

# **NEW APPROACH TO MULTIPLE DATA ASSOCIATION FOR INITIAL ORBIT DETERMINATION USING OPTICAL OBSERVATIONS**

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## **Abstract**

The proposed approach aims to develop a new method of forming and processing of multiple hypotheses for initial orbit determination using optical observations. This method allows us to generalize the existing 2-dimensional flat constrained admissible region (CAR) to a unique 3-dimensional (3D) manifold of points corresponding to the pairs of observed right ascension and declination. Another advantage of this method is that unlike the existing methods of initial orbit determination using CAR, the range, range rate and angular rates are computed analytically using the angle observations, the location coordinates of the observation station, the semi-parameter and semi-major axis corresponding to the CAR. Unlike the existing 2D CAR, the 3D manifold does not include the pairs of range and range rate that do not correspond to the observed angles and computed range rat and angular rates. Given the the semi-parameter and semi-major axis, the proposed approach allows us to analytically compute the orientation angles as

the Keplerian orbital elements, including the longitude of ascending node, inclination and argument of perigee. The resulting method represents a new and computationally efficient procedure for multiple data association processing through multiple hypotheses filter and allows for an uncertainty quantification.

## 1 INTRODUCTION

### 1.1 Orbit determination

A space object's orbit determination using optical observations is one of the fundamental problems of astrodynamics [1]. Although there are many different methods of determining the orbit, the probabilistic approach with multiple hypotheses processing is one of the newest approaches that have demonstrated their efficiency and reliability [2], [3]. About a third of 3,032 designated payloads tend to operate in a limited orbital regime and share that regime with the rocket bodies, upper stages, and associated debris which put them in orbit, the potential exists that any of these payloads might collide with another object in orbit. At relative velocities up to 15 km/s, the results of such a collision would be catastrophic [4]. In the geostationary belt, such a collision would generate debris which would drift around the belt indefinitely, putting at risk all other payloads in that orbit. This situation with possibility of a collision with unknown uncertainty makes

the orbit determination one of the important tasks involving multiple probabilistic data association [5]. This data association relies on the utility of the concept of a constrained admissible region (CAR) which was pioneered in Ref.[6].

Many different nonlinear filtering techniques have been developed up to date for initial orbit determination purposes utilizing the observations of various parameters [7]-[9]. Among these techniques, the probabilistic filtering concepts have demonstrated suitability for multiple data association processing and guaranteed existence of convergence of the probabilistic parameters [10]-[13]. Moreover, the mixture-of-experts approach has successfully been developed for near earth and interplanetary trajectories [14]-[17].

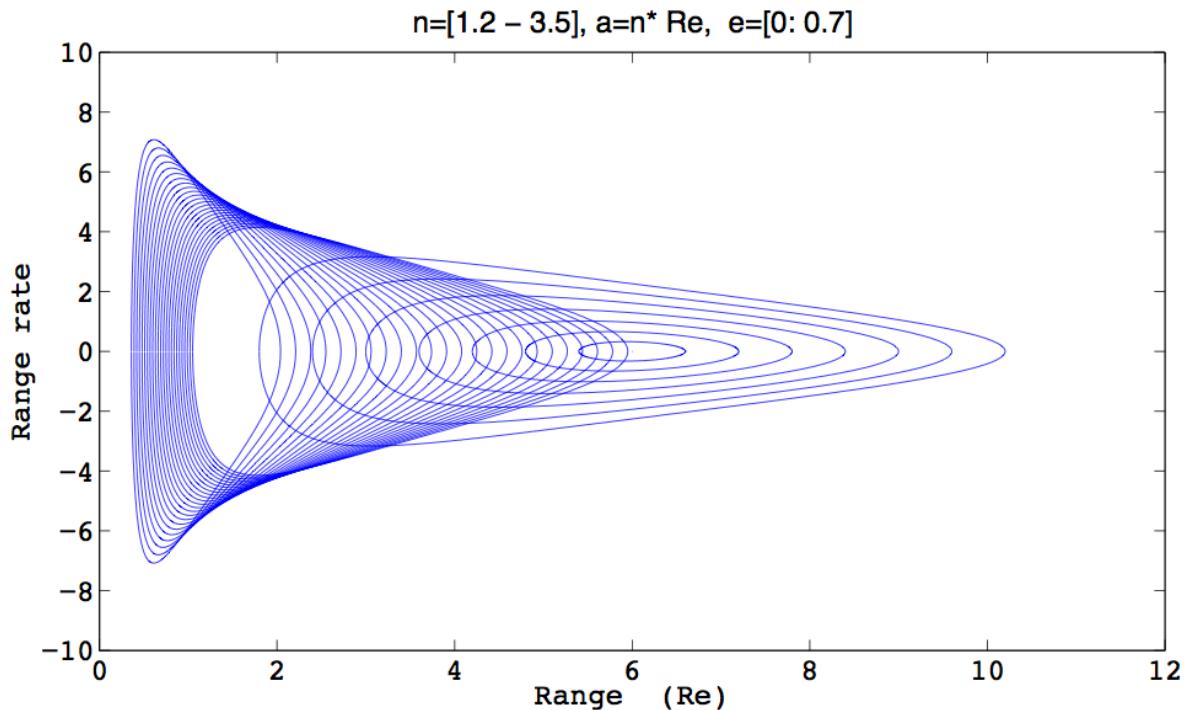


Figure 1: Contours of semi-major axis and eccentricity of a Keplerian orbit.

