Reconstruction of the 1801 Discovery Orbit of Ceres via Contemporary Angles-Only Algorithms

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ABSTRACT

This paper employs contemporary angles-only methods of initial orbit determination (IOD) and differential correction (DC) to revisit how Gauss solved for the orbit of Ceres using observations taken more than two centuries ago. The solutions obtained with these angles-only algorithms are verified by accurate numerically-integrated ephemerides and published U.S. Naval Observatory (USNO) values. These angles-only IOD and DC algorithms are fully applicable to today's space situational awareness for both geocentric and heliocentric objects.

1. INTRODUCTION

Giuseppe Piazzi's observations of the asteroid Ceres during 1801 were both the stimulus and a test case for Gauss's classic orbit determination method using three angles-only observations. While Gauss's method uses just three of Piazzi's observations during January-February 1801, there were actually 19 complete angles-only observations available. But to use all 19 observations would have been prohibitively difficult to accomplish with pen, ink, paper, and logarithm tables back in 1801, even if all of the necessary computational procedures had already been developed.

So this paper presents contemporary solutions that input all 19 complete observations to modern computer programs, and it assesses the quality of the observations and the orbital solutions that could have been obtained with them if contemporary algorithms and computers had been available in 1801. The contemporary solutions rely upon four computer algorithms developed by the two authors: HGM, Der IOD, ORBIT2, and Batch UPM DC, as further described below.

It is important to note that, while this paper solves for the discovery orbit of Ceres using observations taken more than two centuries ago, the algorithms applied herein were developed for use in today's cislunar space situational awareness and interplanetary space probe trajectory modeling.

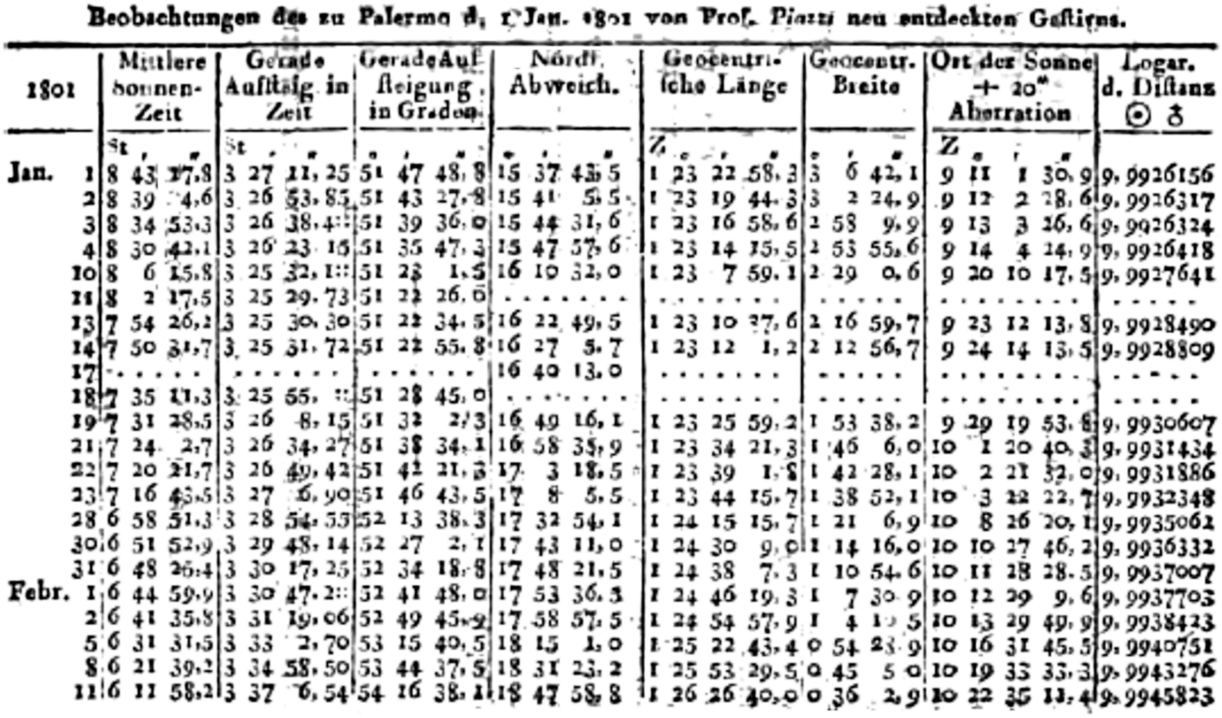
2. PIAZZI'S OBSERVATIONS AND METHODS

Piazzi's observations of Ceres during 1801 January 1 - February 11 are shown in Fig. 1. This table also appears in Fig. 3 in Serio, Manara, and Sicoli [1].

It is a fundamental assumption of this paper that Piazzi's observations were carefully made and usable without further modification. That is, they accurately represent what Piazzi observed relative to his position on Earth's surface.

Piazzi's observations of time, right ascension, and declination were made using a meridian circle. A meridian circle is an astronomical instrument that permits the observer to measure a celestial body's right ascension and declination at meridian transit, i.e., the time at which the body crosses the celestial meridian of the observer (the great semicircle from north celestial pole to south celestial pole, through the observer, that moves with the observer as Earth rotates).

Conceptually, the meridian circle is a circle that lies within the celestial half-plane that includes the celestial meridian. (See Smart [2] for the theory and operation of meridian circle instruments. See [1, p. 18, Fig. 2] for a photograph of the Palermo Circle that was used by Piazzi.)



Beobachtungen des zu Palermo d. 1. Jan. 1801 von Prof. Piazzi neu entdeckten Gestirns.

