300.0

Once the actual initial position and velocity for each object was selected at time t_*, the initial uncertainty used for the uncertainty propagation step was 100 m in position and 1.0 m/s in velocity (both being standard deviations uniformly assigned in all three Cartesian directions). Sigma points were computed for each track and propagated to time t^*. At that point in time, the mean and covariance is computed from the sigma points as one would typically do in the propagation step of the UKF. This propagation is performed for both the information divergence and CBTA solutions. The propagation is made in Cartesian coordinates and no higher orbital perturbations were considered (i.e., we propagated the two-body dynamics only). Later in this section, we show the effect of propagating in Equinoctial orbital elements with and without higher orbital pertubations.

In parallel to this, the true tracks were propagated forward in time to t^*. At which point we simulate an observation process that is equal to the true object state with an added zero mean Gaussian noise signal. The covariance for the observation process assumed a 1.0 m standard deviation in position (same for all three Cartesian directions) and 0.01 m/s for velocity (same for all three Cartesian directions), with no cross-correlation between position and velocity. This covariance matrix is denoted by $\mathbf{\Omega}_j$. Thus the mean of the UCTs is given by the actual measurement $\boldsymbol{\eta}_j$ and covariance is $\mathbf{\Omega}_j$.

The information divergence and CBTA solutions were implemented in Monte Carlo simulations, as mentioned above, for all four orbits with different durations between t_* and t^*. The results are shown in Figures (1), (2), (3) and (4). As can be seen in the figures, the Bhattacharyya divergence outperforms CBTA for all position/velocity initialization parameters and for all propagation periods. In some cases (e.g., LEO and GEO), the Bhattacharyya divergence outperforms CBTA by as much as about 15%-25% in true positive rates. In the case of GTO, the two methods perform nearly equally, with Bhattacharyya divergence only slightly (but consistently) outperforms CBTA. Other things to note, as one would expect, is that performance of both methods reduces as the period between track and UCT increases, and as objects are initialized closer to each other.

We also investigated the impact of higher order orbital perturbations. For this case, we used equinoctial elements for uncertainty propagation. The perturbations that were considered are: solar radiation pressure, 8×8 EGM gravity model, drag, and Sun and Moon third body effects. As one would expect, the numerical simulation in this case would take a much longer time to run. Thus only the MEO case was run with 100 runs per Monte Carlo simulation (instead of 1000) and for only one of the durations (1 day) between track and UCT detection. The number of objects was reduced from 10 to 4. The result is shown in Figure (5). As can be seen, performance is roughly the same as to when no perturbations were considered and the propagation was conducted in Cartesian coordinates. Two effects may have contributed to this, where the perturbations reduced performance but the equinoctial element propagation improved performance. The impact of each effect is not entirely clear yet and more extensive numerical investigations will be conducted in the future to understand these effects.

6. CONCLUSION

In this paper we tackled the TTTA problem using Bhattacharyya divergence as a criterion for association. We have also described the classical CBTA solution and established a mathematical connection between the two. The analysis suggests that the Bhattacharyya divergence is superior to CBTA for the orbital regimes and range of parameters considered in our simulations. We also investigated the impact of using equinoctial elements and orbit perturbations on performance of both methods.

Current research focuses on the development of modified variations of information divergence as well as the use of mutual information for TTTA. We will also focus on adding clutter and new object births as hypotheses to be considered in the TTTA problem. We briefly described one approach to including these phenomena in the analysis. More generally, future work will focus on developing a similar solution when uncertainty is analytic but not Gaussian or when it is completely non-analytic e.g., when the uncertainty is described using a particle cloud using both information divergence and mutual information as criteria for association. From past experience with the OTOA problem, it is expected that a particle-based solution combined with information theoretic criteria will potentially outperform CBTA as well. These ideas are being explored in current and future research towards not just the TTTA and OTOA problems but also towards the classic OTOA problem.

