Shape Estimation from Lightcurves including Constraints from Orbit Determination

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ABSTRACT

Once a Resident Space Object's (RSO) orbit has been determined, further observations are used to characterize the object. Basic information about the object, such as its shape and its attitude motion, are key pieces of information that can be used to infer what the object is and what it is doing – the foundation of Space Situational Awareness (SSA). A common type of measurement that is used to determine information about an RSO shape and attitude are lightcurves, which simply put are time series measurements of an object's brightness. Although this information is widely used to characterize asteroid shapes and attitude, there are many assumptions made as the problem of determining the shape and attitude from the lightcurve information is technically ill-posed and unsolvable. Thus, it is highly desirable to fuse other information into the characterization problem in order to further constrain the RSO properties. This paper discusses a general method for processing lightcurve observations, and how detailed estimation of solar radiation pressure (SRP) forces obtained from the orbit determination process can be used to further constrain the characterization results.

1. INTRODUCTION

Space situational awareness doesn't end when a Resident Space Object's orbit is determined – it is also desirable to determine whatever we can about the shape and attitude state of the object. In many cases, the measurement types available to characterize this information is the optical lightcurve, which gives the observed brightness of the object over time. For most objects, the lightcurve varies in magnitude with time, and this periodicity is useful for determining the spin period and/or testing to see if an object is tumbling. Unfortunately, estimating both the shape and attitude of the RSO from lightcurves is impossible - it has been rigorously proven that these features are unobservable in [1]. This is due to the fact that the brightness measurement is an integral of all of these parameters and states, so that an infinite number of possible combinations could produce the same measurements.

Although the inversion problem is technically ill-posed, and thus not uniquely solvable, researchers have still approached this problem and had success in characterizing different pieces of information from lightcurves by making certain assumptions. The most common and successful application of this theory is for estimating the shape and spin state of asteroids, using the method developed in [2, 3]. This method assumes there is a fixed spin pole and spin rate and a simple bidirectional reflectance distribution functions (BRDF) that is constant for the entire surface. Under these assumptions, it is possible to estimate the spin pole, spin rate, and shape. Even with these strong assumptions, this is a difficult non-linear optimization problem, and thus the answer is generally a non-unique local minima. Other methods have relied on posing various hypotheses which constrain the shape and spin state sufficiently to allow a neighboring solution to be found, however these results are clearly limited by the assumptions made when constructing the hypotheses [8, 9, 10].

However, there is extra information that can be leveraged to improve the situation due to the coupling of attitude and translation in the dynamics through SRP perturbations. The SRP force at any given time is determined by the integral of the momentum exchange from all portions of the surface lit by the Sun. Meanwhile, the observed brightness obtained at a telescope as part of a lightcurve is some component of the reflected light that contributed to the SRP force. This relationship is extremely complicated due to the strong dependency on the shape definition, integrals over the shape, and the fact that the integrals are over different regions. However, there is coupling here, so using information gained about the SRP force necessarily provides more information about the body which simultaneously generates a given SRP force and lightcurve observation.

The focus of the proposed work concerning lightcurves is to understand the fundamental information content available in lightcurves alone and when combined with estimates of solar radiation pressure, and to use this information in order to provide constraints on the characteristics of an RSO.

2. BASICS OF LIGHTCURVE INVERSION

Lightcurves are measurements of the brightness of an object over time. Lightcurves have historically been important for a number of astronomical investigations including for asteroids [5] among many applications. An example lightcurve from [5] is shown in Fig. 1.