1801	Mittlere Sonnen-Zeit	Gerade Aufsteig. in Zeit	Gerade Aufsteigung in Graden	Nördl. Abweich.	Geocentrische Länge	Geocentr. Breite	Ort der Sonne + 20" Aberration	Logar. d. Distanz ☉ ♁
	St ′ ″	St ′ ″	° ′ ″	° ′ ″	Z ° ′ ″	° ′ ″	Z ° ′ ″	
Jan. 1	8 43 27,8	3 27 11,25	51 47 48,8	15 37 43,5	1 23 22 58,3	3 6 42,1	9 11 1 30,9	9,9926156
2	8 39 4,6	3 26 53,85	51 43 27,8	15 41 5,5	1 23 19 44,3	3 2 24,9	9 12 2 28,6	9,9926317
3	8 34 53,3	3 26 38,4::	51 39 36,0	15 44 31,6	1 23 16 58,6	2 58 9,9	9 13 3 26,6	9,9926324
4	8 30 42,1	3 26 23,15	51 35 47,3	15 47 57,6	1 23 14 15,5	2 53 55,6	9 14 4 24,9	9,9926418
10	8 6 15,8	3 25 32,1::	51 23 1,5	16 10 32,0	1 23 7 59,1	2 29 0,6	9 20 10 17,5	9,9927641
11	8 2 17,5	3 25 29,73	51 22 26,6	. . . .	. . . .	. . . . .	. . . .	. . . . .
13	7 54 26,2	3 25 30,30	51 22 34,5	16 22 49,5	1 23 10 27,6	2 16 59,7	9 23 12 13,8	9,9928490
14	7 50 31,7	3 25 31,72	51 22 55,8	16 27 5,7	1 23 12 1,2	2 12 56,7	9 24 14 13,5	9,9928809
17	. . . . . .	. . . .	. . . .	16 40 13,0	. . . .	. . . . .	. . . .	. . . . .
18	7 35 11,3	3 25 55,::	51 28 45,0	. . . .	. . . .	. . . . .	. . . .	. . . . .
19	7 31 28,5	3 26 8,15	51 32 2,3	16 49 16,1	1 23 25 59,2	1 53 38,2	9 29 19 53,8	9,9930607
21	7 24 2,7	3 26 34,27	51 38 34,1	16 58 35,9	1 23 34 21,3	1 46 6,0	10 1 20 40,3	9,9931434
22	7 20 21,7	3 26 49,42	51 42 21,3	17 3 18,5	1 23 39 1,8	1 42 28,1	10 2 21 32,0	9,9931886
23	7 16 43,5	3 27 6,90	51 46 43,5	17 8 5,5	1 23 44 15,7	1 38 52,1	10 3 22 22,7	9,9932348
28	6 58 51,3	3 28 54,55	52 13 38,3	17 32 54,1	1 24 15 15,7	1 21 6,9	10 8 26 20,1	9,9935062
30	6 51 52,9	3 29 48,14	52 27 2,1	17 43 11,0	1 24 30 9,0	1 14 16,0	10 10 27 46,2	9,9936332
31	6 48 26,4	3 30 17,25	52 34 18,8	17 48 21,5	1 24 38 7,3	1 10 54,6	10 11 28 28,5	9,9937007
Febr. 1	6 44 59,9	3 30 47,2::	52 41 48,0	17 53 36,3	1 24 46 19,3	1 7 30,9	10 12 29 9,6	9,9937703
2	6 41 35,8	3 31 19,06	52 49 45,9	17 58 57,5	1 24 54 57,9	1 4 10,5	10 13 29 49,9	9,9938423
5	6 31 31,5	3 33 2,70	53 15 40,5	18 15 1,0	1 25 22 43,4	0 54 23,9	10 16 31 45,5	9,9940751
8	6 21 39,2	3 34 58,50	53 44 37,5	18 31 23,2	1 25 53 29,5	0 45 5,0	10 19 33 33,3	9,9943276
11	6 11 58,2	3 37 6,54	54 16 38,1	18 47 58,8	1 26 26 40,0	0 36 2,9	10 22 35 11,4	9,9945823

Fig. 1. Piazzi's Discovery Observations of Ceres. These observations were first published by F. X. von Zach in *Monatliche Correspondenz* [3]. The English translation of the caption in the figure is, "Observations of the newly discovered star by Prof. Piazzi at Palermo on 1 January 1801." The column headings in the figure are "1801," "Mean Solar Time," "Right Ascension in Hours, Minutes, and Seconds of Time," "Right Ascension in Degrees, Minutes, and Seconds of Arc," "Northern Declination," "Geocentric Longitude," "Geocentric Latitude," "Position of the Sun + 20 Seconds Aberration," "Logarithm of the Distance, Sun to Earth." As can be seen from the figure, only 19 observations are complete in the sense that they have all three of time, right ascension, and declination.

The key facts to remember about the Palermo Circle as a meridian circle instrument, as regards reduction of Piazzi's observations to heliocentric ecliptic orbital elements, are these:

(a) the measured right ascension of the celestial object is precisely the local apparent sidereal time,
(b) it was customary in Piazzi's time to record or calculate the local mean solar time (LMT) at each measurement of right ascension and declination, and
(c) it was also customary to reckon LMT from local noon rather than the day's beginning (zero hours LMT).

So by (c), 12 hours must be added to each of Piazzi's observation times to reckon time from zero hours LMT, i.e., from the local midnight immediately preceding the LMT of the observation. (This last fact will be reflected in the LMTs provided in Table 1 at the end of the next section.)

3. HGM, DER IOD, ORBIT2, AND BATCH UPM DC ALGORITHMS

HGM is an Initial Orbit Determination (IOD) method documented for geocentric orbits in [4] and [5]. "HGM" is shorthand for "Herget/UPM," where UPM stands for "Uniform Path Mechanics," a universal variables method of orbit propagation based upon the formulations of Karl J. Stumpff and William H. Goodyear. Given n observations, where n >= 3, HGM holds the line-of-sight directions of the first and last observations fixed. Given estimates of the ranges of the first and last observations, ρ_1 and ρ_n, HGM fits these two parameters, in the least squares sense, to the P and Q residuals of the remaining n-2 observations. The P and Q residuals formulas are

$$P = \rho \cos(\delta)\, \Delta\alpha \quad \text{and} \quad Q = \rho\, \Delta\delta .$$

Here α is right ascension, δ is declination, and Δ denotes the residual in the sense "observed minus computed."

