Efficient Conjunction Assessment using Modified Chebyshev Picard Iteration

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ABSTRACT

Conjunction Assessment is one of the most important and computationally expensive components of modern SSA efforts. Timely warnings of potential conjunctions are critical for the protection of valuable space assets. As upgrades to the US Space Surveillance Network (SSN) such as the Space Surveillance Telescope and the new Space Fence become operational, the influx of newly trackable objects will exacerbate the current issues of computational tractability. Modified Chebyshev Picard Iteration (MCPI) is a numerical method for solving ordinary differential equations that can be utilized to efficiently approximate orbits with high accuracy. Unlike, more traditional stepping based integrators; MCPI uses recursive approximation using Chebyshev polynomials to estimate segments of an orbit. The end result of the propagation is orthogonal Chebyshev polynomial approximation of the orbital trajectory; this approximation is analytically differentiable and potentially accurate to machine precision. Once computed, these approximations provide an efficient method for evaluating and comparing the positions of space objects. The reduced cost of catalog propagation and subsequent conjunction probability analysis when using MCPI, allows for significant reduction in the cost to perform high fidelity conjunction assessment. A method for catalog propagation and conjunction assessment using MCPI is presented, along with results from implementation running in a compute cluster environment are presented.

1. INTRODUCTION

In order to maintain the hundreds of billions dollars in space assets and the capabilities they represent, effective methods for assessing the probability of collisions for resident space objects (RSOs) are required. A key aspect of this problem, known as conjunction analysis or conjunction assessment, is one of the most important and simultaneously resource intensive elements of current efforts in space situational awareness (SSA). Conjunction analysis is necessary for detecting and preventing (if possible) impact events between RSOs. This challenge has substantial ramifications beyond a single collision. One impact event can result in the colliding RSOs separating into from hundreds up to hundreds of thousands of separate objects, thus increasing the likelihood of future impact events as well as the complexity of future tracking and conjunction analysis. The most prominent historical example of this, the debris resulting from the 2009 Iridium-Kosmos Collision, is shown in Fig. 1.

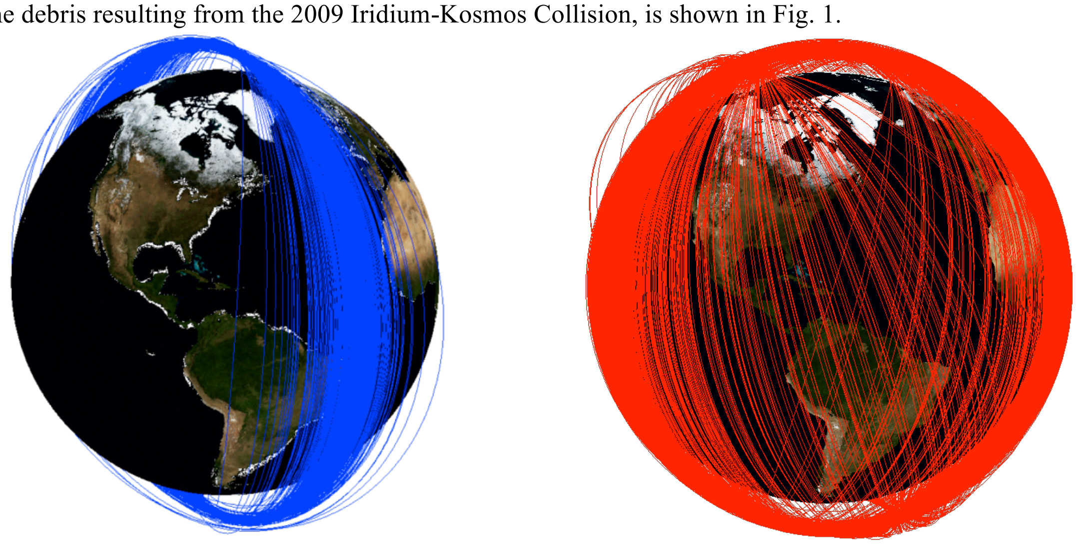

Fig. 1: Debris from Iridium-33 (left) and Kosmos-2251 (right) after the 2009 Iridium-Kosmos Collision

