A Fast Method for Embattling Optimization of Ground-Based Radar Surveillance Network

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ABSTRACT

A growing number of space activities have created an orbital debris environment that poses increasing impact risks to existing space systems and human space flight. For the safety of in-orbit spacecraft, embattling optimization of ground-based radar surveillance network is needed to catalog the large number of space objects. This paper proposed a fast method for embattling optimization of ground-based radar surveillance network. Firstly established a space coverage projection model of radar facilities, and then optimized the embattling of ground-based radar surveillance network with the artificial intelligent algorithm, which can greatly simplifies the computational complexities. Comparing with the traditional method, simulation results demonstrates the proposed approach not only greatly improved the computational efficiency but also can guarantee very high accuracy.

1. INTRODUCTION

A growing number of space activities have created an orbital debris environment that poses increasing impact risks to existing space systems and human space flight[1-3]. For the safety of in-orbit spacecraft, lots of observation facilities are needed to catalog the large number of space objects. Space objects observation are mainly rely on ground-based radars[4]. Due to the ability limitation of existing facilities, many new ground-based facilities will be built in the next few years. How to optimize the embattling of ground-based radar surveillance network is a problem to need to be solved.

The traditional method for embattling optimization of ground-based radar surveillance network can be divided into two steps, which first conducts detection simulation of all possible stations with cataloged data and makes a comprehensive comparative analysis of various simulation results with the combinational method, and then selects a sub-optimal result as the station layout scheme. This method is time consuming and high computational complexity, when the number of stations increases, the complexity of optimization problem will be increased exponentially, and cannot be solved with traditional method. There is no better way to solve this problem till now.

PSO introduced by Kennedy and Eberhart [6,7] is one of the most recent met heuristics, which is inspired by the swarming behavior of animals and human social behavior. Scientists found that the synchrony of animal's behavior was shown through maintaining optimal distances between individual members and their neighbors. Thus, velocity plays the important role of adjusting each member for the optimal distance. Furthermore, scientists simulated the scenario in which birds search for food and observed their social behavior. The PSO algorithm has shown its robustness and efficacy for solving function value optimization problems in real number spaces. Only a few researches have been conducted for extending PSO to combinatorial optimization problems.

This paper firstly established a space coverage projection model of radar facilities and then optimized the embattling of ground-based radar surveillance network with the PSO algorithm, which can greatly simplifies the computational complexities. Comparing with the traditional method, simulation results demonstrates the proposed approach not only greatly improved the computational efficiency but also can guarantee very high accuracy.

2. BASIC MODELS

2.1 Radar Coverage Model

Radar coverage[4] is mainly determined by maximal detection range $R_{\max}$ and field of view, which can be simplified as the longitude coverage of its coverage's ground projection.

The longitude coverage of radar at special latitude can be computed with geometry method. Three coordinate system are involved in the computation: station coordinate system (RAE), topocentric horizon system (SEZ) and earth fixed coordinate system (ECEF).

Assume location of site is $(\beta_s, \lambda_s, r_s)$, first translate from station coordination system (φ, θ, ρ) to topocentric horizon system (SEZ)

$$\vec{\rho}_{SEZ} = \begin{bmatrix} -\rho\cos(\theta)\cos(\varphi) \\ \rho\cos(\theta)\sin(\varphi) \\ \rho\sin(\theta) \end{bmatrix} \tag{1}$$

Then translate from topocentric horizon system to earth fixed coordinate system

$$\vec{r}_{ECEF} = \vec{r}_{site,ECEF} + \vec{\rho}_{ECEF} \tag{2}$$

where

$$\vec{\rho}_{ECEF} = [\mathrm{ROT3}(-\lambda)][\mathrm{ROT2}(\phi - 90^\circ)]\vec{\rho}_{SEZ}$$

$$\mathrm{ROT2}(\alpha) = \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$

$$\mathrm{ROT3}(\alpha) = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\vec{r}_{site,ECEF}$ has relation with earth oblateness, and earth oblateness can be neglected in rough simulations, then $\vec{r}_{site,ECEF}$ can be rewrite as

$$\vec{r}_{site,ECEF} \cong r_s \cdot \begin{bmatrix} \cos(\beta_s)\cos(\lambda_s) \\ \cos(\beta_s)\sin(\lambda_s) \\ \sin(\beta_s) \end{bmatrix}$$