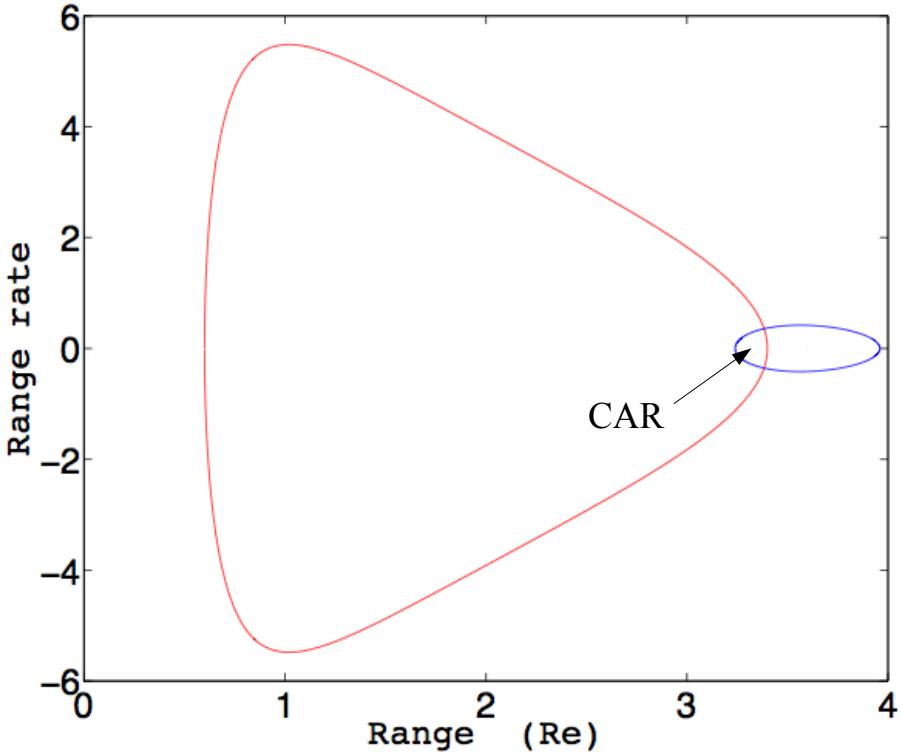


Figure 2: Example of orbital constraints to obtain a CAR.

## 1.2 Remarks about Constrained Admissible Region- Multiple Hypothesis Filter

The Constrained Admissible Region- Multiple Hypothesis Filter (CAR-MHF) is a new recent approach to the initial orbit determination, and has been shown viable for initialization of the unknown space objects' tracks and for association of hypotheses on orbit and drag parameters in the case of a breakup [3], [18]. This is a necessary prerequisite to completely assess the breakup circumstances. This filter initializes the orbit and parameter states using only Keplerian orbital motion parameters. This filter works in the framework of a probabilistic data association procedure which takes observed parameters with uncertainties and associates them with the pairs of range and range rate from

the CAR thereby forming hypothesized states with uncertainties (see Fig.1). Here the angular rates are obtained based on the "smoothing" technique. But this CAR, which is currently in use, is formed based on the expressions of the range and range rate in terms of semi-major axis and eccentricity of a Keplerian orbit [3] (see Fig.2). The contours of these latter two parameters, computed separately by the range and range rate, represent boundaries of the CAR. But the components of the total velocity of the object other than the rage rate are not considered in the construction of the CAR. The objectives of this study are to (1) derive analytical computational formulas to compute range rate, angular rates and the Keplerian orbital elements that can be associated with or correspond to the optical observations with no measurement errors and to a given range to the object of interest or its inertial position vector magnitude, and (2) create a unique 3D manifold of points corresponding to the pairs of right ascension and declination observed at a time instant using all possible boundaries of all orbital elements.

## 2 DATA MIS-ASSOCIATION IN MULTIPLE DATA ASSOCIATION PROCESSING

The current version of CAR-MHF has been shown as a powerful tool for initial orbit determination using optical observations [3], [5]. But it should be noted that this process accumulates uncertainties inherited from the observations and data associations from the processing at

previous observation times.

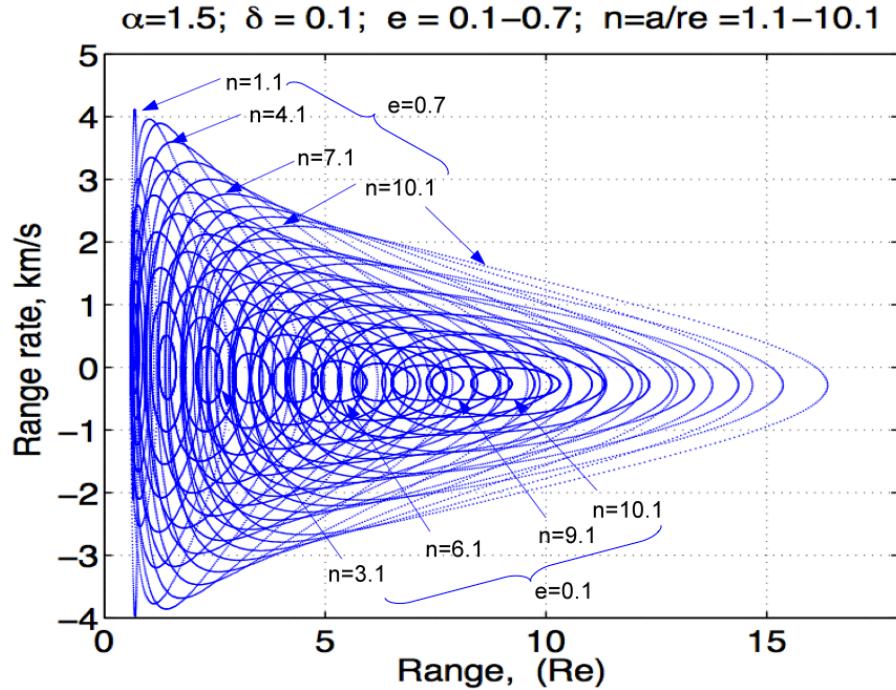


Figure 3: Relationship between the range and range-rate currently in use in CAR-MHF.