The computational challenge of conjunction analysis arises mainly form the scale of the problem. The computational order of the problem, shown in the Eqn. 1, is equal to the order of the conjunction analysis method multiplied by your catalog size choose two. This means that there are two driving factors for the costs of performing conjunction analysis, the number of objects being considered and the complexity of your algorithm for finding close conjunctions. Decreasing the complexity of your conjunction analysis algorithm linearly reduces the total computational complexity, while decreasing the number of objects that have to be considered has a more substantial effect due to the nature of the binomial coefficient calculation shown in Eqn. 2.

$$O(ConjuntionAnalysis) = O(ConjunctionCheck)\left(\frac{n}{2}\right) \quad (1)$$

$$\left(\frac{n}{k}\right) = \frac{n!}{k!(n-k)} for 0 \leq k \leq n \quad (2)$$

Due to the greater payoff in reducing the number of projects being considered, most efforts to decrease the computational requirements of conjunction analysis have focused on limiting which parts of the catalog should be considered together. A simple example is a perigee–apogee filter where only satellites that have orbital trajectories that pass between the perigee and apogee of the RSO of interest are checked for possible conjunctions. Despite resulting in lower gains, improvements in the efficiency of the close conjunction check are still important, especially if they can be achieved without reducing accuracy. The efforts presented in this paper focus on providing a high fidelity check for close conjunctions based on leveraging several useful properties of the Chebyshev polynomial based ephemerides produced by Modified Chebyshev Picard Iteration (MCPI) catalog propagation.

2. MCPI CATALOG PROPAGATION

MCPI has proven to be an effective method for solving the initial value problem for smooth and continuous Ordinary Differential Equations (ODEs), which is a large class of systems that includes orbital propagation. MCPI is a technique for numerical quadrature for ODEs that uses a trajectory approximation, generated from a set of high-order orthogonal Chebyshev polynomials, and recursively refines it using Picard iteration. The second order vector matrix implementation of the MCPI algorithm, shown in Fig. 2, consists of two major stages: initialization and iteration. The initialization stage includes the determination of the time span and number of function evaluation nodes for the trajectory segment approximation, the creation of certain constant matrices required for iteration, the necessary time transformation, and the generation of an initial trajectory guess. The iteration stage evaluates the forcing function at each of the nodes along the trajectory, and improves upon the trajectory approximation with an update of the velocity and subsequent update of the position. The algorithm then repeats the iteration phase until either the accuracy requirement or iteration limit is met. This formulation allows for low overhead and efficient iterations while also allowing for massive parallelization because all function evaluations can be completed independently and simultaneously.