The fit is nonlinear, i.e., ρ_1 and ρ_n must be iterated. This is the essence of the non-linear, least-squares IOD method as published by Paul Herget [6] in the *Astronomical Journal* in February 1965. HGM improves upon the original Herget method by (a) adopting the c-function-based, universal-variables formulas of Goodyear [7], as published in the *Astronomical Journal* in April 1965, and by (b) reformulating Gauss's hypergeometric X-function (needed in the Lambert solver) as a quotient of c-functions. The end result is closed-form (no infinite series in the fast variable), uniform formulas (no branching on the cases eccentricity < 1, eccentricity = 1, or eccentricity > 1).

HGM uses all of the available observations, subject to a central angle (geocentric or heliocentric) limit of about 1/3 orbit. So HGM is a so-called "short-arc" IOD method. When HGM is used for space situational awareness relative to a deep-space UCT (Uncorrelated Target) of high interest, it can determine the orbit quickly, as soon as three suitably-spaced observations become available. Then, when more than three observations become available, HGM can continue to process all of the available observations without having to pick just three. This can be, but is not necessarily an operational advantage.

Since HGM uses all of the available observations, its interface is similar to that of the follow-up differential correction (DC) that it precedes and "seeds" (provides the starting estimate of orbital state for the DC). HGM uses USNO precession and nutation matrices as documented in [13] and [16].

Der IOD is the name herein associated with Der's angles-only method of initial orbit determination, as documented in [9] and [11]. Although Der IOD is classical, like Gauss's method, in the sense that it uses three and only three observations, it is a "range solving" method. This is in distinction to HGM, which is a "range guessing" method. That is, Der IOD solves for the range at the middle (second) observation without any a priori knowledge, whereas HGM must make initial estimates of the first and last ranges, ρ_1 and ρ_n . (Starting HGM range estimates for geocentric motion are typically 1 Earth radius. Starting HGM ranges for heliocentric motion are typically one astronomical unit, 1AU.)

Because (a) Der IOD is a range-solving IOD method, together with the facts that (b) Der IOD can handle central angle spreads from a few degrees up to 360 degrees, and (c) Der IOD is generally faster than HGM, we assert that, in the final analysis, Der IOD is the more general, more robust, and faster IOD method.

ORBIT2 numerically integrates the equations of motion of the major solar system planets and a user-specified additional body called "Spare." Spare is, in this case, the asteroid Ceres. ORBIT2 is further documented in [10] and applied in [11].

The initial state vectors of the major planets were taken from the Jet Propulsion Laboratory's DE405 ephemerides and are therefore referred to the mean equator and equinox of J2000.0 (Julian date 2451545.0). The initial state vector for Ceres was obtained from Hilton [14] for an epoch in 1997 and referred to the mean equator and equinox of J2000.0.

Thus ORBIT2 was used to numerically integrate the state vectors of the major planets and Ceres back to the dates of the Piazzi observations during 1801 January 1 - February 11, but in the J2000.0 reference frame. The resulting J2000 state vectors therefore had to be transformed to Mean-of-Date (MOD) by precession and to True-of-Date (TOD) by nutation ("Date" being here the date of the observation in 1801).The precession and nutation matrices were taken from Montenbruck and Gill [15].

Batch UPM DC (the heliocentric version will be referred to as "HDC") is documented for artificial Earth satellites in [5], [8], [12], and [13]. Given n observations consisting of two measurements each (topocentric right ascension and declination), the basic vector-matrix equation of batch least squares estimation is

$$\mathbf{X}o' = \mathbf{X}o + (\mathbf{A}^\mathbf{T}\mathbf{W}\mathbf{A})^{-1}\, \mathbf{A}^\mathbf{T}\mathbf{W}\, [\mathbf{Y} - \mathbf{F}(\mathbf{X}o)]$$

where $\mathbf{X}o'$ is the updated 6-by-1 state estimate (position and velocity at epoch), $\mathbf{X}o$ is the previous or initial state estimate to be iterated, and $\mathbf{Y}$ is the measurements vector with N = 2n components consisting of the topocentric right ascension and declination measurement quantities

$$\cos(\delta)\, \Delta\alpha \quad \text{and} \quad \Delta\delta\ .$$

A is the N-by-6 matrix of partials of the components of **F**(**X**) with respect to the components of **X**o and **W** is the N-by-N diagonal weight matrix consisting of the reciprocals of the variances in the measurements. (For the purposes of this application, **W** is the identity matrix.) It is seen then that [**Y** - **F**(**X**o)] is the residuals vector for the current iteration.

The modeling for a heliocentric trajectory, vs. a geocentric trajectory, must additionally include a model of Earth's motion around the Sun and a way to transform geocentric coordinates to/from heliocentric coordinates. Fig. 2 illustrates the key vectorial concepts.