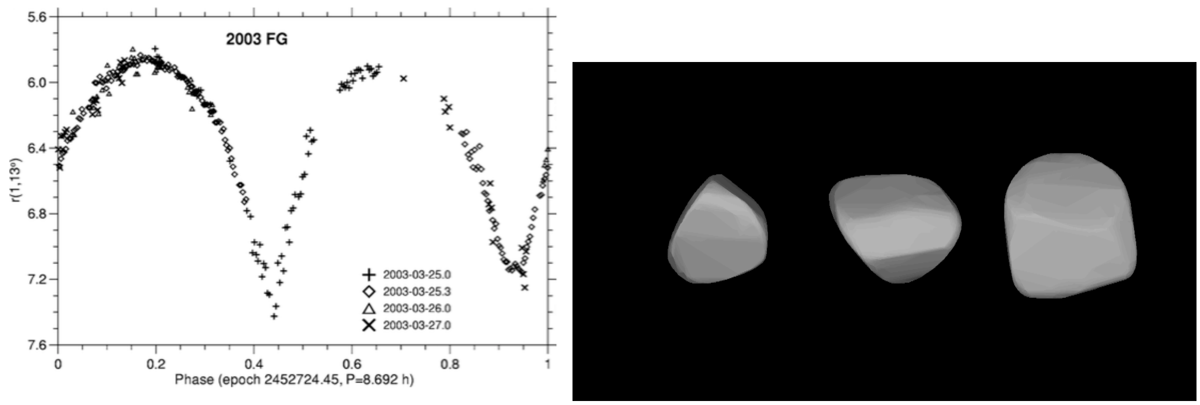

Figure 1 - Example lightcurve of asteroid 2003 FG taken over 3 nights (from [5]). On the right, an example convex hull asteroid shape of 2003 EX2 (from [6]).

The observed brightness of an object is given by

$$B = F_s(\lambda) \int_{\mathscr{B}} \int_{\phi_0} \int_{\theta_o} f_r \cos\theta_o \mathrm{d}\theta_o \mathrm{d}\phi_o \mathrm{dA} \tag{1}$$

where $F_s(\lambda)$ is the solar flux at the observation wavelength, λ, f_r is the BRDF, which can be different for different portions of the surface, and θ_o and ϕ_o define the observer direction with respect to the current portion of the surface, dA. B indicates that this integration is carried out over the entire body surface. Note that if a section of the body surface is neither illuminated (by the sun or by secondary reflections) or in line-of-sight to the observer, it doesn't contribute to B.

The BRDF describes the amount of reflected light in any given direction (θ_o, ϕ_o) when a surface is illumination from any other direction (θ_s, ϕ_s). It has the form

$$f_r(\theta_s, \phi_s; \theta_o, \phi_o; \lambda) = \frac{\mathrm{d}L_r(\theta_o, \phi_o; \lambda)}{\mathrm{d}E_i(\theta_s, \phi_s; \lambda)} \tag{2}$$

where θ_s and ϕ_s are the angles describing the sun direction, $\hat{\mathbf{u}}$, and θ_o and ϕ_o are the same but for the observer direction, $\hat{\mathbf{o}}$, $\mathrm{d}L_r$ is the reflected radiance in W m^{-2} sr^{-1}, $\mathrm{d}E_i$ is the irradiance in W m^{-2}. Generally, the BRDF is assumed to be understood for a planar section of surface of the body, so that the observer and sun angles are measured with respect to a local surface normal frame. Thus, at a given observation, although the sun, observer, and object are at a given location and attitude, these angles vary over the body with respect to different sections of the surface, and are a function of the body shape and attitude. Thus, the observed brightness is a function of four independent parameters (the sun and observer location with respect to the body) as well as a possibly infinite number of parameters which describe the shape and optical properties (BRDF) over the surface. The geometry of the lightcurve observation is illustrated in Fig. 2. As the sun and observer directions change with respect to the body (through the body's attitude motion, orbital motion, or Earth motion), a different brightness will be observed.

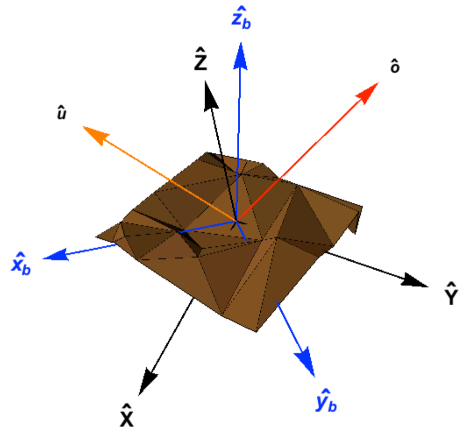

Figure 2 - Illustration of the lightcurve geometry showing the sun direction ($\hat{\mathbf{u}}$), the observer direction ($\hat{\mathbf{o}}$), the body-fixed frame (blue axes) and a reference frame from which the RSO attitude can be defined (black axes).

Eq. 1 shows that lightcurves contain information about an observed body's attitude, shape, and optical properties, although the information is convolved into a series of brightness measurements. Thus, lightcurves are a powerful tool for characterization of RSOs.