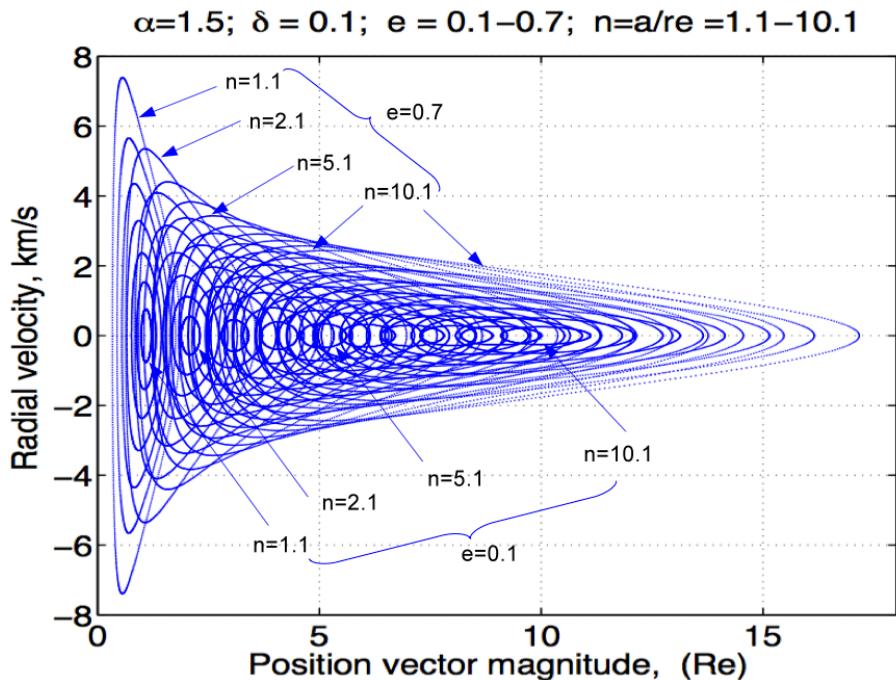


Figure 4: Relationship between the position vector magnitude and radial velocity (similar to range and range rate currently in use in CAR-MHF).

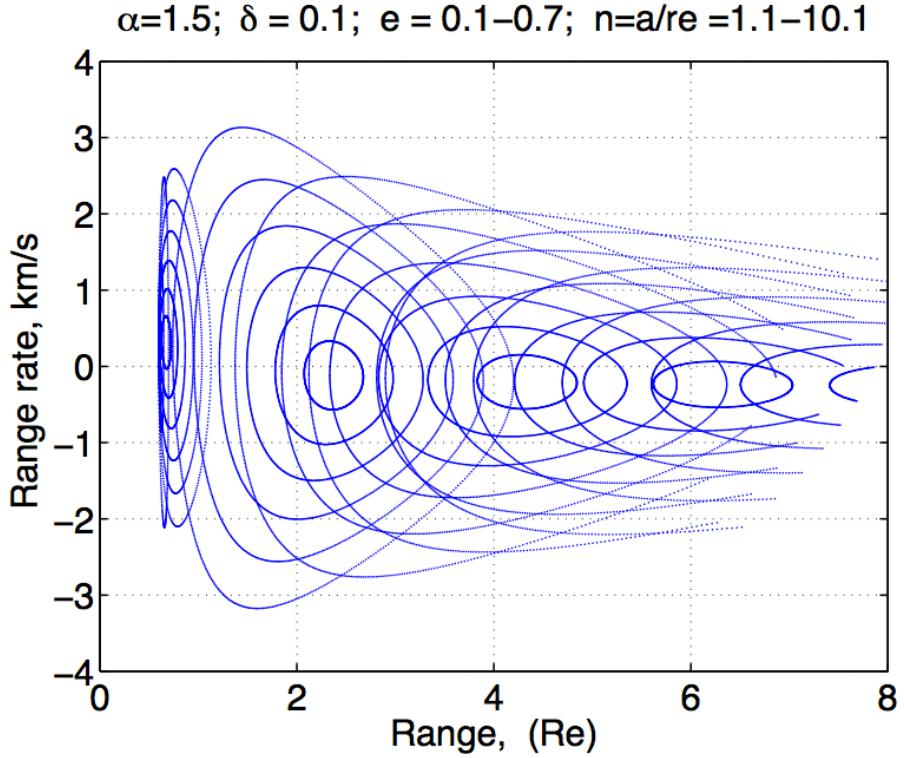


Figure 5: Actual relationships that correspond to a valid pair of  $\alpha$  and  $\delta$ . Range to SO vs range rate (with larger increments in  $n$ )

This filter computes (smoothes) angular rates based on the angle measurements (right ascension and declination) and time between the measurements, thereby "deviating" from the actual instantaneous values of the angular rates. The errors (differences between these and actual values) resulting from these computations are incorporated into the hypothesized state vector computations [3].

Note also that when determining an initial orbit through a CAR-MHF processing, all possible hypotheses created by pairs of range and range-rate are considered without making a distinction between which of these pairs may represent an orbit and which pairs are not really associated with any orbit. This means that 100% certainty is assigned to orbit hypotheses. However, it is unknown which hypothesized state

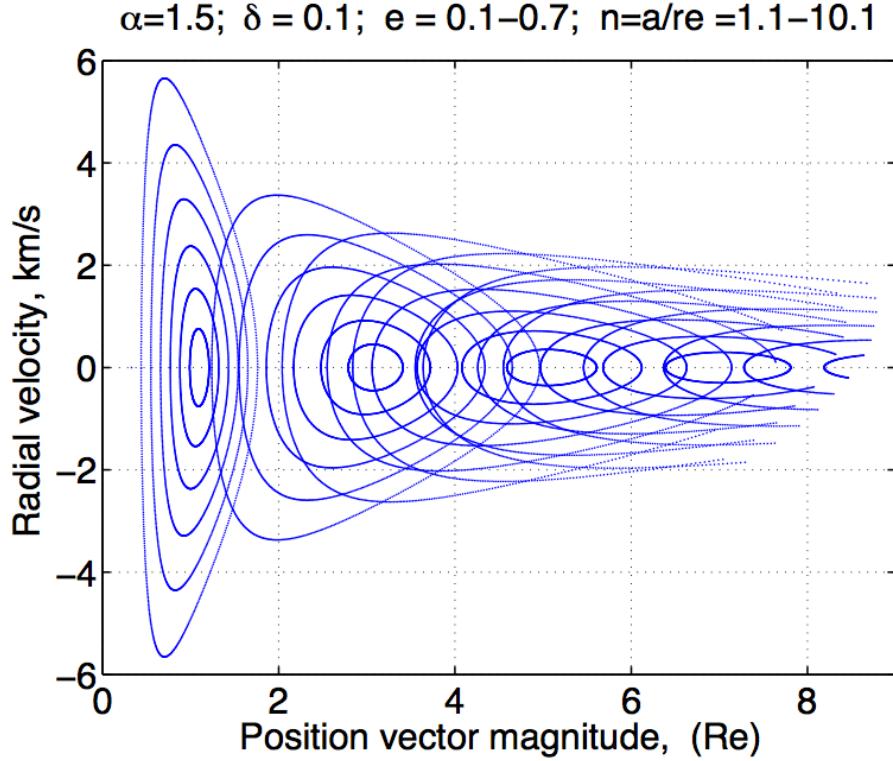


Figure 6: Actual relationships that correspond to a valid pair of  $\alpha$  and  $\delta$ . The SO's inertial position vector magnitude vs radial velocity (with larger increments in  $n$ ).