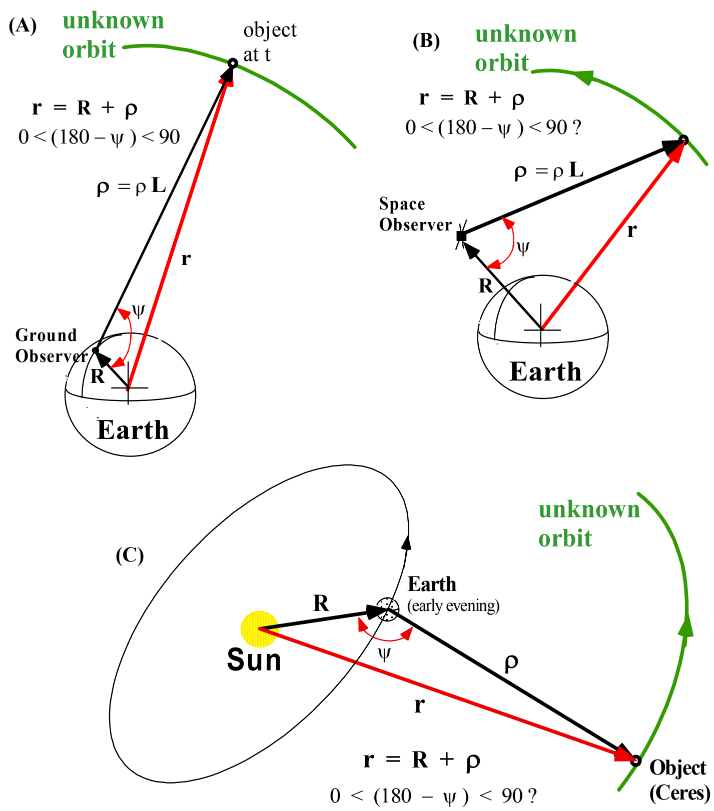


Fig. 2. Vector Geometry for Geocentric (A, B) and Heliocentric (C) Orbit Determination. The dynamical relationships among Geocenter-EarthSatellite-Observer and Heliocenter-CelestialObject-Observer can be reduced to simple vector triangles.

The necessary models and transformations of heliocentric motion are illustrated and employed in [13]. That is, [13] completely specifies, in the mathematical notations just described, a batch UPM DC of the heliocentric state vector of the asteroid Ceres, starting with (a) a preliminary HGM orbital state estimate with epoch on 1801 January 1, (b) Piazzi's observations, and (c) Palermo's geographical location.

Table 1 shows the 19 actual Piazzi observations treated in this paper and in [13], along with the ORBIT2-computed Piazzi observations. The full precision of the ORBIT2-generated angles was retained, rather than to convert to degrees, minutes, and seconds, in order to provide traceability back to the ORBIT2 output.

(We should state here again, for emphasis, that ORBIT2 was developed for contemporary cislunar and interplanetary space probe trajectory modeling applications. It was never intended to integrate for centuries or more into the past or future. We do realize that numerical and modeling errors have inevitably accumulated to a measurable degree over such a long span of integration as, in this case, 196 years.)

Table 1. 19 Actual Piazzi vs. 19 ORBIT2-Computed Piazzi Observations

Gregorian Date	Actual Piazzi Observations		ORBIT2-Computed Piazzi Observations	
	Right Ascension	Declination	Right Ascension	Declination
year mo da hh mm ss.s	deg mn ss.s	deg mn ss.s	degrees	degrees
1801 01 01 20 43 17.8	051 47 48.8	+15 37 43.5	50.905648004007	15.241303823571
1801 01 02 20 39 04.6	051 43 27.8	+15 41 05.5	50.836834177521	15.297177162270
1801 01 03 20 34 53.3	051 39 36.0	+15 44 31.6	50.774895406006	15.354369796511
1801 01 04 20 30 42.1	051 35 47.3	+15 47 57.6	50.719846776442	15.412868536518
1801 01 10 20 06 15.8	051 23 01.5	+16 10 32.0	50.534774974763	15.790688009299
1801 01 13 19 54 26.2	051 22 34.5	+16 22 49.5	50.535641801957	15.996379645443
1801 01 14 19 50 31.7	051 22 55.8	+16 27 05.7	50.549656156521	16.067338052830
1801 01 19 19 31 28.5	051 32 02.3	+16 49 16.1	50.720976455445	16.439021475898
1801 01 21 19 24 02.7	051 38 34.1	+16 58 35.9	50.835948660084	16.595112613678
1801 01 22 19 20 21.7	051 42 21.3	+17 03 18.5	50.903177509577	16.674650913710
1801 01 23 19 16 43.5	051 46 43.5	+17 08 05.5	50.976819801394	16.755152556106
1801 01 28 18 58 51.3	052 13 38.3	+17 32 54.1	51.438831321195	17.171163084143
1801 01 30 18 51 52.9	052 27 02.1	+17 43 11.0	51.666098499373	17.343351650463
1801 01 31 18 48 26.4	052 34 18.8	+17 48 21.5	51.788556743156	17.430572641938
1801 02 01 18 44 59.9	052 41 48.0	+17 53 36.5	51.916811048065	17.518506217319
1801 02 02 18 41 35.8	052 49 45.9	+17 58 57.5	52.050805986090	17.607128616391
1801 02 05 18 31 31.5	053 15 40.5	+18 15 01.0	52.486678087124	17.876881808883
1801 02 08 18 21 39.2	053 44 37.5	+18 31 23.2	52.972258828131	18.152005392992
1801 02 11 18 11 58.2	054 16 38.1	+18 47 58.8	53.505940477799	18.431881790325

4. HGM, BATCH UPM DC, AND DER IOD SOLUTIONS WITH ORBIT2-COMPUTED OBSERVATIONS

Table 2 shows the HGM and Batch UPM DC (HDC) solutions for all 19 ORBIT2-computed Piazzi observations, and the Der IOD solution for three ORBIT2-computed Piazzi observations. (The three ORBIT2-computed Piazzi observations correspond to those Piazzi made on 1801 January 2, January 22, and February 11.)

Table 2. HGM, Batch UPM DC, and Der IOD Solutions with ORBIT2-Computed Piazzi Observations

Orbital Element/Quantity	HGM with 19 ORBIT2- Computed Observations	HDC with 19 ORBIT2-Computed Observations	Der IOD with 3 ORBIT2-Computed Observations
Semimajor axis, AU	2.76752797	2.76813708	2.76896376
Eccentricity	0.07876754	0.07890051	0.0787479
Inclination, deg	10.57956185	10.58067	10.584
Long. of Asc. Node, deg	80.97327171	80.97143	80.819
Arg. of Perihelion, deg	70.08341586	69.99616	69.986
Mean Anomaly, deg	285.7934066	285.8938371	286.247
Mean Arg. of Latitude, deg	355.8768225	355.889997	356.233
Mean Daily Motion, deg/day	0.214075046	0.21400439	0.21390856
RMS of residuals, arc-seconds	0.237	0.195	0.0 (3 perfect obs)

All three solutions show remarkably good agreement, in view of the fact that the HGM and HDC solutions are based upon analytical models, whereas the Der IOD solution is a two-body plus third-body (Jupiter) solution for three almost perfect, angles-only, computed observations derived from highly precise numerical integration. The differences that do exist are explained as follows.