Unfortunately, estimating the shape and attitude of the RSO from lightcurves is impossible -- these features are unobservable as was rigorously proven in [1]. This is due to the presence of the integral over the body in Eq. 1 of all of these parameters and states, so that an infinite number of possible combinations could produce the same measurements. This is the same issue that is commonly faced with estimating drag in orbit determination -- the area, mass, and coefficient of drag all appear in the drag force equation together and cannot be separated, so the ballistic coefficient (the combination of these parameters) is estimated instead.

Although the inversion problem is ill-posed, and thus not uniquely solvable, people have still approached this problem and had success in characterizing different pieces of information from lightcurves by making certain assumptions. One straightforward application of lightcurves is to estimate the spin pole and spin period of an asteroid in a principal axis rotation state. Initially, the spin period can be quickly deduced from a small number of lightcurves by recognizing the periodicity in the data. This is what is illustrated in Fig. 1. It is clear that as an observed object (in this case the asteroid 2003 FG) rotates so that different parts are exposed to sunlight, as well as different parts are seen by the observer, the brightness of the object changes in a periodic manner. In this case, by stacking lightcurves from several nights on top of one another the period can be more accurately determined. Determining a constant spin pole requires several such apparitions with a varied viewing geometry (spreading $\hat{\mathbf{o}}$ around in the asteroid body-fixed frame) so that the pole direction can be constrained when the period is assumed constant over all apparitions. Carrying out this determination of the pole and spin period doesn't require any information about the shape or BRDF of the asteroid, although it is clearly a highly constrained application.

The asteroid community has taken this process a step further to attempt to estimate the shape of asteroids as well, through a process known as lightcurve inversion. The most common and successful application of this theory is for estimating the shape and spin state of asteroids, using the method developed by Kaasalainen [2, 3]. This method assumes there is a fixed spin pole and spin rate and a simple diffuse BRDF that is constant for the entire surface, and under these assumptions will estimate the spin pole, spin rate, and shape. Even with these strong assumptions, this is a difficult non-linear optimization problem, and thus the answer is generally a non-unique local minima. Initially the spin pole is solved for, but in later iterations it is left constant and only the shape is estimated. The shape returned in this case is a convex hull, which may or may not represent the actual shape depending on the level of concavities present in the true shape. An advantage of performing this inversion for asteroids is that the fixed spin pole can be assumed the same or very similar over multiple apparitions, even if they are separated by a timespan on the order of years due to the relatively slow change caused by the YORP effect.

The lightcurve inversion technique solves the following problem using an iterative least squares methodology [2, 3, 4]

$$\mathbf{L} = [A]\mathbf{g} \tag{3}$$

$$A_{ij} = S(\mu_{ij}, \mu_{0,ij}, \alpha_i)\rho_j \tag{4}$$

$$\mathbf{g}_j = e^{a_j} \tag{5}$$

where **L** contains the *i* lightcurve measurements, the *[A]* matrix represents the Lommel-Seeliger/Lambertian BRDF used, as shown in Eq. 4, and $\mathbf{g}_j$ is the exponential of the facet areas which ensures positive values in the least-squares solution. Note that this solution can not be carried out until the attitude motion is already known – typically a constant spin axis and period are found from the lightcurves before this method is used to constrain the shape. Also note that the shape is somewhat fixed to start – a fixed number of facets and a priori orientations are used to start the iterative process. Given the extensive non-linearities in the full problem, a solution close to the truth is not guaranteed.

3. SOLAR RADIATION PRESSURE

The forces due to SRP and drag depend on both the translation and attitude states,

$$\mathbf{a}_{SRP} = \mathbf{h}(\mathbf{\Theta}, \mathbf{r}, \mathscr{P}) \tag{6}$$

where $\mathscr{P}$ is a parameter vector that defines the shape and optical parameters (those parameters needed to define the BRDF(s) of the surface) of the body. The SRP force is intricately related to one another through the shape of the body so that these functions are not independent.

However, the attitude motion is typically not known for RSOs (thus the purpose of this paper), so simplified models are sought. The most commonly used attitude independent SRP model is known as the cannonball model. This model represents the SRP force as a constant magnitude force acting directly away from the Sun, and is typically expressed as

$$\mathbf{a}_{SRP} = -C_R \frac{G_1}{R^2}\frac{A}{m}\hat{\mathbf{u}} \tag{7}$$

where $\hat{\mathbf{u}}$ is the direction of the Sun from the body, G_1 is the solar radiation constant and is equal to 1e14 kg km/ s^2, and R is the distance from the Sun (in units of km), A is the nominal area of the body exposed to sunlight, m is the mass of the object, and C_R is the coefficient that scales the body reflectivity.