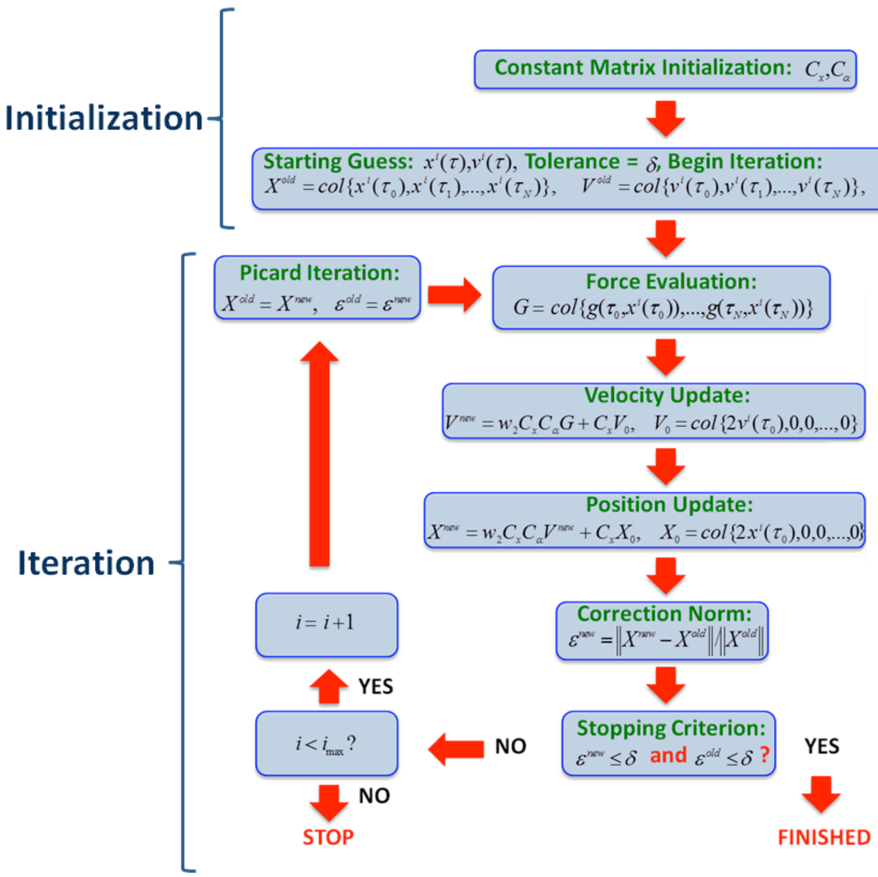

Fig. 2: Second Order Cascade MCPI Algorithm for Solving Initial Value Problems

Substantial refinements have been made to the standard MCPI algorithm to improve its efficiency and accuracy for application to orbital propagation. [1][2][3] This refined version of MCPI has been used to implement high fidelity catalog propagation on the TAMU/LASR SSA Compute Cluster. The LASR SSA Cluster is an approximately two Teraflop compute cluster that was provided by AFRL under the GEO Odyssey Program to provide a collaborative, high performance computational environment for testing and validation of SSA and astrodynamics research. This implementation allows for propagation of the full, publically available catalog of RSOs much faster than real time with perturbations such as spherical harmonic gravity and drag, a rendering of which is shown in Fig. 3. This propagator can be used to provide two types of ephemeris outputs from its propagation; the standard ephemeris type with position and velocity outputs at defined time steps and a second type that consists of Chebyshev polynomial approximations of the position, velocity, and acceleration trajectories. This second type can be extremely useful for conjunction analysis.

3. CHEBYSHEV POLYNOMIAL EPHEMERIDES

Chebyshev polynomial ephemerides are simply the coefficients for the Chebyshev polynomial approximation for the orbital position, velocity, and acceleration trajectory. Chebyshev polynomials are a type of orthogonal polynomial that can be used to approximate arbitrary functions to levels of accuracy approaching machine precision. The mathematical definitions needed for discreet Chebyshev polynomial approximation are shown in Eqns. 3-5.