(a) HGM uses an analytical model of the Sun's motion in the ECI equatorial, J2000 reference frame along with precession and nutation models, all as obtained from the USNO's annual *Astronomical Almanac* [16] or the Explanatory Supplement [17], whereas

(b) ORBIT2 uses more accurate models, i.e., highly precise numerical integration of the motions of all of the solar system bodies of interest, also in the ECI equatorial, J2000 reference frame, and with precession and nutation as obtained from the equations in [15, Section 5.3].

(c) The HDC solution is more accurate than the HGM solution because the HGM solution is a two-parameter fit (HGM fits the range estimates for the first and last observations), whereas the HDC orbit is a two-body, six-parameter fit (HDC fits the 6-by-1 state vector at epoch).

Figs. 3 and 4 plot the final-iteration residuals for the HGM and HDC solutions using 19 ORBIT2-computed Piazzi observations. Note that for HGM there are only 17 P and Q residuals, since the directions of the first and last observations are held fixed.

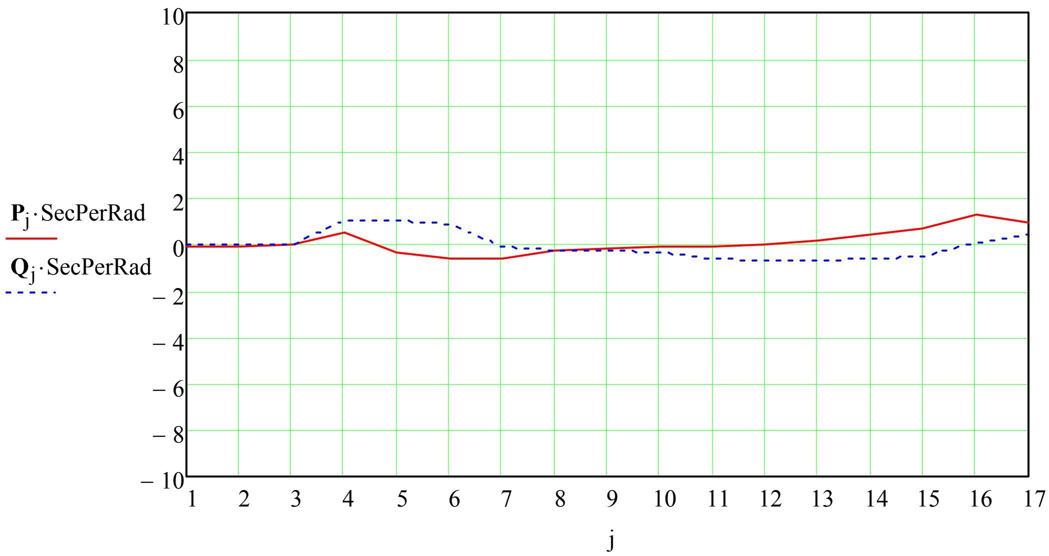


Fig. 3. Plot of HGM P and Q Residuals for n-2=17 "Middle" ORBIT2-computed Piazzi Observations. There are n-2 = 17 **P** and **Q** residuals for n =19 ORBIT2-computed Piazzi observations because HGM holds the topocentric directions of the first and last observations fixed and iterates on the first and last observations' range estimates. (Red colors P residuals plot and blue colors Q residuals plot. Vertical axis units are arc-seconds.)

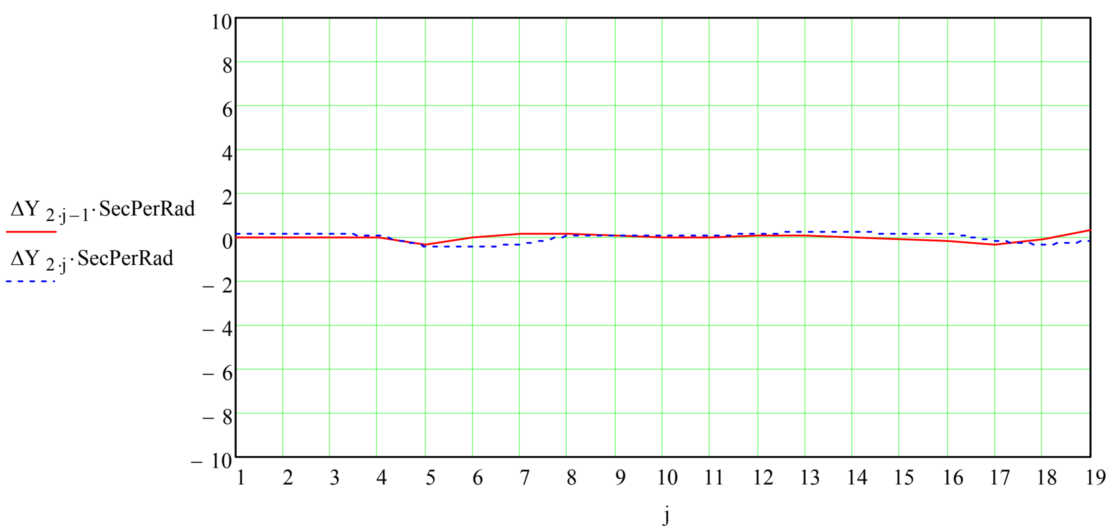


Fig. 4. Plot of HDC RA and DEC Residuals vs. Observation Number for n = 19 ORBIT2-computed Piazzi Observations. (Red colors right ascension residuals plot and blue colors declination residuals plot. Vertical axis units are arc-seconds.)

These figures show, again, that both the HGM and HDC solutions, based upon approximate analytical models, agree very well with the highly-precise ORBIT2 numerically-integrated solution.

5. HGM AND HDC SOLUTIONS USING 19 ACTUAL PIAZZI OBSERVATIONS

HGM and HDC were now run with the 19 actual Piazzi observations. As will be shown, when the solutions are compared with the ORBIT2 orbit, there are noticeable differences.