REFERENCES

[1] I. I. Hussein, C. W. T. Roscoe, M. P. Wilkins, and P. W. Schumacher, Jr., “Information Theoretic Criteria for Observation-to-Observation Association,” *Proceedings of the Advanced Maui Optical and Space Surveillance Technologies Conference*, Wailea, HI, September 9–12 2014.

[2] I. I. Hussein, C. W. T. Roscoe, M. P. Wilkins, and P. W. Schumacher, Jr., “On Mutual Information for Observation-to-Observation Association,” *Space Flight Mechanics Meeting*, Williamsburg, VA, January 11–15 2015.

[3] R. Linares, V. Kumar, P. Singla, and J. L. Crassidis, “Information Theoretic Space Object Data Association Methods using an Adaptive Gaussian Sum Filter,” *Advances in the Astronautical Sciences*, Vol. 140, 2011, pp. 665–680. (Proceedings of the 21st AAS/AIAA Space Flight Mechanics Meeting, New Orleans, LA, February 13–17 2011, Paper AAS 11-148).

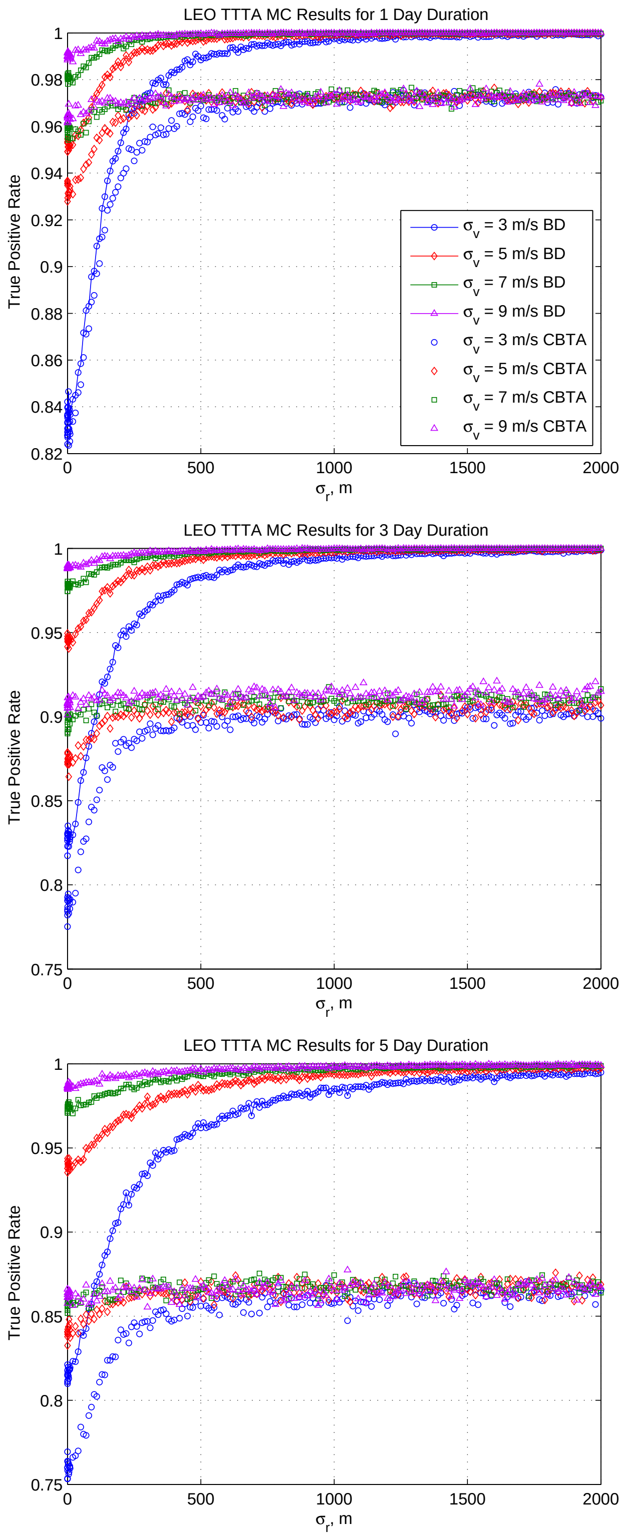

Fig. 1. Information Divergence vs. CBTA True Positive Rates for LEO.