It is instructive to point out that this perturbation scales linearly with the area-to-mass ratio, which is why HAMR objects are more strongly affected by SRP perturbations than massive satellites. The combination of this ratio with the scale factor C_R also looks eerily similar to the ballistic coefficient commonly seen in simple drag models and has inspired people to in fact use the same value for both effects in some orbit determination applications (i.e. SGP4 used to propagate TLEs [11] Effectively, this model assumes that the object is a sphere with uniform optical properties because the area that intersects the sunlight is the same from any direction and the force is directed entirely away from the Sun, which only happens if the optical properties are identical around the surface of the sphere.

In general, the incoming radiation pressure from the Sun exerts a force (and a torque) in a direction that is not parallel to $\hat{\mathbf{u}}$. This force depends on the shape of the body and the optical properties of the body that govern how light is reflected and absorbed as it intersects the surface. The SRP force produced at any time is determined by integrating the exchange of momentum over the surface area of the body that are illuminated by the Sun, which is in direction $\hat{\mathbf{u}}$ from the body, as well as those areas that are not directly illuminated by the Sun, but receive energy as it is re-emitted and/or reflected from Sun-lit portions of the body. In order to make these computations tractable, the body of interest is sub-divided into a finite number of differential areas, termed facets, that are planar and usually triangular.

The SRP acceleration can be expressed for any location of the Sun in the body-fixed frame as in McMahon and Scheeres [12, 13]

$$\mathbf{F}_{SRP} = P(R) \sum_{n=0}^{\infty} [\mathbf{A}_n(\delta_s)\cos(n\lambda_s) + \mathbf{B}_n(\delta_s)\sin(n\lambda_s)] \tag{8}$$

by fitting the computed SRP acceleration (based on any model) as a set of Fourier series in the body-fixed frame via the equations

$$\mathbf{A}_0 = \frac{1}{2\pi}\int_0^{2\pi} \mathbf{F}_{SRP}\ \mathrm{d}\lambda_s \tag{9}$$

$$\mathbf{A}_n = \frac{1}{\pi}\int_0^{2\pi} \mathbf{F}_{SRP}\cos(n\lambda_s)\ \mathrm{d}\lambda_s \tag{10}$$

$$\mathbf{B}_n = \frac{1}{\pi}\int_0^{2\pi} \mathbf{F}_{SRP}\sin(n\lambda_s)\ \mathrm{d}\lambda_s \tag{11}$$

using a chosen BRDF model to compute an a priori for the force from the fundamental physics,

$$\mathbf{F}_{SRP} = -\frac{\Phi}{c}\int_{\mathscr{B}} (\hat{\mathbf{u}}\cdot\hat{\mathbf{n}}) \int_0^{\infty} F_s(\lambda)\left[\hat{\mathbf{u}} + \int_0^{2\pi}\int_0^{\pi/2} f_r \cos\theta_o \hat{\mathbf{o}}\mathrm{d}\theta_o\mathrm{d}\phi_o\right]\mathrm{d}\lambda\mathrm{dA} \tag{12}$$

where c is the speed of light, $\hat{\mathbf{n}}_i$ is the surface normal associated with the differential area, dA, and Φ is the shadow factor. Basically, the SRP force is produced by the momentum exchange with incoming light, as well as with reflected/re-emitted light. The important relationship here is due to the term inside the double integral over the observer position - this term is precisely the same as the term in Eq. 1, however in this case it is integrated over all "observer" locations instead of for an actual observation. This is the reflected/re-emitted component of light. Therefore, there is a direct correlation between a lightcurve observation and the SRP force at that time.

This coupling between the SRP and observed brightness is depicted in Fig. 3. The SRP force at any given time is determined by the integral of the momentum exchange from all portions of the surface lit by the Sun. Meanwhile, the observed brightness obtained at a telescope as part of a lightcurve is some component of the reflected light that contributed to the SRP force. This relationship is extremely complicated due to the strong dependency on the shape definition, integrals over the shape, and the fact that the integrals are over different regions. If one could observe the RSO from all possible observer locations at a given time, the SRP force could be accurately predicted. The inverse also provides information - if the SRP force is known, then some constraint is placed on the distribution of brightness for the various observer locations.