Piazzi was a careful observer, but accurate, standard reference star catalogs were not in place in the early 1800s. Indeed, Piazzi himself was in the process of constructing more accurate star catalogs when he discovered Ceres, and it was not until the mid-1800s that star catalogs could be standardized.

HGM and HDC were also run with the "17 Best Actual Piazzi Observations," as shown in Table 3. How this came about will be explained in the narrative and figures that follow the table.

Table 3. HGM and HDC Solutions with all 19 and Best 17 Actual Piazzi Observations

Orbital Element/ Quantity	HGM with all 19 Piazzi Observations	HDC with all 19 Piazzi Observations	HGM with 17 Best Piazzi Observations	HDC with 17 Best Piazzi Observations
Semimajor axis, AU	2.76515688	2.78369045	2.756729	2.77185262
Eccentricity	0.08409732	0.09175257	0.080789	0.08716516
Inclination, deg	10.61067	10.61467	10.60992	10.61659
Long. of Asc. Node, deg	81.028387	81.01542	81.037908	81.02084
Arg. of Perihelion, deg	66.337427	64.57815	67.635910	65.71636
Mean Anomaly, deg	290.7618546	293.263317	289.15625	291.6910488
Mean Arg. of Latitude, deg	357.099282	357.841467	356.79216	357.407409
Mean Daily Motion, deg/day	0.214350455	0.21221333	0.2155687	0.21357424
RMS of residuals, arc-seconds	5.117	3.599	3.347	2.155

It can be seen in Table 3 that the RMS of residuals trends downward from column to column, as expected. Compare with final column in Table 2, which is the ORBIT2 numerical solution toward which they all trend.

Figure 5 plots the residuals for the HGM run with all 19 actual Piazzi observations. Analysis of the results showed that there were two bad observations. But since the HGM run was a preliminary IOD, and the two bad observations did not prevent convergence, they were left in the HDC run with all 19 actual Piazzi observations. The HDC results, plotted in Figure 6, confirm that the same two observations were spoiling the fit.

So HDC was run again with these two bad observations omitted from the input stream. Figure 7 plots the results of this final HDC run with the "17 best actual Piazzi observations."

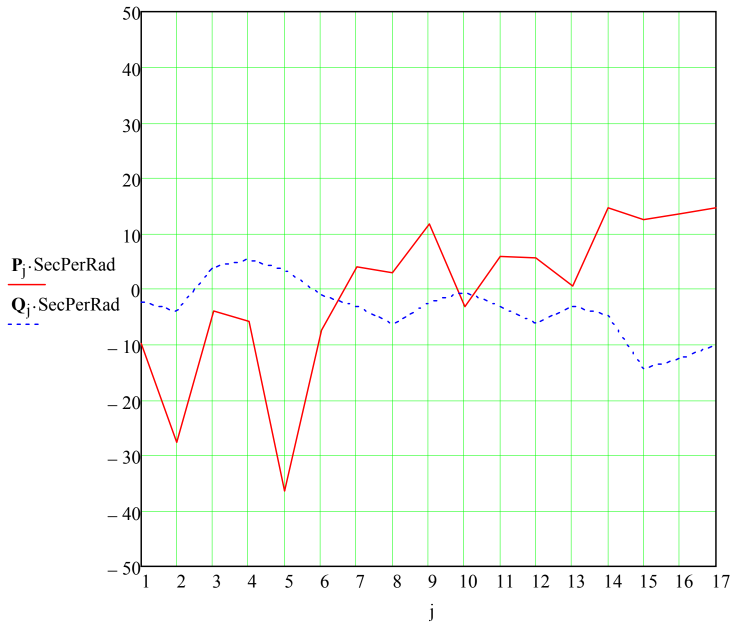


Fig. 5. Plot of HGM P and Q Residuals for n-2=17 Actual Piazzi Observations. Note that observations 2 and 5 in the set of n-2 = 17 observations are observations 3 and 6 in the complete set of n =19 observations, and that they spoil the fit. But since HGM is preliminary to and seeds the full Batch UPM DC, we leave them in. (Red colors right ascension residuals plot and blue colors declination residuals plot. Vertical axis units are arc-seconds.)

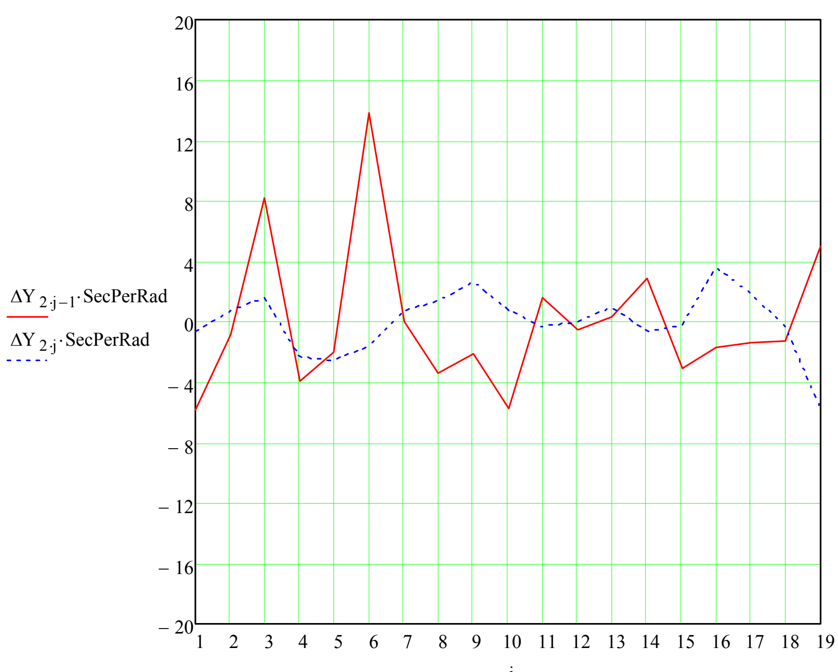


Fig. 6. Plot of HDC RA and DEC Residuals for n=19 Actual Piazzi Observations. Note that observations 3 and 6 spoil the fit. (Red colors right ascension residuals plot and blue colors declination residuals plot. Vertical axis units are arc-seconds.)