needs to be associated with which pair of the range and range rate. Another aspect of the current version of CAR-MHF is that the range and range-rate are computed independently of the angle measurements to form hypothesized states (see Fig.3). It should be noted that the similar relationship between the position vector magnitude and radial velocity (similar to range and range rate currently in use in CAR-MHF) can be obtained (see Fig.4). Consequently, these hypothesized states potentially include those pairs of the range and range rate which are not actually associated with the measurements and the resulting states. The proposed work will relate these parameters to the angular observations. In particular, the preliminary studies show a significant

difference between the computation of the range and range-rate independently from the angular observations, and the computation of the range and range rate as a function of the the angular observations. Indeed, the actual relationships between the range and range rate that correspond to a valid pair of the angular observation, as well as the relationships between the object's inertial position vector magnitude and radial velocity are given in figures 5 and 6. The differences between these figures and figures 3 and 4 are obvious. This means that if a given hypothesis is not valid for this object, then this creates a data mis-association. This data mis-association then inherently propagates to the next measurement time, but the existing CAR-MHF does not quantify the uncertainties as well as the initial uncertainty related to this data mis-association. Therefore, the existing initial orbit determination using angular observations carries the uncounted uncertainties of various origins [5].

### **3 NEW APPROACH TO MULTIPLE DATA ASSOCIATION PROCESSING**

The proposed work aims to develop a new method of forming and processing of multiple hypotheses for initial orbit determination using optical observations [18]. This method is based on the utility of orbital parameters to create a unique 3-dimensional (3D) manifold of points corresponding to the pairs of right ascension and declination observed at a time instant. This manifold represents generalization of the ex-

isting 2-D flat constrained admissible region (CAR) to a 3D manifold. Unlike the existing version of CAR-MHF and conventional way of orbit determination, which use two or more observations, the proposed work uses one observation of the angles, and the proposed manifold does not include the pairs of range and range rate that does not correspond to the observed angles and angular rates. Another advantage of this method is that the angular rates are computed analytically using the angle observations, location coordinates of the observation station and Keplerian orbital parameters. The preliminary results include formation of new forms of hypotheses based on a triple of the parameters consisting of position vector magnitude, radial and transversal velocities of an object, derivation of the range and range rate formulas using the object's position and velocity as well as the position vector of the observation station with respect to the Earth's center of mass, and derivation of the formulas for the angular rates for the object's right ascension and declination angles. These results allow us to form a 3D manifold bounded by the curves of eccentricity and semi-major axis of all possible Keplerian orbits thereby improving the accuracy of the orbit determination. Also, given these two angles at a given time, that is for a single observation, these results allow us to explicitly compute the angular rates and range rate, and consequently, to form hypothesized states thereby reducing the time of multiple data association processing [18]. Consideration of the measurement and other errors in the analytical formulas obtained in this paper can serve as a starting

point of uncertainty quantification. Future work involves the qualitative and quantitative analyses of the analytical formulas for the orbital parameters with a priori covariances.

## 4 ANALYTICAL DETERMINATION OF ANGULAR RATES

If  $e$  and  $a$  are the eccentricity and semi-major axis of the orbit of a space object (SO), and a polar coordinate system  $Or\theta$  is introduced with the origin  $O$  at the Earth's center of mass, then it is known that

$$r = \frac{p}{1 + e \cos f}, \quad (1)$$

$$v_r = \dot{r} = \pm \sqrt{\frac{\mu}{p}} e \sin f \quad (2)$$

$$v_\theta = r\dot{\theta} = \pm \sqrt{\frac{\mu}{p}} (1 + e \cos f) \quad (3)$$

where

$$p = a(1 - e^2), \quad \theta = f + \omega,$$

and  $r$  is the position vector of the object,  $\theta$  and  $f$  are the polar angle true anomaly of the object, and  $\omega$  is the angular distance of the perigee of the SO's orbit. By eliminating  $f$  from these three equations, one can show that

$$\dot{r} = \pm \sqrt{\frac{\mu}{p} \left( e^2 - \frac{p^2}{r^2} + 1 \right)}, \quad (4)$$

$$r\dot{\theta} = \pm \frac{\sqrt{\mu p}}{r^3}. \quad (5)$$

Given the range of values of  $a$  and  $e$ , that is  $a_1 \leq a \leq a_2$  and  $e_1 \leq e \leq e_2$ , one can construct a 3-D constrained admissible region (CAR) by using a triple of the parameters, that is  $r$ ,  $\dot{r}$  and  $r\dot{\theta}$  computed by Eqs. (1), (4) and (5).

Here it will be assumed that the observations of the right ascension,  $\alpha$ , and the declination,  $\delta$ , are available at a given time (see Fig.7). Also, the geocentric position vector of the observation site,  $\mathbf{R}_s$  is assumed given. Then the question of interest is how to compute the range rate,  $\dot{r}$  and the angular rates,  $\dot{\alpha}$  and  $\dot{\delta}$ , and how to form hypothesized states utilizing the recently developed CAR multiple hypothesis filter (MHF) for the purposes of orbit determination.