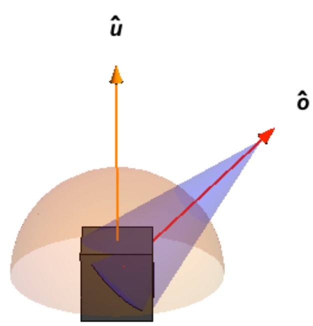

Figure 3 - Illustration of the relationship between integral bounds when determining the SRP force and the measured lightcurve brightness. The Sun is in the $\hat{\mathbf{u}}$ direction, and the SRP force depends on an integration over the orange region. The observer is in the $\hat{\mathbf{o}}$ direction, and the measured brightness is due to the light reflected in the telescope direction illustrated by the blue cone.

When discussing unresolved optical observations of RSOs can also be used to estimate parameters defining the force models, most notably here the SRP model. It has been shown in previous papers [13] that the Fourier coefficients can be observed and estimated; the degree to which this is possible depends on the data quality and how well the orbit is estimated. For example, it was shown that with sparse data (5 measurements at 30 second intervals with 2-100 hour observation gaps) the coupled HAMR trajectory illustrated in the previous section could be estimated to a 3D RMS error on the order of the measurement noise while estimating all Fourier coefficients of orders 0 and 1 [13].

4. THE BRIGHTNESS MAP

The brightness map is a concept used to encompass all of the information available about an objects shape and optical properties from unresolved measurements. The brightness map is similar in concept to the BRDF, but instead of being defined for one material and/or one facet, the brightness map represents the response of the entire body. In short, for any given RSO attitude, sun position, and observer position, the brightness map gives the observed brightness. A conceptual illustration of the brightness map of an RSO is shown in Figs. 4 - 5.

It is natural to first consider the brightness map in a reference frame where it is natural to define the sun and observer positions, such as the Earth's Hill frame (e.g. the black frame in Fig. 2), as pictured in Fig. 4. In this frame, the sun position is actually fixed, so that there are no degrees of freedom associated with it. Thus, any lightcurve taken can be placed into the map over the range of observing conditions. Unfortunately, the map is not single-valued in this frame due to the fact that the RSO's attitude has not been accounted for – there are 3 degrees of freedom associated with the RSO's attitude, but there is one constraint so that in total there are 4 degrees of freedom to map to each brightness value. By matching different locations at which the same brightness is seen, and enforcing continuous attitude dynamics, a model for the attitude motion can be gleaned once enough measurements have been taken (see Seciton 5). This is what lightcurve inversion does for the simple attitude motion discussed above.

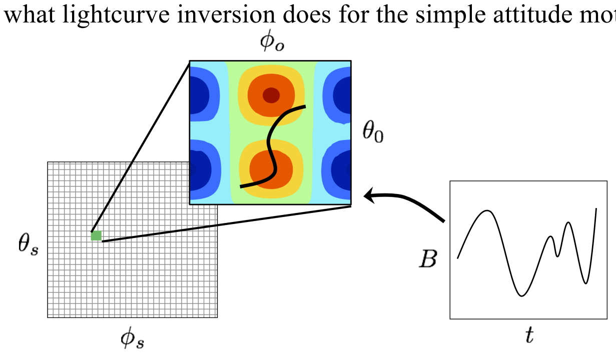

Figure 4 - Notional illustration of an RSO's brightness map. A lightcurve is mapped into the 4-dimensional space defined by the sun and observer positions. In this case, the sun position is constant during this lightcurve, but the observer position varies over time. The color in the brightness map indicates the varying brightness due to the RSO shape and material properties captured in the BRDFs.

Once the attitude motion has been estimated, the map can be converted to the RSO body-fixed frame, as seen in Fig. 5. Now the Sun position is no longer fixed, but the attitude motion has been removed. This is the fundamental form of the brightness map as the brightness information is now tied to the body. This becomes an interesting map in that it maps from a bi-spherical domain (the unit vectors defining the sun and observer positions, which are independent) onto a scalar, $S^2 \times S^2 \rightarrow R$. Constructing this map is key to characterizing the RSO shape.