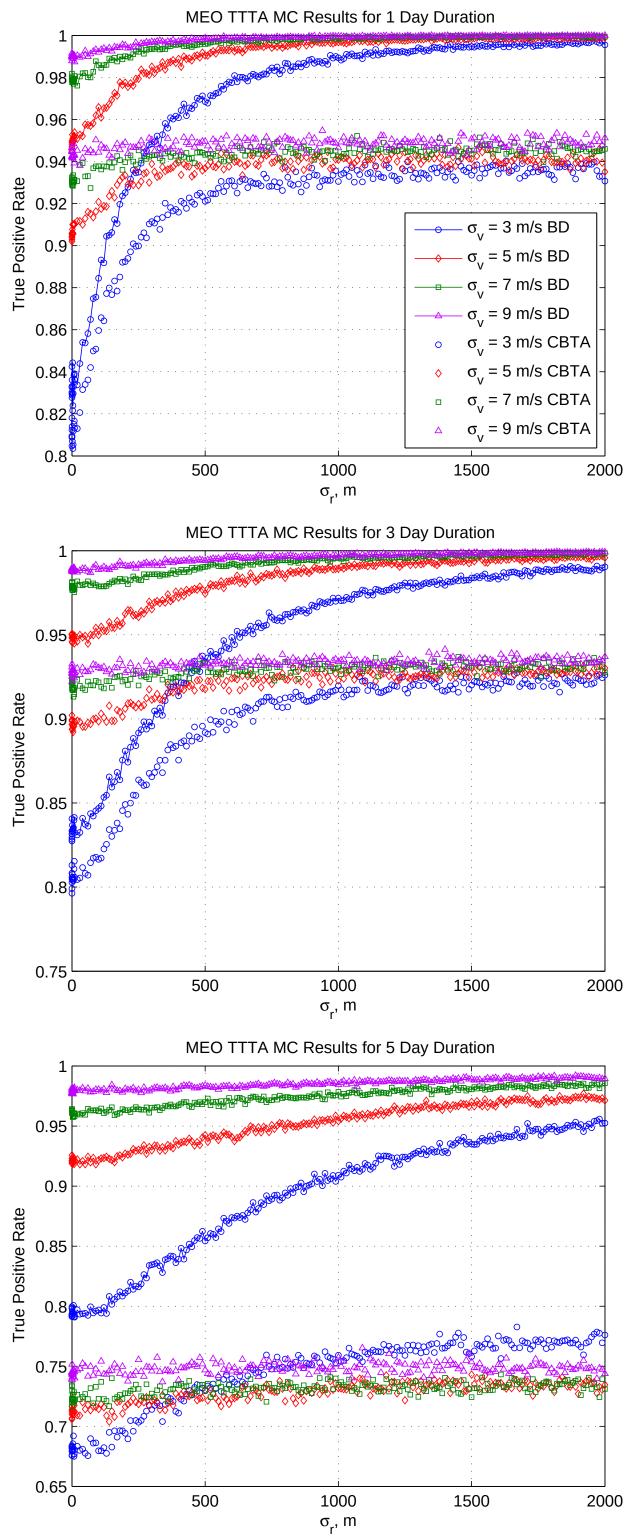

Fig. 2. Information Divergence vs. CBTA True Positive Rates for MEO.