The SO's inertial position vector,  $\mathbf{r}(x, y, z)$  and its rate can be computed as (see Fig.7)

$$\mathbf{r} = \boldsymbol{\rho} + \mathbf{R}_s, \quad (6)$$

$$\dot{\mathbf{r}} = \dot{\mathbf{R}}_s + \dot{\boldsymbol{\rho}} = \dot{\mathbf{R}}_s + \dot{\rho}\mathbf{u} + \rho\dot{\mathbf{u}}, \quad (7)$$

where

$$\mathbf{r} = \mathbf{r}(x, y, z), \quad \boldsymbol{\rho} = \boldsymbol{\rho}(\xi, \eta, \zeta) = \rho\mathbf{u}, \quad \mathbf{R}_s = (R_{sx}, R_{sy}, R_{sz})^T, \quad (8)$$

$$u_x = \cos \delta \cos \alpha, \quad u_y = \cos \delta \sin \alpha, \quad u_z = \sin \delta. \quad (9)$$

Using the polar and cartesian coordinates,  $x$ ,  $y$  and  $z$ , one can obtain the expression that relates the angular rates to the orbital parameters,  $a$  and  $e$  through  $r$  and  $p$ :

$$\dot{\rho}^2 + \rho^2 \dot{\delta}^2 + \rho^2 \cos^2 \delta \dot{\alpha}^2 + a_1 \dot{\rho} + a_2 \rho \dot{\delta} + a_3 \rho \dot{\alpha} + a_4 = \frac{\mu}{p} \left[ e^2 + 2 \frac{p}{r} - 1 \right], \quad (10)$$

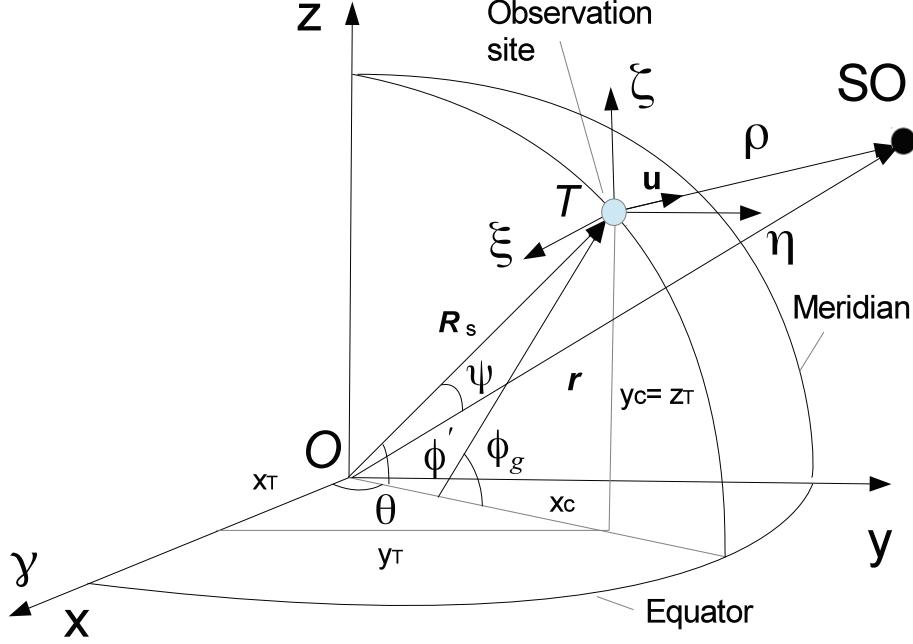


Figure 7: Cartesian coordinates of the point of observation and the SO

where  $a_j = a_j(\alpha, \delta, \dot{R}_{sx}, \dot{R}_{sy}, \dot{R}_{sz})$ ,  $j = 1, \dots, 4$ . Note that  $(\dot{\mathbf{u}}\mathbf{R}_s) = a_5\dot{\delta} + a_6\dot{\alpha}$ , where  $a_5$  and  $a_6$  are the functions of  $\alpha$ ,  $\delta$  and the components of  $\mathbf{R}_s$ . Below the second expression which relates the angular rates to the orbital parameters is obtained. If  $\boldsymbol{\omega}_e$  is the Earth rotation rate, then (see Fig.7)

$$\dot{\mathbf{R}}_s = \boldsymbol{\omega}_e \times \mathbf{R}_s \quad (11)$$

$$\mathbf{r}\dot{\mathbf{R}}_s = rR_s \cos \psi, \quad (12)$$

where  $\psi$  is the angle formed by  $\mathbf{r}$  and  $\mathbf{R}_s$ , and  $r = \|\mathbf{r}\|$ ,  $R_s = \|\mathbf{R}_s\|$ . Then the range,  $\rho$  from the observation site to the SO, the range rate and the range acceleration in terms of  $\alpha, \delta, a$  and  $e$  are given by

$$\rho = \sqrt{(\mathbf{u}\mathbf{R}_s)^2 + r^2 - R_s^2} - (\mathbf{u}\mathbf{R}_s), \quad (13)$$

$$\dot{\rho} = a_7\dot{\delta} + a_8\dot{\alpha} + a_9, \quad (14)$$

$$\ddot{\rho} = \frac{-\rho(\ddot{\mathbf{u}}\mathbf{R}_s)}{\rho+x} - \frac{\rho(2\dot{\mathbf{u}}\dot{\mathbf{R}}_s + \mathbf{u}\ddot{\mathbf{R}}_s)}{\rho+x} + k_3, \quad (15)$$

where

$$\begin{aligned} x &= \mathbf{u}\mathbf{R}_s, \quad a_q = a_q(\alpha, \delta, R_{sx}, R_{sy}, R_{sz}, a, e, f), \quad q = 7, \dots, 9 \\ k_1 &= r^2 - R_s^2, \quad k_2 = \frac{1}{2}\dot{k}_1 = rr\dot{r} - R_s\dot{R}_s, \\ k_3 &= \frac{\rho\dot{x}^2 - \rho x\dot{x} + \dot{k}_2(\rho + x) - k_2(\dot{\rho} + \dot{x})}{(\rho + x)^2}. \end{aligned}$$

Note that in Eq.(13)

$$(\mathbf{u}\mathbf{R}_s)^2 + r^2 - R_s^2 > 0. \quad (16)$$

This condition means that no all hypothesized  $a$  and  $e$  in combination with observed  $\alpha$  and  $\delta$  may result in a true orbit. Using the two-body dynamics equation,  $\ddot{\mathbf{r}} = \mu/r^3\mathbf{r}$ , it can be shown that

$$\left(\ddot{\rho} + \frac{\mu}{r^3}\rho\right)\mathbf{u} + 2\dot{\rho}\dot{\mathbf{u}} + \rho\ddot{\mathbf{u}} = -\left(\ddot{\mathbf{R}}_s + \frac{\mu}{r^3}\mathbf{R}_s\right), \quad (17)$$