The HDC residuals plot suggests that observations 3 and 6 have right ascension residuals much larger than those for the other 17 observations. So HDC was rerun with the 17 "best" actual Piazzi observations (19 original observations minus observations 3 and 6). The results are shown in Fig. 7.

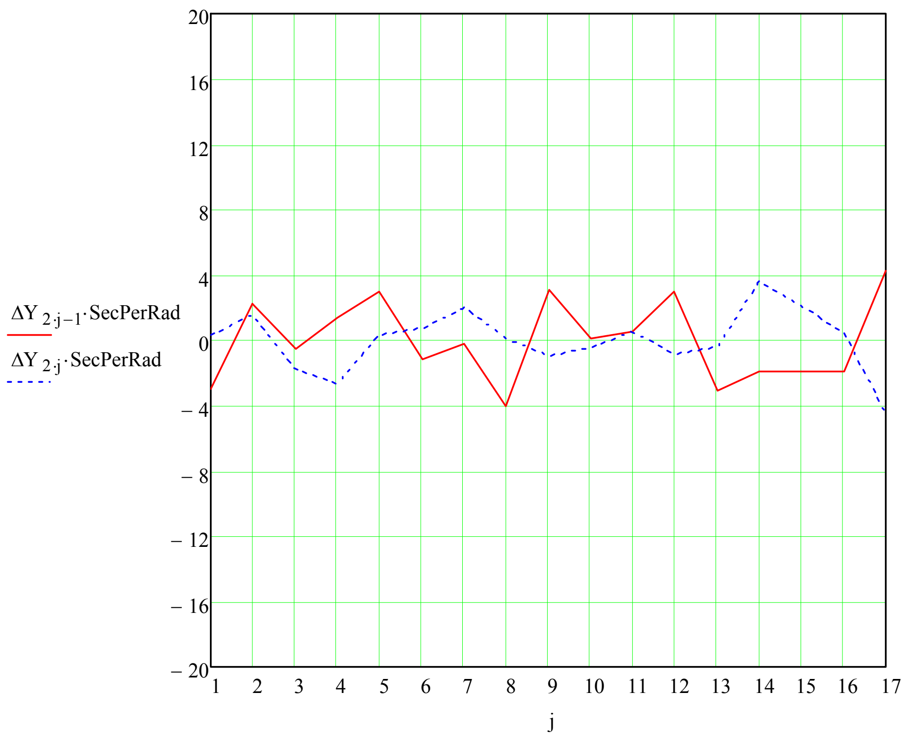


Fig. 7. Plot of HDC RA and DEC residuals for n=17 Best Actual Piazzi Observations. The best 17 actual Piazzi observations are the 17 observations that remained from the original set of 19 observations after observations 3 and 6 were deleted. (Red colors right ascension residuals plot and blue colors declination residuals plot. Vertical axis units are arc-seconds.)

6. CERES EPHEMERIDES AT 30-DAY INTERVALS FROM 1801 JAN 1 TO 1802 MAR 27

The HDC-predicted right ascensions and declinations of Ceres were plotted at equal 30-day intervals in Fig. 8 using Software Bisque's *TheSkyX Professional* [18] (this same program was used to generate Figs. 8, 9, and 10). The HDC predictions here were based upon the osculating two-body orbit that resulted when the 17 best Piazzi observations were input to HDC.

The ORBIT2-predicted right ascensions and declinations of Ceres were also plotted at equal 30-day intervals in Fig. 9, together with the HDC-predicted right ascensions and declinations, for comparison. The ORBIT2 predictions are based upon the perturbed Ceres orbit, vs. the osculating HDC orbit at the 1801 January 1 epoch.