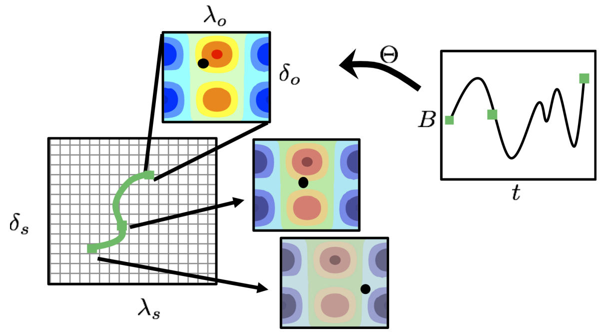

Figure 5 – The body-fixed brightness map representation.

5. FITTING THE BRIGHTNESS MAP

The brightness map can most directly be filled out by viewing the RSO at every possible sun and observer postions, which is generally impractical. Thus we desire a way to determine a representation of the brightness map with less than full information. Fundamentally, we seek a way to represent the brightness curves analytically, similar to how gravity fields are represented with spherical harmonics. In this case, a 4-dimensional spherical harmonic basis is not correct, however, because the sun and observer positions are independent and thus do not reside on a 4-dimensional hypersphere surface. An appropriate basis set is being investigated.

Given some basis set, we still require sufficient information to fit the parameters of the basis functions over the 4-dimensional domain. While this can theoretically be done via many lightcurves, this could take a long time. Thus, we propose to use constraints from the Fourier SRP model, which are functionally related to the brightness, to improve this fitting process. Basically, the SRP model can provide a set of constraints on the brightness map of the form

$$\sum_{i \in \mathscr{B}} B_i = C|\mathbf{F}_{SRP}| \tag{13}$$

which represents the fact that the SRP force is produced by the scaled integral of the reflected light over the body. Using this constraint, it should be possible to estimate the parameterized brightness map faster than with lightcurves alone.

6. SHAPE ESTIMATION

Once a brightness map is obtained, the task of characterization actually takes place. Producing the 4-dimensional brightness map is a constrained estimation problem, however characterization is not estimation per se - it is a hypothesis testing problem. As previously discussed, even if the brightness map is known exactly, there are an infinite number of shape and surface property combinations that can produce that brightness map.

Some methods use this by positing a number of hypotheses about possible shapes, BRDFs and qualitative spin rates 8, 9, 10], and picks the best fit from the available library in a multiple hypothesis method. Other methods have attempted to numerically identify the attitude behavior of an RSO through the use of particle filters [7].

The process we propose is pictured in Fig. 6. The typical lightcurve inversion process follows the light colored boxes; in this case we propos to add the constraints from estimation of the SRP force to create an augmented brightness map. From this, a hypothesis testing method can be used to gain insight into constraints on the shape and optical properties of the body.

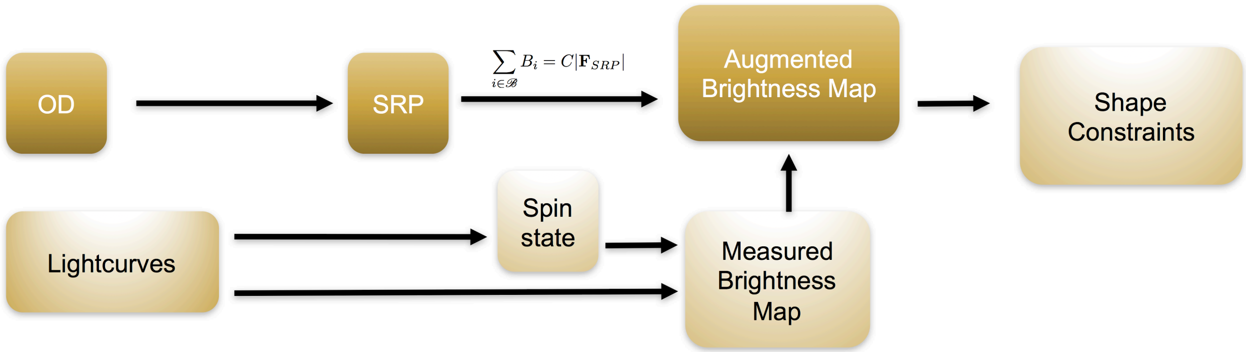

Figure 6 – Schematic of proposed process

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