Then Eqs. (15) with (17) yield

$$\dot{\alpha}_{ij} = \frac{1}{2} \left[ -\left(\frac{a}{2} \mp 1\right) \pm \sqrt{\left(\frac{a}{2} \mp 1\right)^2 - 4 \left[\frac{y^*}{2} \mp h\right]} \right], \quad (18)$$

$$\dot{\delta}_{ij} = \frac{d_{11}\dot{\alpha}_{ij}^2 + d_{12}\dot{\alpha}_{ij} + d_{13}}{d_{14}\dot{\alpha}_{ij} + d_{15}}, \quad (19)$$

$$\dot{\rho}_{ij} = a_7\dot{\delta}_{ij} + a_8\dot{\alpha}_{ij} + a_9, \quad i, j = 1, 2. \quad (20)$$

Here

$$h = \frac{\frac{a}{2}y^* - c}{2\left(\frac{a^2}{4} - b + y^*\right)}$$

and  $y^*$  is a real root of the cubic equation with respect to  $y$ :

$$y^3 - by^2 + (ac - 4d)y - a^2d + 4bd - c^2 = 0 \quad (21)$$

and

$$a = \frac{d_{22}}{d_{21}}, \quad b = \frac{d_{23}}{d_{21}}, \quad c = \frac{d_{24}}{d_{21}}, \quad d = \frac{d_{25}}{d_{21}}, \quad d_{21} \neq 0.$$

$$d_{2w} = d_{2w}(\alpha, \delta, R_{sx}, R_{sy}, R_{sz}, \dot{R}_{sx}, \dot{R}_{sy}, \dot{R}_{sz}, a, e, f),$$

with  $w = 1, \dots, 5$ . Eqs.(18)-(20) represent the functions  $\dot{\delta}_{ij}$ ,  $\dot{\alpha}_{ij}$  and  $\dot{\rho}_{ij}$  in terms of  $\alpha, \delta, \rho, r, e, p$ , where  $\rho = \rho(\alpha, \delta, r)$  and  $r = r(a, e)$  and  $p = p(a, e)$ . Consequently, if the observations  $\alpha$  and  $\delta$  are given, then for the range of values of  $a$  and  $e$ , that is  $a_{min} \leq a \leq a_{max}$  and  $0 \leq e \leq 1$ , one can compute the following parameters valid for an entire revolution of the SO's orbit of interest ( $0 \leq f \leq 2\pi$ ):

$$r = r(a, e), \quad v_r = \dot{r} = \dot{r}(a, e), \quad v_\theta = r\dot{\theta} = v_\theta(a, e),$$

$$\rho = \rho(\alpha, \delta, a, e), \quad \dot{\rho} = \dot{\rho}(\alpha, \delta, a, e),$$

$$\dot{\alpha} = \dot{\alpha}(\alpha, \delta, a, e), \quad \dot{\delta} = \dot{\delta}(\alpha, \delta, a, e).$$

Given the observations  $\alpha$  and  $\delta$ , the CAR-MHF can be executed using the hypothesized state variables,  $x, y, z, \dot{x}, \dot{y}, \dot{z}$  formed by using (mapped from)  $\rho, \dot{\rho}, \dot{\alpha}$  and  $\dot{\delta}$ . The results of the preliminary studies and simulations are presented in figures 8 - 10.

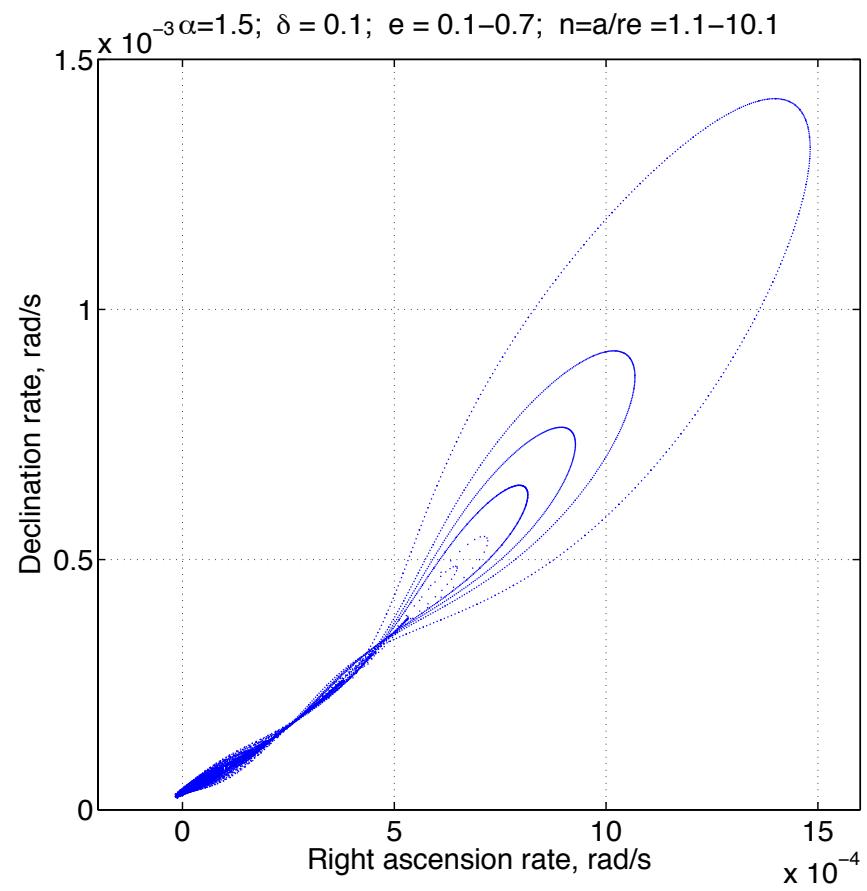


Figure 8: Functions of relationship between the angular rates.