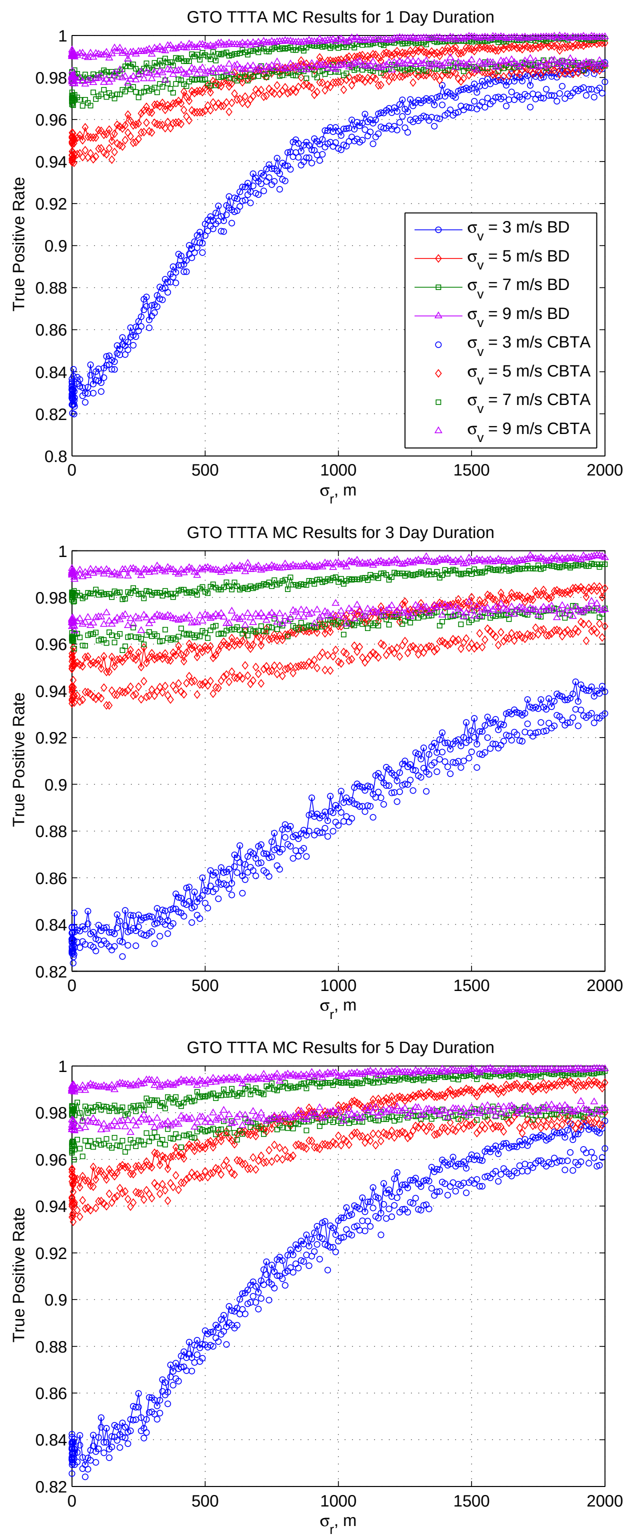

Fig. 3. Information Divergence vs. CBTA True Positive Rates for GTO.