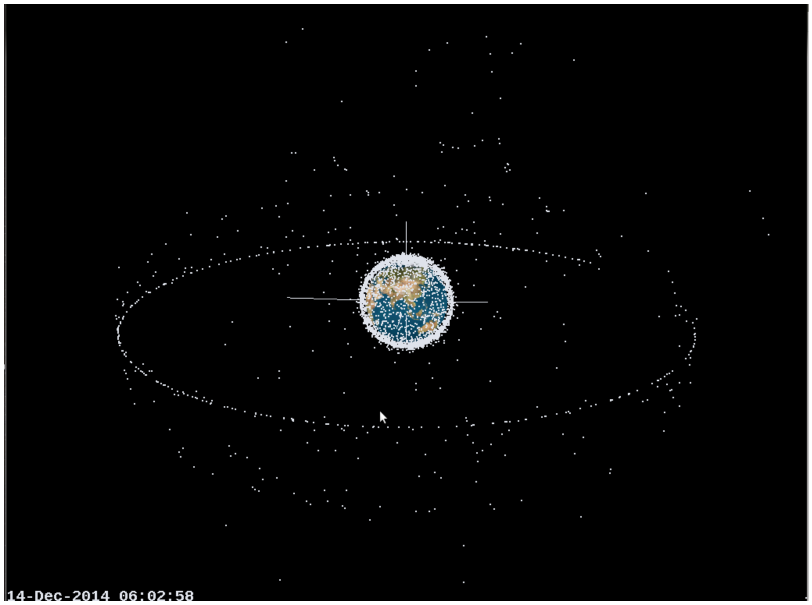

Fig. 3: Propagation of the Public RSO Catalog using MCPI

$$f(\tau) \approx \sum_{k=0}^{N-1} c_k T_j(\tau) - \frac{1}{2} c_0 \tag{3}$$

$$c_j = \frac{2}{N} \sum_{k=1}^{N} f(\tau_k) T_j(\tau_k) \tag{4}$$

$$T_0(\tau) = 1 \tag{5.1}$$

$$T_1(\tau) = \tau \tag{5.2}$$

$$T_{k+1}(\tau) = 2\tau T_k(\tau) - T_{k-1}(\tau). \tag{5.3}$$

The orthogonality of the polynomials allows for them to be computed efficiently to arbitrary accuracy without requiring a matrix inverse. However, there are two stipulations for this polynomial approximation to function. The first is that it only has a valid range from -1 to 1 so it requires a time transformation to a time variable, τ, defined on that range. Secondly, the samples required to form the polynomial approximation while satisfying the orthogonality conditions must be made at Chebyshev-Gauss-Lobatto (CGL) nodes, as defined by Eqn. 6. A comparison of uniform sampling to sampling at GCL nodes (cosine sampling) is shown in Fig. 4.

$$\tau_j = \cos(j\pi / N), j = 0,1,2...,N \tag{6}$$

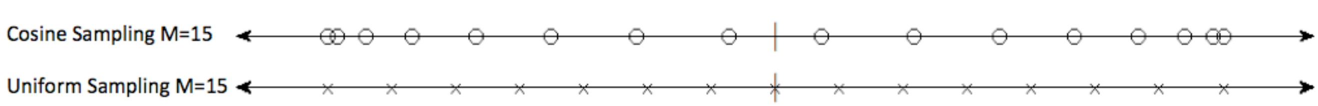

Fig. 4: Comparison of Cosine and Uniform Sampling

Both of these stipulations are automatically accounted for in MCPI because it relies on Chebyshev polynomial approximation for its formulation. This means that MCPI integration is capable of producing Chebyshev polynomial based ephemerides more efficiently than it can produce a standard set of evenly spaced position and velocity outputs.

These Chebyshev polynomial ephemerides provide an easily evaluated approximation of the orbital trajectory that is correct to whatever level of accuracy the initial propagation with MCPI was performed. Additionally, if the ephemerides have the same time span then the coefficients of multiple approximations can be easily combined or differenced to give results that are kinematically consistent with the original trajectories because they are based on polynomials. This helps provide a smooth time variable model for the distance between RSOs and its derivatives, which are useful for conjunction assessment.