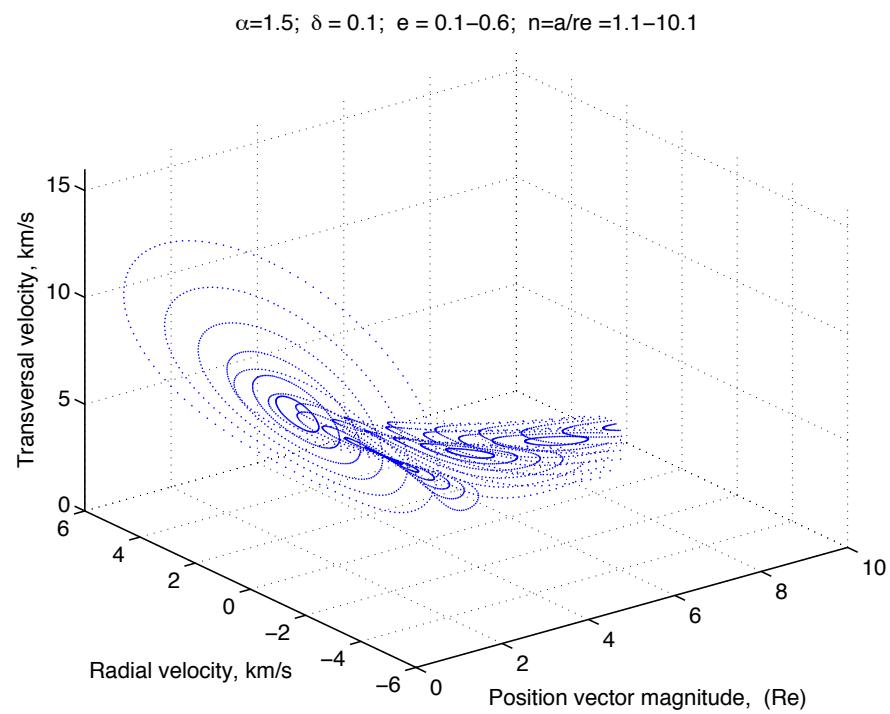
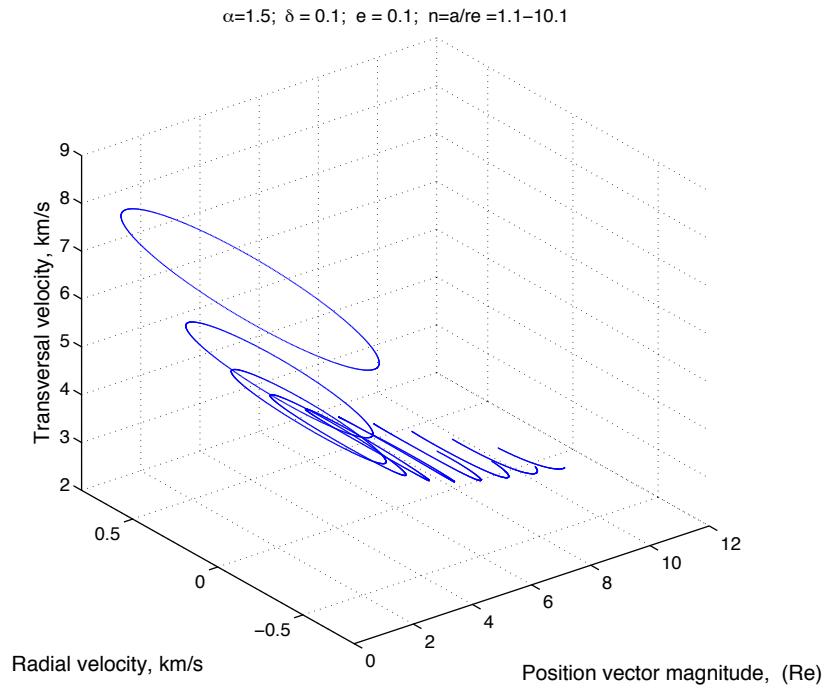


Figure 9: Contours of 3-D CAR (two different views) formed by the magnitudes of the SO's position and velocity vectors for a specific eccentricity,  $e = 0.1$  and for the range of semi-major axis,  $1.1 - 10.1 Re$ .