The transform from RAE to ECEF can be expressed as follows

$$\vec{r}_{ECEF} = \begin{bmatrix} r_s\cos(\beta_s)\cos(\lambda_s) + \rho(\cos(\lambda_s)\cos(\beta_s)\sin(\theta) - \cos(\theta)\cos(\varphi)\cos(\lambda_s)\sin(\beta_s) - \sin(\lambda_s)\cos(\theta)\sin(\varphi)) \\ r_s\cos(\beta_s)\sin(\lambda_s) + \rho(\sin(\lambda_s)\cos(\beta_s)\sin(\theta) - \cos(\theta)\cos(\varphi)\sin(\lambda_s)\sin(\beta_s) + \cos(\lambda_s)\cos(\theta)\sin(\varphi)) \\ r_s\sin(\beta_s) + \rho(\sin(\beta_s)\sin(\theta) + \cos(\theta)\cos(\varphi)\cos(\beta_s)) \end{bmatrix} \tag{3}$$

The longitude λ and latitude β of the observed data can be computed as follows

$$\lambda = \arctan\left(\frac{x_{ECEF}}{y_{ECEF}}\right) \tag{4}$$

$$\beta = \arcsin\left(\frac{z_{ECEF}}{\sqrt{x_{ECEF}^2 + y_{ECEF}^2 + z_{ECEF}^2}}\right) \tag{5}$$

Assume the radar is located at $(\beta_s, \lambda_s, r_s)$ with maximal detection range $R_{\max}$ and scanning field FOV, then the maximal longitude coverage at height *h* of the facility can be calculated with the following steps:

(1) Calculation of elevation $\theta(\mathrm{FOV}, R_{\max}, h)$ and slot range $\rho(\mathrm{FOV}, R_{\max}, h)$ of maximal longitude coverage in station coordinate system

$$\theta(\mathrm{FOV}, R_{\max}, h) = \begin{cases} 90 - \mathrm{FOV}/2 & h \le R_{\max}\cdot\cos(\mathrm{FOV}/2) \\ \arcsin(h/R_{\max}) & R_{\max}\cdot\cos(\mathrm{FOV}/2) < h < R_{\max} \\ 90 & h \ge R_{\max} \end{cases} \tag{6}$$

$$\rho(\mathrm{FOV}, R_{\max}, h) = \begin{cases} h/\cos(\mathrm{FOV}/2) & h \le R_{\max}\cdot\cos(\mathrm{FOV}/2) \\ R_{\max} & R_{\max}\cdot\cos(\mathrm{FOV}/2) < h < R_{\max} \\ 0 & h \ge R_{\max} \end{cases} \tag{7}$$

(2) Calculation of longitude coverage at any azimuth angle $\Lambda_{\mathrm{cov}}(\beta_s, \lambda_s, r_s, \varphi, \theta, \rho)$

$$\Lambda_{\mathrm{cov}}(\beta_s, \lambda_s, r_s, \varphi, \theta, \rho) = \arccos\left(\frac{x_1\cdot x_2 + y_1\cdot y_2}{\sqrt{x_1^2 + y_1^2}\cdot\sqrt{x_2^2 + y_2^2}}\right) \tag{8}$$