4. CONJUNCTION ASSESSMENT ALGORITHM

Given a defined sub-set of the RSO catalog to be analyzed for possible conjunctions, the orbital elements are computed and the RSO with the shortest period is identified. MCPI is then used to propagate this catalog subset for a time span based on that period. The Chebyshev polynomial ephemerides generated for that timespan are then passed to the conjunction assessment module. For each RSO being considered, the Chebyshev polynomial ephemerides are subtracted to provide the difference in position, velocity, and acceleration. The time of closest approach and any conjunction that occurs must be where Δr and the square of Δr (Eqn. 7 and 8) are minimum, and this must be a root of the derivative of the square of the distance between RSOs, shown in Eqn. 9. Therefore the delta Chebyshev polynomial approximations are passed to a Newton-Raphson solver that identifies roots in the derivative of the norm of the delta position, using Eqn. 10. Once the root has been identified, the distance at that point is evaluated. If the distance is below the specified threshold it is flagged as a conjunction. The process for MCPI aided conjunction analysis is shown in Fig. 5.

$$\Delta r_{ij} = r_j(t) - r_i(t) \tag{7}$$

$$f(t) = \frac{1}{2}\Delta r_{ij}^T(t)\Delta r_{ij}(t) \tag{8}$$

$$g(t) = \frac{df(t)}{dt} = \Delta r_{ij}^T(t)\Delta \dot{r}_{ij}(t) \tag{9}$$

$$t_{k+1} = t_k - \frac{g(t)}{\frac{df(t)}{dt}} = t_k - \frac{\Delta r_{ij}^T(t)\Delta \dot{r}_{ij}(t)}{\Delta \dot{r}_{ij}^T(t)\Delta \dot{r}_{ij}(t) + \Delta r_{ij}^T(t)\Delta \ddot{r}_{ij}(t)} \tag{10}$$

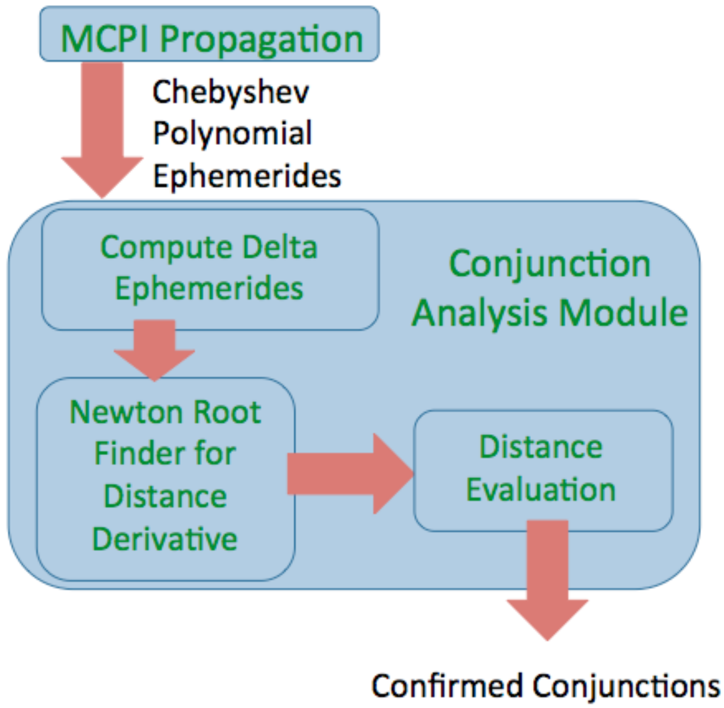

Fig. 5: MCPI Aided Conjunction Analysis Process

5. CONJUNCTION ASSESSMENT TESTING

5.1 Conjunction Identification

To validate the performance of the proposed conjunction assessment algorithm it was implemented in Matlab and a simple Monte Carlo analysis was performed. The RSOs being considered for this test were the satellites in the Iridium constellation and the time period considered was two days. Additionally, a fake RSO was inserted to insure that a conjunction was present. To insert this fake conjunctor a random time was chosen as the conjunction time and a random satellite number was selected from the constellation for the object to impact. The selected satellite was propagated to the selected time and its state was duplicated. A random perturbation was chosen with a magnitude up to 5% of the total velocity of the satellite at the conjunction time. This perturbation was added to the velocity of the duplicated state perpendicular to the orbital plane to set the conjunctor velocity at that time. The conjunctor was then back propagated to the initial time and inserted into the catalog. MCPI was then used to propagate the entire catalog for the time under consideration and it provided Chebyshev polynomial ephemerides as outputs. These ephemerides were then passed into the conjunction analysis module to be processed. The conjunction analysis module was then used to identify any conjunctions as well as the time of impact. This test was repeated 100 times with different conjuncting satellite and conjunction times for each trial.