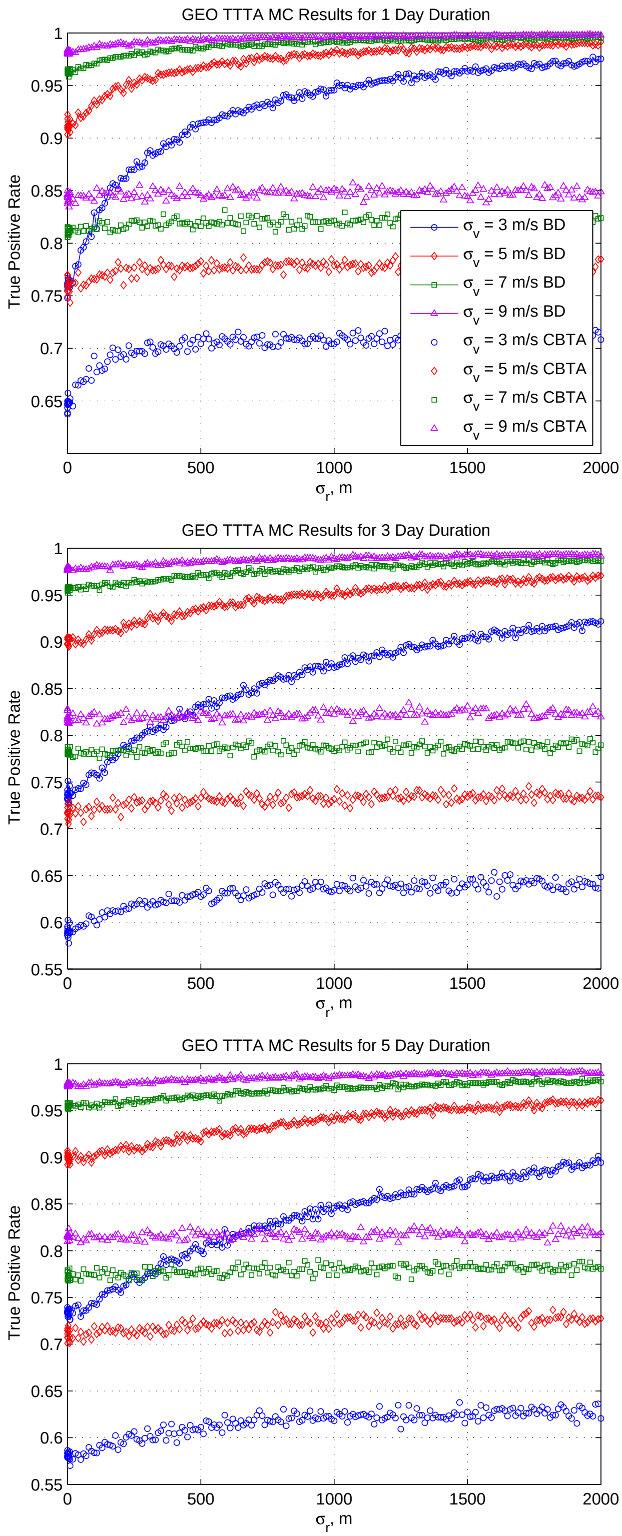

Fig. 4. Information Divergence vs. CBTA True Positive Rates for GEO.

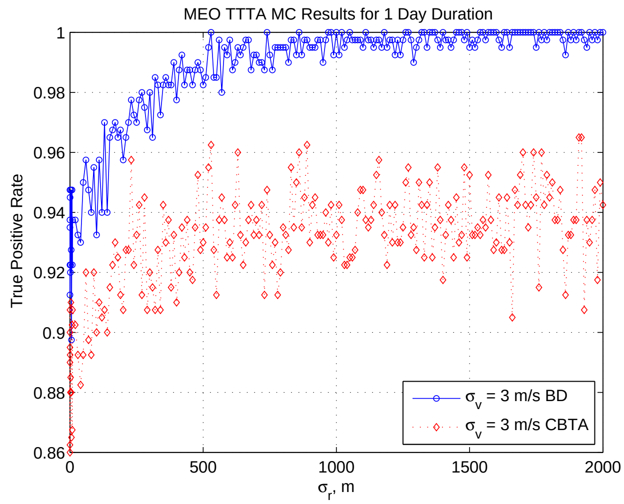

Fig. 5. Information Divergence vs. CBTA True Positive Rates for MEO with propagation in equinoctial elements and with orbital perturbations.

[4] S. Julier and J. Uhlmann, "A New Method for the Nonlinear Transformation of Means and Covariances in Nonlinear Filters," *IEEE Transactions on Automatic Control*, Vol. 44, March 2000, pp. 477–482.

[5] K. Hill, K. T. Alfriend, and C. Sabol, "Covariance-based Uncorrelated Track Association," *AIAA/AAS Astrodynamics Specialist Conference*, 2008.

[6] A. Rényi, "On measures of entropy and information," *Proceedings of the fourth Berkeley Symposium on Mathematics, Statistics and Probability 1960*, 1961, pp. 547–561.

[7] A. O. Hero, D. Castañón, D. Cochran, and K. Kastella, *Foundations and Applications of Sensor Management*. Springer, 2007.

[8] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley-Interscience, 1991.