where

$$x_1 = r_s\cos(\beta_s)\cos(\lambda_s) + \rho(\cos(\lambda_s)\cos(\beta_s)\sin(\theta) - \cos(\theta)\cos(\varphi)\cos(\lambda_s)\sin(\beta_s) - \sin(\lambda_s)\cos(\theta)\sin(\varphi))$$
$$x_2 = r_s\cos(\beta_s)\cos(\lambda_s) + \rho(\cos(\lambda_s)\cos(\beta_s)\sin(\theta) - \cos(\theta)\cos(\varphi)\cos(\lambda_s)\sin(\beta_s) + \sin(\lambda_s)\cos(\theta)\sin(\varphi))$$
$$y_1 = r_s\cos(\beta_s)\sin(\lambda_s) + \rho(\sin(\lambda_s)\cos(\beta_s)\sin(\theta) - \cos(\theta)\cos(\varphi)\sin(\lambda_s)\sin(\beta_s) + \cos(\lambda_s)\cos(\theta)\sin(\varphi))$$
$$y_2 = r_s\cos(\beta_s)\sin(\lambda_s) + \rho(\sin(\lambda_s)\cos(\beta_s)\sin(\theta) - \cos(\theta)\cos(\varphi)\sin(\lambda_s)\sin(\beta_s) - \cos(\lambda_s)\cos(\theta)\sin(\varphi))$$

(3) Calculation of maximal longitude coverage $\Lambda_{\max}$

Usually the height of radar (h_{site}) is about several hundred meters, which is far less than earth's radius (R_e) and can be ignored during the analysis, so we can get $r_s \approx R_e = 6378.137$。 With simplified computation, the maximal longitude coverage is

$$\Lambda_{\max} = \arccos\left(\frac{D_0 x_0^2 + B_0 x_0 + F_0}{A_0 x_0^2 + B_0 x_0 + C_0}\right) \tag{9}$$

where

$$\begin{aligned}
A_0 &= -\rho^2 \cos^2\theta \cos^2\beta_s \\
B_0 &= -R_e \rho \sin(2\beta_s)\cos\theta - \rho^2 \sin(2\beta_s)\sin(2\theta)/2 \\
C_0 &= R_e^2 \cos^2(\beta_s) + \rho^2 \cos^2(\beta_s)\sin^2(\theta) + \rho^2\cos^2(\theta) + 2R_e\rho\cos^2(\beta_s)\sin\theta \\
D_0 &= \rho^2\cos^2\theta(1+\sin^2\beta_s) \\
E_0 &= -R_e\rho\sin(2\beta_s)\cos\theta - \rho^2\sin(2\beta_s)\sin(2\theta)/2 \\
F_0 &= R_e^2\cos^2(\beta_s) + \rho^2\cos^2(\beta^s)\sin^2(\theta) - \rho^2\cos^2(\theta) + 2R_e\rho\cos^2(\beta_s)\sin\theta \\
x_0 &= \left(-(A_0+C_0) + \sqrt{(A_0+C_0)^2 - B_0^2}\right)/B_0 \\
\varphi_0 &= \arccos(x_0)
\end{aligned}$$

(4) Calculation of the corresponding latitude of maximal longitude coverage

With location of site $(\beta_s, \lambda_s, r_s)$ and observed data $(\varphi_0, \theta, \rho)$ corresponding to maximal longitude coverage, the latitude β corresponding maximal longitude coverage can be calculated with equation (3) and (5)

$$\begin{aligned}
\beta &= \arcsin\left(\frac{z_{ECEF}}{\sqrt{x_{ECEF}^2 + y_{ECEF}^2 + z_{ECEF}^2}}\right) \\
&= \arcsin\left(\frac{R_e\sin(\beta_s) + \rho(\sin(\beta_s)\sin(\theta) + \cos(\theta)\cos(\varphi_0)\cos(\beta_s))}{\sqrt{\rho^2 + R_e^2 + 2\rho R_e \sin\theta}}\right)
\end{aligned} \tag{10}$$

(5) Calculation the longitude coverage at special latitude β_t

Slot range ρ_t and elevation angle θ_t in station coordination system can be computed with equation (6) and (7), and then its azimuth angle and longitude coverage can be compute with equation (10) and (8) respectively.