5.2 Timing Comparison

To get a baseline for timing performance the Single Satellite conjunction detection method proposed by Healy [4] was implemented in Matlab using Valado's Simplified General Perturbation model (SGP4) as a propagator [8]. Briefly, this method uses a propagator to output ephemeris at a specified interval, and finds the point among the ephemeris where the two satellites are closest and computes the time of closest approach using Eqn. 11, where t_k is the time of the ephemeris with the closest distance and x_k is the distance between RSOs at that time. If greater accuracy is desired the $t_{closest}$ can be refined using Newton's method solver. The implementation selected used 2-minute ephemeris output and without the subsequent Newton iteration to refine the time of closest approach because the simplicity means that the timing results would be as fast as possible. This method was run along side the proposed MCPI aided method to analyze a pair of satellites over a one orbit time period. The timing results from both methods were averaged over ten runs and were compared. These timing tests did not reflect the initialization time for SGP4 or the initial catalog propagation using MCPI.

$$t_{closest} = t_k - \frac{x_k \cdot \dot{x}_k}{\dot{x}_k \cdot \dot{x}_k} \tag{12}$$

6. RESULTS

6.1 Conjunction Identification

The results of the Monte Carlo testing of the algorithm were completely successful, with the correct pair of conjunction RSOs and the approximate conjunction time being identified in every case. The average error for time of closest approach calculation was ~0.032 seconds. Results for a single test when a conjunction was detected are shown below in Figs. 6-7. Fig. 6 shows the identification of roots in the derivative of the distance and the corresponding points on the graph of distance, with the conjunction highlighted with a green x. Fig. 7 shows the two orbital trajectories with the evaluation points for possible time of closest approach shown. The pair of evaluation points at the top of the image shows an identified conjunction.