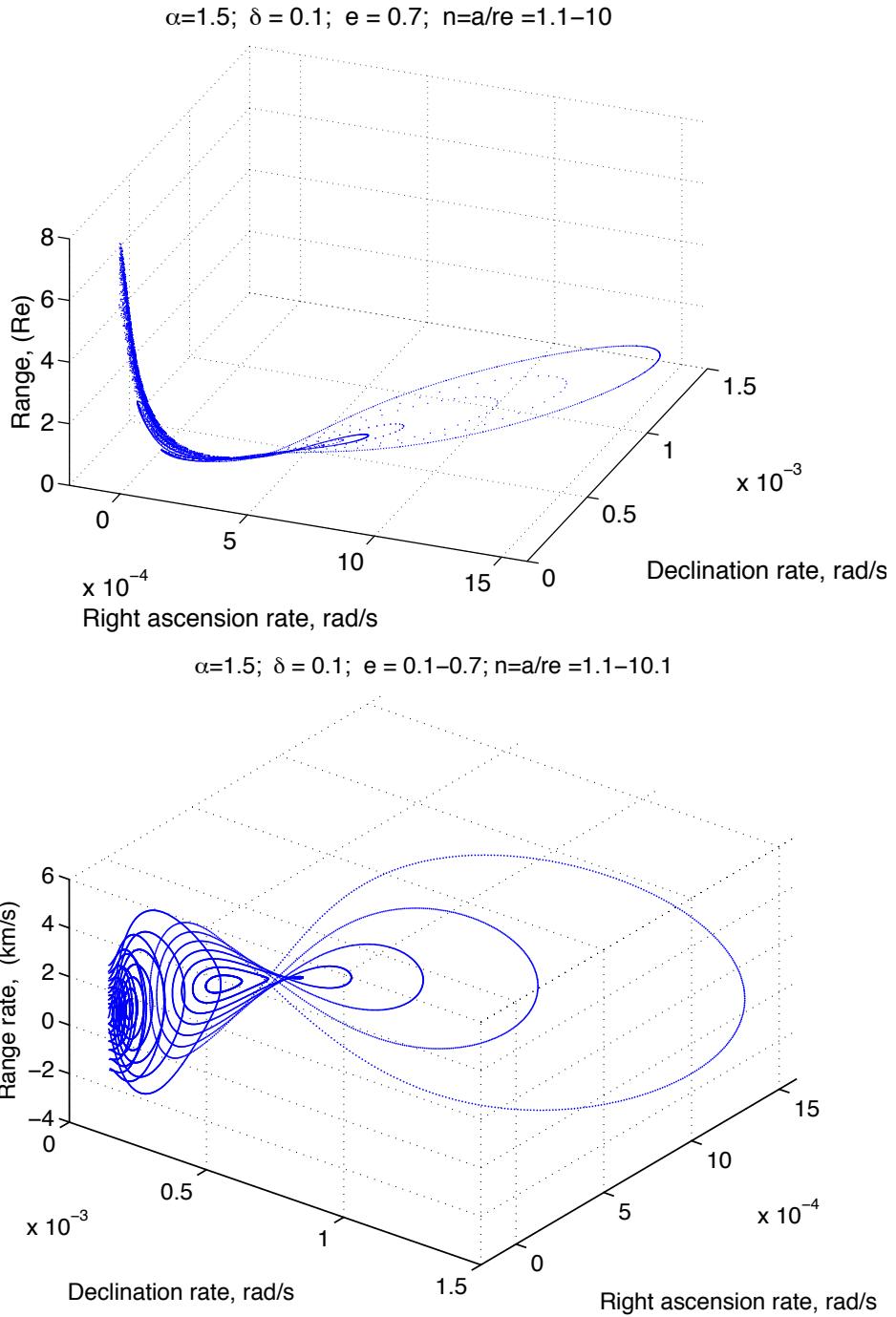


Figure 10: Functions of relationship between the angular rates and the range -1.

## References

- [1] Directive. Department of Defense. Number 3100.10. October 18, 2012. Subject: Space Policy. 12 p.
- [2] Moriba, J. US Keynote Address: Current Problems of SSA and Required areas of Research. April 10, 2013. 19 p.
- [3] Kelecy, T., Shoemaker, M. and Jah, M. Application of the constrained Admissible Region Multiple Hypothesis Filter to Initial Orbit Determination of a Break-Up. LA-UR-13-22380. Los Alamos National Laboratory. Los Alamos. NM. 2013. 8p.
- [4] Kelso, T.S. and Alfano, S. Satellite Orbital Conjunction Reports Assessing Threatening Encounters in Space (SOCRATES). 2012. <http://celestrak.com/SOCRATES/>. 4 p.
- [5] Schumacher, P. Parallel Track Initiation for Optical Space Surveillance using Range and Range-Rate Bounds. Advanced Maui Optical and Space Surveillance Technologies Conference (AMOS). Abstract of Technical Papers. September 10-13, 2013. Kihei, Maui, HI, p.9.
- [6] Milani, A.G et all. “*Orbit Determination with Very Short Arcs. I. Admissible Regions*”, Celestial Mechanics and Dynamical Astronomy. 2004, V.90, July, pp.59-87.

- [7] Frazer, G. Orbit Determination using a Decametric Line-of-Sight Radar. Advanced Maui Optical and Space Surveillance Technologies Conference (AMOS). September 10-13, 2013. Kihei, Maui, HI.
- [8] Julier, S. and Uhlmann, J., Unscented Filtering and Nonlinear Estimation. Proceedings of the IEEE, Vol. 92, March 2004.
- [9] Bishop, R.H. and Azimov, D.M., Dubois-Matra, O., Chomel, C. “*Analysis of Planetary Re-entry Vehicles With Spiraling Motion,*” New Trends in Astrodynamics and Applications III. Princeton University. New Jersey. August 16, 2006. 20 p.
- [10] Magill, D.T. “*Optimal adaptive estimation of sampled stochastic processes,*” IEEE Transactions on Automatic Control, V.AC-10, October, 1965, pp.434-39.
- [11] Mehra, R.K. “*Approaches to adaptive filtering,*” IEEE Transactions on Automatic Control, V.AC-15, April 1970, pp.175-184.
- [12] Ito, K. and Xiong, K. “*Gaussian Filters for Nonlinear Filtering Problems,*” IEEE Transactions on Automatic Control, Vol. 45, No. 5, May 2000, pp. 910927.
- [13] Horwood, J. T., Aragon, N. D., and Poore, A. B. “*Gaussian Sum Filters for Space Surveillance: Theory and Simulations,*” Journal of Guidance, Control, and Dynamics, Vol. 34, No. 6, November-December 2011, pp. 18391851.

- [14] Linares, R., Jah, M., Crassidis, J., Leve, F., Kelecy, T. “*Astrometric and Photometric Data Fusion for Inactive Space Object Feature Estimation*,” Journal of the International Academy of Astronautics: Acta Astronautica. 2012. Accepted (08/01/12).
- [15] Shoemaker, M.B., Wohlberg, B. and Koller, J. Atmospheric Density Reconstruction using Satellite Orbit Tomography. Proceedings of the 23rd AAS/AIAA Space Flight Mechanics Meeting, Kauai, HI. February 14, 2013.
- [16] Chaer, W.S., Bishop, R.H., Ghosh, J. “A Mixture of Experts Framework for Adaptive Kalman Filtering,” IEEE Transactions on Systems, Man, and Cybernetics, V.27, N.3, June 1997, pp.452-464.
- [17] Azimov, D.M. and Bishop, R.H. An Overview of Mixture of Experts Structure and Gating Network Characteristics. Technical Report. UT-GNC-TR-12-06-5. The University of Texas at Austin. December 16, 2006. Austin, TX. 24 p.
- [18] Azimov, D.M. Modifications to Constrained Admissible Region-Multiple Hypothesis Filter for Orbit Determination using Optical Observations. Final Report. Air Force Office of Scientific Research. Summer Faculty Fellowship Program (SFFP) 2013. Air Force Research Laboratory. September 15, 2013. Albuquerque, NM. 10 p.