2.2 Simplified Object Detection Model

The ground projection of satellite's trajectory was shown in figure 1. As it can be seen from figure 1, the longitude spans of trajectories between adjacent revaluations are approximately the same. The longitude span denotes the change RAAN of satellite orbit, which is mainly determined by two factors: the earth's rotation and earth oblateness.

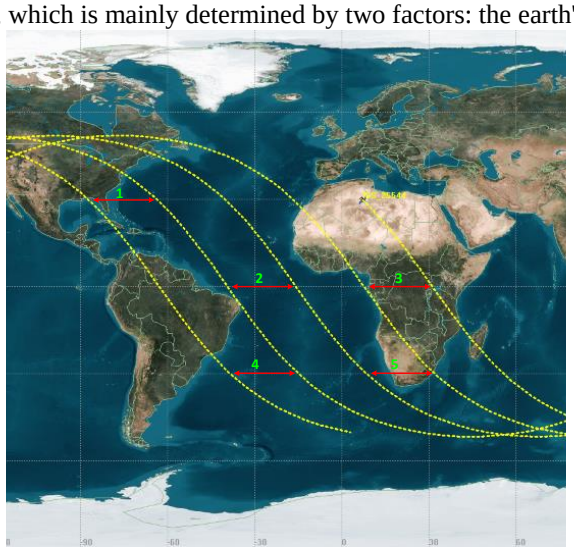

Fig. 1 Ground Projection of Satellite's Trajectories

The RAAN change caused by earth's rotation can be expressed as

$$\lambda_B = \lambda_{B_0} - \omega_E (t - t_B) \tag{11}$$

where ω_E is the angular velocity of the earth, and t_B is the epoch when spacecraft pass the ascend point.
After one revolution, the RAAN change caused by earth's rotation is

$$(\Delta\lambda_B)'_{2\pi} = -\omega_E T = -2\pi\omega_E\sqrt{\frac{a^3}{\mu}} \tag{12}$$

The second factor caused the drift of RAAN is earth oblateness, the RAAN change caused by earth oblateness in one revolution is

$$(\Delta\Omega)_{2\pi} = -3\pi J_2 R_E^2 \frac{\cos i}{a^2(1-e^2)^2} \tag{13}$$

The total RAAN change in one revolution is

$$\begin{aligned}(\Delta\lambda_B)_{2\pi} &= (\Delta\lambda_B)'_{2\pi} + (\Delta\Omega)_{2\pi} \\ &= -2\pi\omega_E\sqrt{\frac{a^3}{\mu}} - 3\pi J_2 R_E^2 \frac{\cos i}{a^2(1-e^2)^2}\end{aligned} \tag{14}$$

To simplify the object detection, the trajectory of sub-satellite point and the maximal longitude coverage of radar facility are computed, if they intersect each other in the revolution, then assume the object can be detected during this pass, otherwise the object cannot be detected.

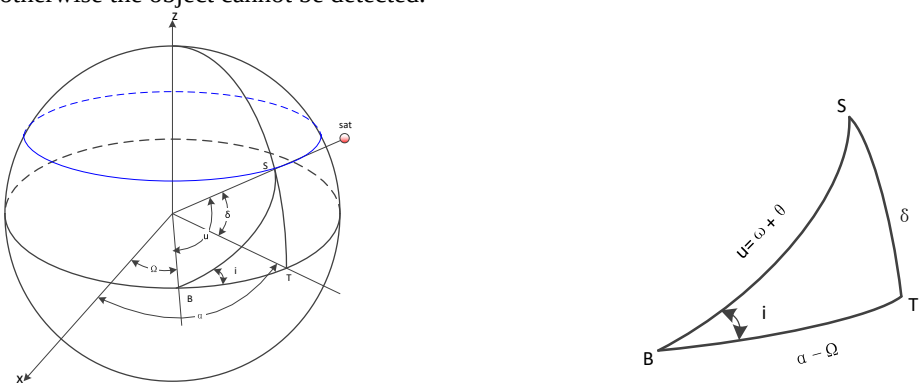

Fig. 2 Spherical Trigonometry

With spherical trigonometry, the following equations can be obtained[5]

$$\begin{cases} \sin\delta = \sin i \sin(\omega+\theta) \\ \sin(\alpha-\Omega) = \cos i \sin(\omega+\theta)/\cos\delta \\ \cos(\alpha-\Omega) = \cos(\omega+\theta)/\cos\delta \end{cases} \tag{15}$$