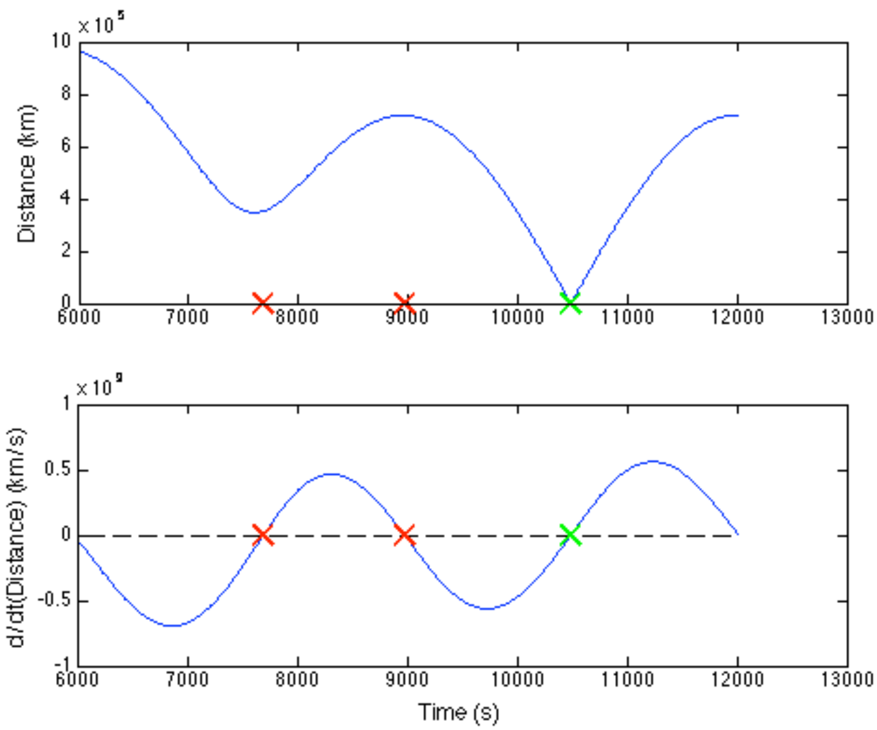

Fig. 6: Distance between RSOs and its Derivative

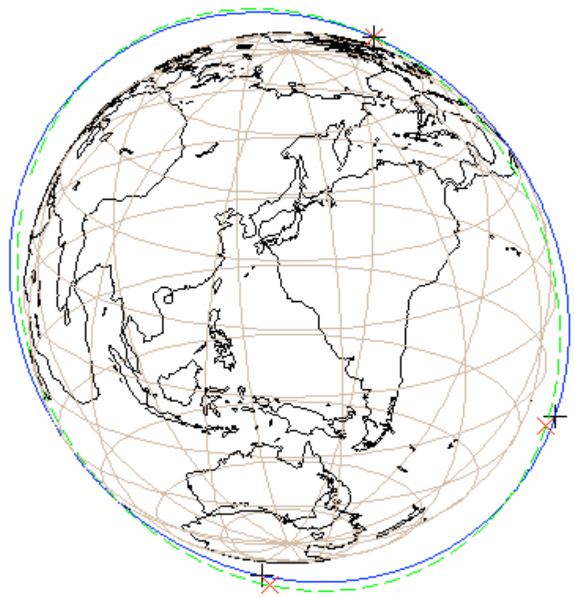

Fig. 7: Potential Times of Closest Approach (with Conjunction)

6.2 Timing Comparison

The timing results between the proposed method and the Single Satellite Method using SGP4 are shown in Fig. 8. MCPI aided conjunction analysis provides an approximately 50% increase in speed when compared to the Single Satellite Method using SGP4. This increase in speed is also essentially independent of the perturbations being considered for the MCPI based propagation as it relies only on the Chebyshev polynomial ephemerides. This time comparison does not include the time required to perform the catalog propagation, which has a non-trivial initial cost, so this method would not be faster for small sets of objects. To determine the point where the total computational cost of catalog propagation and conjunction analysis using MCPI falls below the computational cost for the Single Satellite method an approximate timing analysis was computed. This analysis evaluated the cost of the conjunction analysis (Eqn. 1) plus the cost of the catalog propagation for the MCPI based method and compared it to the cost of only the conjunction analysis for the Single Satellite method. The relative cost of MCPI propagation compared with SGP4 can vary greatly based on the physical models being used so characteristic values of 100 and 1,000 times greater cost were considered. The results for this timing analysis are shown in Fig. 9. The case with a 100 times greater cost for MCPI propagation would result in a lower total cost when over approximately 1,300 RSOs are considered and for the case where the relative propagation cost is 1,000 times greater the total cost becomes less at approximately 4,500 RSOs. While the MCPI based conjunction analysis method has been implemented for use on the LASR SSA Cluster and can be used to identify potential conjunctions in the full orbit catalog, there is currently no publically available baseline to test it against.