Then we can get

$$\alpha = \Omega + \operatorname{arc\,cot}\left(\frac{\sqrt{(\sin \mathrm{i}/\sin\delta)^2 - 1}}{\cos \mathrm{i}}\right) \tag{16}$$

3. EMBATTLING OPTIMIZATION METHOD

3.1 Particle swarm optimization

The general principles of the PSO[6,7] algorithm are stated as follows. Let *m* be the size of the swarm. Each particle *i* can be represented as an object with several characteristics. Suppose that the search space is a *n*-dimensional space, then the *i*th particle can be represented by a *n*-dimensional vector, $X_i = \{x_{i1}, x_{i2}, \cdots, x_{in}\}$, and velocity $V_i = \{v_{i1}, v_{i2}, \cdots, v_{in}\}$, where i = 1, 2, ..., m. In PSO, particle *i* remembers the best position it visited so far, referred as $P_i = \{p_{i1}, p_{i2}, \cdots, p_{in}\}$, and the best position of the best particle in the swarm, referred as $G = \{G_1, G_2, \cdots, G_n\}$ PSO is similar to an evolutionary computation algorithm and, in each generation *t*, particle *i* adjusts its velocity v_{ij}^t and position x_{ij}^t for each dimension *j* by referring to, with random multipliers, the personal best position p_{ij}^{t-1} and the swarm's best position g_{ij}^{t-1}, using Eqs. (17) and (18), as follows:

$$v_{ij}^t = v_{ij}^{t-1} + c_1 r_1 (p_{ij}^{t-1} - x_{ij}^{t-1}) + c_2 r_2 (g_{ij}^{t-1} - x_{ij}^{t-1}) \tag{17}$$

And

$$x_{ij}^t = x_{ij}^{t-1} + v_{ij}^t \tag{18}$$

Where c_1 and c_2 are the acceleration constants and r_1 and r_2 are random real numbers drawn from [0, 1].
In the latest versions of the PSO, Eqs. (17) is changed into the following one:

$$v_{ij}^{t} = \chi(\omega v_{ij}^{t-1} + c_1 r_1 (p_{ij}^{t-1} - x_{ij}^{t-1}) + c_2 r_2 (g_{ij}^{t-1} - x_{ij}^{t-1})) \tag{19}$$

ω is called inertia weight and is employed to control the impact of the previous history of velocities on the current one. χ is a constriction factor, which is used to limit the velocity.

3.2 Embattling Optimization via PSO

Radar facility embattling optimization is a question to choose a better location for the new facility to improve the radar network performance. It is coordinated with the basic concept of PSO, so PSO can be used for this question. The basic particle of PSO is defined as a vector which composed of facility's longitude and latitude. Using the amount increased number of detected target as a standard of the merits, its fitness function *f* is defined as follows:

$$f = \sum_i w_i \cdot \frac{D_N(i)}{D_N(i) + D_O(i)} \tag{20}$$

where ω_i is weight coefficient of target *i*, $D_N(i)$ and $D_O(i)$ are both statistic function, they are used to count the detected times of object *i* for new facility and other facilities.
The embattling optimization algorithm via PSO can be divided into the following steps:

Step 1. Initialize parameters, include maximal velocity, position and so on.
Step 2. Evaluate the fitness function of all particles in the population using Eq. (20), find particles best position (Pbest) of each particle and update its objective value. Similarly, find the global best position (Gbest) among all the particles and update its objective value.
Step 3. If stopping criterion is met go to step 6. Otherwise continue.
Step 4. Update the velocity using Eq. (19).
Step 5. Update the position of each particle according to Eq. (18).
Step 6. Output the Gbest particle and its objective value.