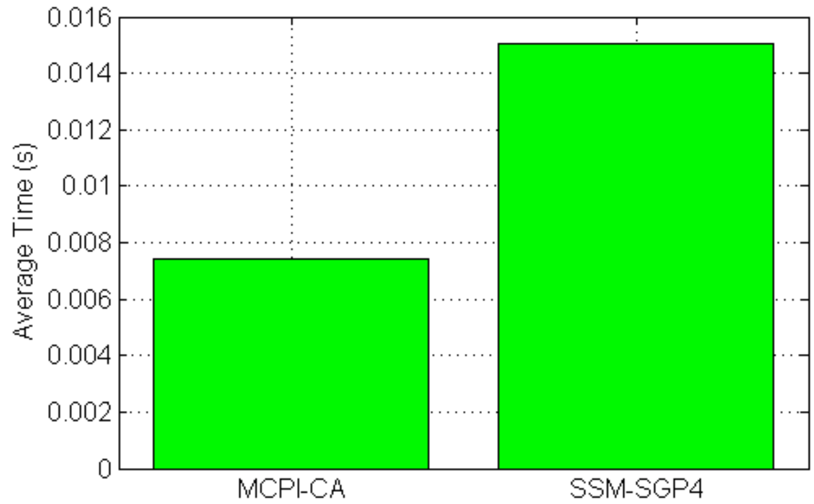

Fig. 8: Timing Comparison between MCPI Aided Conjunction Analysis and the Single Satellite Method with SGP4

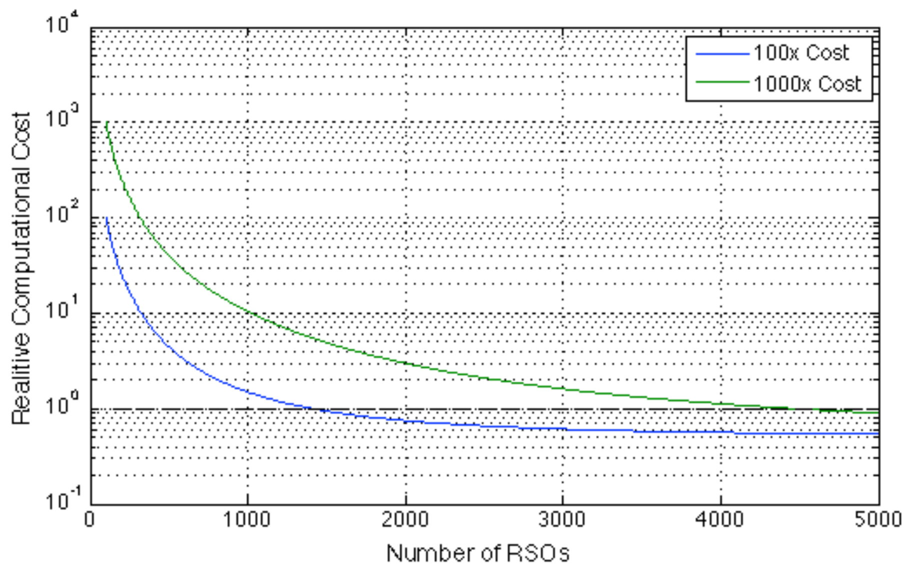

Fig. 9: Cost Savings of MCPI Aided Conjunction Analysis and the Single Satellite Method with SGP4 versus Number of RSOs

7. CONCLUSIONS

An algorithm for MCPI aided conjunction analysis is presented. MCPI provides an efficient method for catalog propagation and can provide Chebyshev polynomial ephemerides as an output. The conjunction analysis algorithm described utilizes Chebyshev polynomial ephemerides to reduce conjunction assessment for each pair of satellites to a series of Newton-Raphson evaluations to identify possible times of closest approach and check for a conjunction. This algorithm uses high fidelity approximations of orbital trajectories and can therefore provide significantly greater accuracy than methods that rely on general-perturbation based methods such as SGP4. A simple Monte Carlo analysis was performed to validate the method and it demonstrated 100% success. The computational cost of the method was also compared to the Single Satellite method utilizing SGP4 as a propagator and demonstrated an up to 50% increase in speed for large number of objects while maintaining a higher level of accuracy.

8. FUTURE WORK

The first extension of these efforts will include the integration of probability of collision calculation using propagation of the state transition matrix. Secondly, the single satellite method and other competing methods conjunction assessment must be implemented in C/C++ and MPI for comparison using the LASR SSA Cluster.

9. ACKNOWLEDGEMENTS

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