4. NUMERICAL SIMULATION

4.1 Radar Coverage Experiments

According to the analysis in section 2, radar coverage is mainly determined by its maximal detection range, field of view, altitude of detected orbit and latitude of site, but has no relation with longitude of site.
In these experiments, assume the maximal detection range of radar facility is 3000 km/m^2. First, we assume the facility is located at latitude 40 degrees, Fig. 3 shows its maximal longitude coverage vs. FOV in different orbital altitude, Fig. 4 is a two-dimensional map of maximal longitude coverage versus FOV and orbital altitude. Fig. 5 is a two-dimensional map of maximal longitude coverage versus FOV of facility and latitude of radar site, where the orbital altitude is set as 1000km. Fig. 6 is a two-dimensional map of maximal longitude coverage versus latitude of radar site and orbital altitude with facility's FOV fixed at 100 degrees.

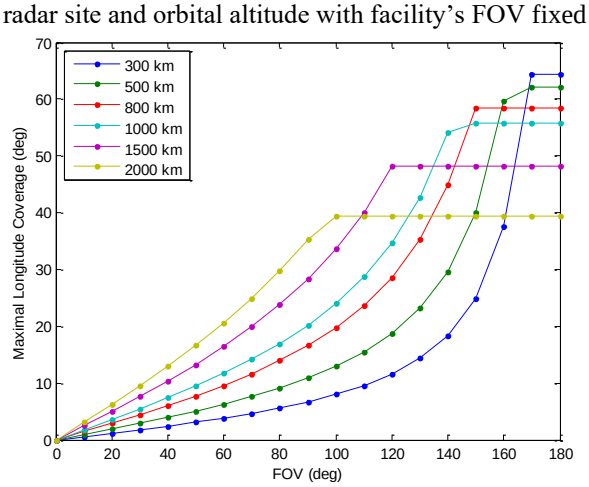

Fig. 3 Maximal Longitude Coverage vs. FOV

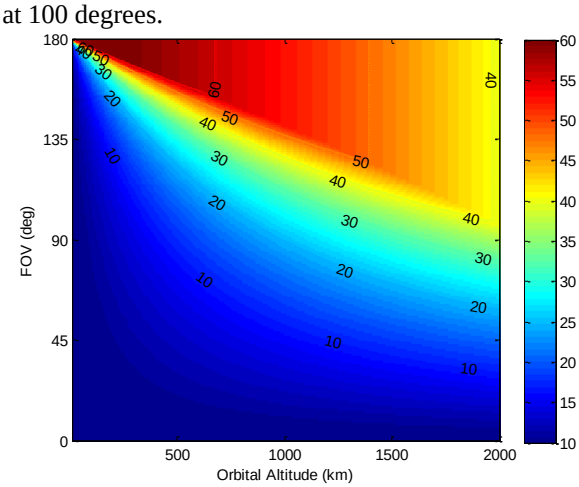

Fig. 4 Maximal Longitude Coverage vs. FOV and Orbital Altitude

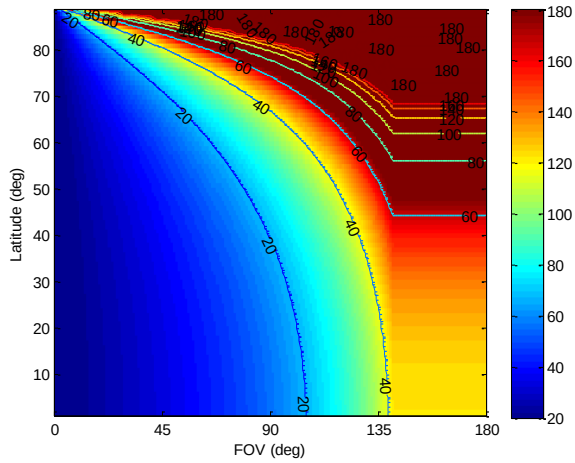

Fig. 5 Maximal Longitude Coverage vs. FOV and Latitude of radar site

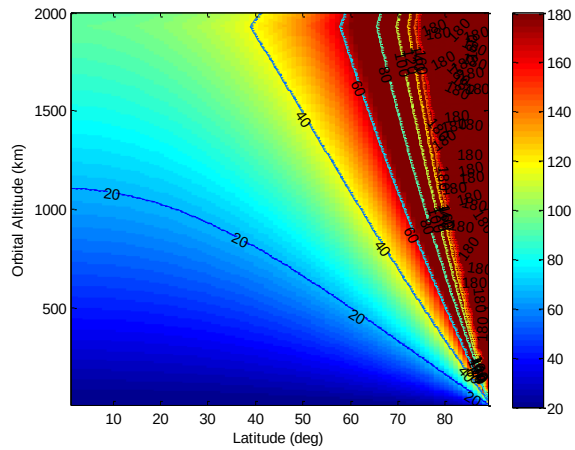

Fig. 6 Maximal Longitude Coverage vs. Orbital Altitude and Latitude of radar site

4.2 Radar Detection Experiment

In these experiments, the radar site is located at (120E30N), maximal detection range is 3000km/m^2 and FOV is 120 degrees. The orbital data used in the simulations are January 5, 2015 online publication of NOARD number less than 10000, and the simulation period is 2015-01-05 00:00:00 ~ 2015-01-06 00:00:00, simulations are both conducted with ROOF[8] software and our the proposed method respectively, and the simulation results are listed in the following table

Tab. 1 Results of Radar Detection Experiment

	PROOF	Proposed Method	The Same	Correctness
Detect Objects	1805	1906	1660	90.86%
Detect Times	4682	4850	4032	86.12%

From the table above, we can see that the results of the proposed method can achieve more than 85% of the accuracy when compared with the results of PROOF. The result of the proposed method is a statistic result, which does not vary with the simulation period, while the result of PROOF will vary with simulation period, so the proposed method can basically meet the requirement for the embattling of the new equipment. Additionally, according to the analysis in section 2, the method proposed in this paper only needs one propagation for object's detection in one revolution while more than 20 propagations are needed for the PROOF method. In conclusion, the proposed method not only greatly improved the computational efficiency but also can guarantee very high accuracy.

5. CONCLUSION AND FUTURE WORK

This paper established a simplified radar coverage computation model and proposed an embattling optimization method for ground-based radar surveillance network with the PSO algorithm. Comparing with the traditional method, simulation results demonstrates the proposed approach not only greatly improved the computational efficiency but also can guarantee very high accuracy. There are still much left for improvement, such as object's RCS is not considered in radar detection, these will incorporated into the models in our future work.

6. REFERENCES

1. Du Heng, Zhang Wenxiang, Pang Baojun, and Liu Jing. Space Debris. Beijing: China Astronautic Publishing House, 2007.
2. J.-C. Liou and N. L. Johnson. Risks in space from orbiting debris. SCIENCE, 2006, 311(5759): 340-341.
3. Klinkrad H. Space debris. Wiley Online Library, 2010.
4. Merrill I. Skolnik. Introduction to radar systems, Third Edition. New York: McGraw-Hill Companies, Inc. 2002.
5. Vallado, D. A. Fundamentals of Astrodynamics and Applications, Microcosm Press, 2013.
6. Kennedy J and Eberhart R C. Particle swarm optimization. IEEE International Conference on Neural Networks. Piscataway: IEEE Press, 1995: 1942-1948.
7. Shi Y H and Eberhart R. A modified particle swarm optimizer. IEEE World Congress Computational Intelligence. Piscataway: IEEE Press, 1998: 69-73.
8. J. Gelhaus, S. Flegel, and C. Wiedemann. Program for radar and optical observation forecasting. Report of European Space Agency, 2011.