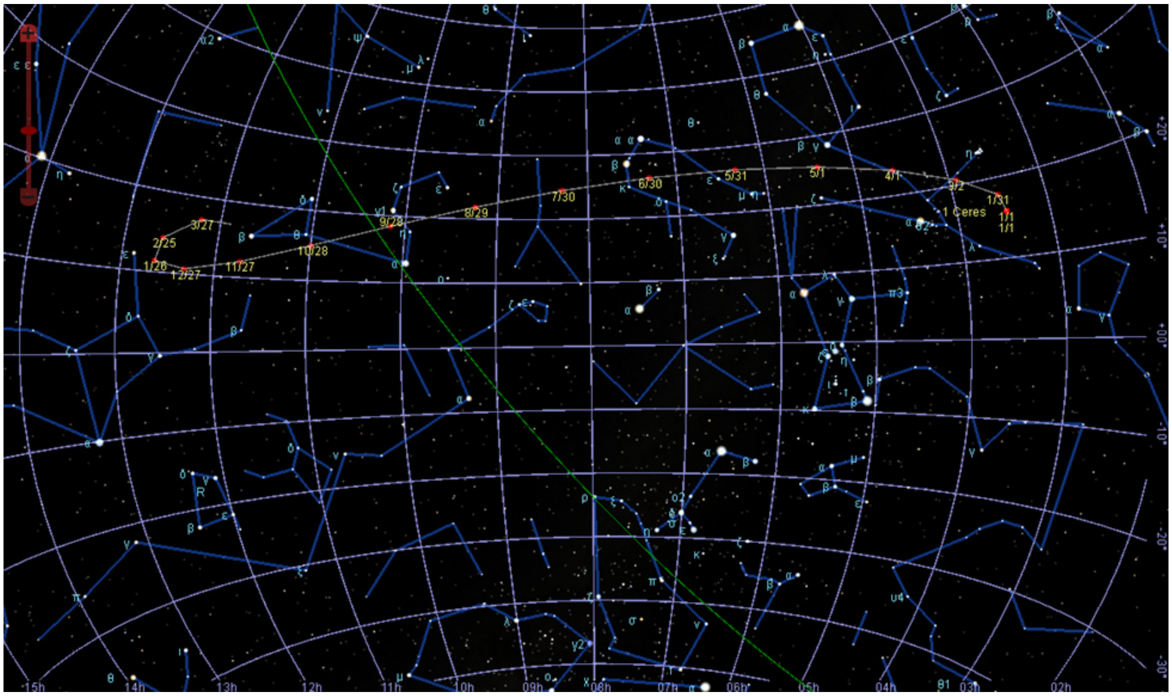


Fig. 8. Path of Ceres from 1801 January 1 to 1802 March 27. Predicted positions at equal 30-day intervals as obtained via Batch UPM DC solution with 17 best Piazzi observations. Sky chart is azimuthal equidistant projection and was generated via *TheSkyX Professional* for an observer at Palermo, Italy. Green arc depicts horizon at Palermo on 1801 January 1 at 8:43 p.m. local mean time. The bright zodiacal star *Regulus,* in constellation Leo, has just risen above eastern horizon.

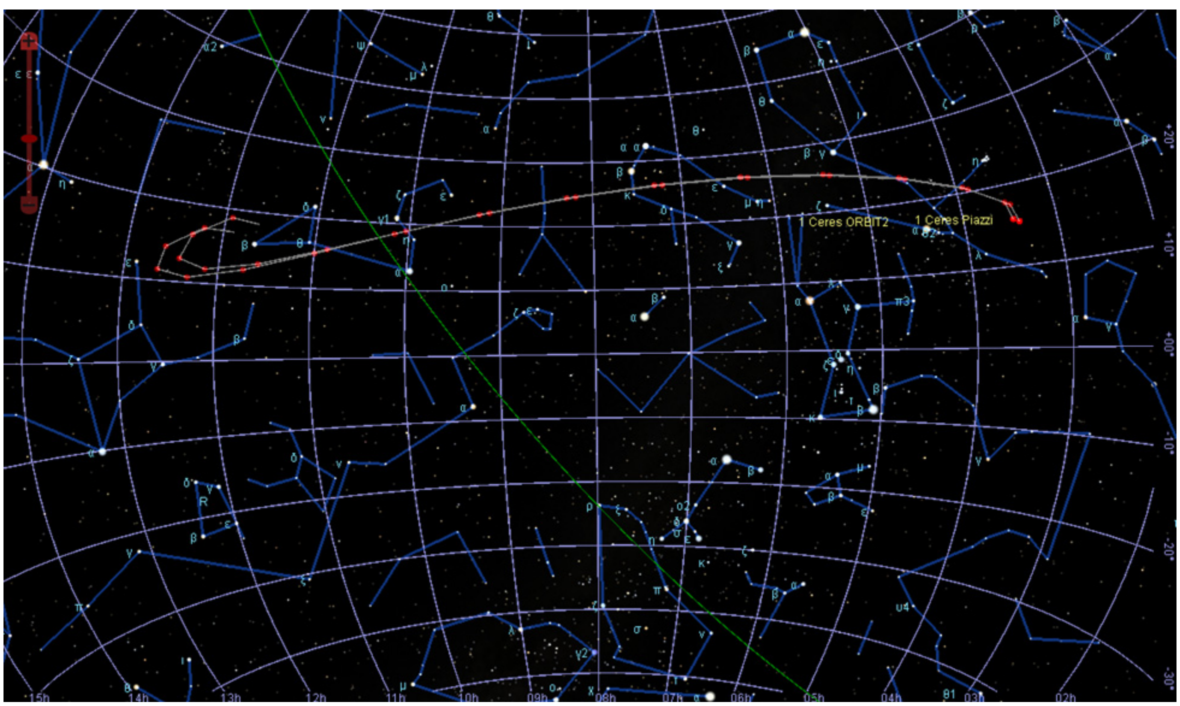


Fig. 9. Path of Ceres, "17 Best Piazzi Obs" DC Solution vs. ORBIT2 Solution. Previous figure displayed path of Ceres for 17-Best-Piazzi-Obs DC solution alone. This figure adds ORBIT2-computed path for comparison. The two sky traces are virtually identical until Ceres moves into Leo. But the 17-Best-Piazzi-Obs positions are seen to lead the ORBIT2 positions. (The month/day labels were suppressed in this figure for the sake of visual clarity.)

Fig. 8 depicts likely the best solution that could have been obtained from the 17 best Piazzi observations based upon contemporary models. We did not have available to us, at the time that we first plotted Figs. 8 and 9, the Ceres search ephemeris that Gauss provided to von Zach, which von Zach used to recovered Ceres on 1801 December 31. So we relied upon the ORBIT2 ephemeris as a second, independent indication of where Ceres actually was.

Fig. 9 additionally depicts, for comparison, the geocentric path that Ceres was determined to have taken from 1801 January 1 to 1802 March 27, based upon the ephemeris generated by ORBIT2, starting with the Ceres state vector obtained from [14].

We can readily see from both Figs. 8 and 9 that the ORBIT2-computed Ceres orbit and the HDC solution that we obtained using Piazzi's 17 best observations both follow the same trace in the sky during most of 1801. The solution that we can obtain from Piazzi's observations only differs as to the position of Ceres on that trace.

Consider now that following our construction of these two figures for the first time in early March 2016, we were able to obtain (in mid-March 2016) our own copy of von Zach [3]. There on p. 647 was the Ceres search ephemeris that Gauss gave to von Zach. It was only necessary now to convert the geocentric ecliptic longitudes and latitudes in the table to right ascensions and declinations, using the equations in Smart [2, p. 40], with the obliquity of the ecliptic of date as obtained from [16, p. B52]. The original geocentric ecliptic ephemeris of Gauss and the corresponding geocentric right ascensions and declinations are provided in the columns of Table 4.

Table 4. Gauss's 1801 November-December Search Ephemeris for Ceres

Gregorian Date	Ecliptic Longitude*	Ecliptic Latitude	Right Ascension	Declina- tion
year mo da	deg mn	deg mn	hours	degrees
1801 11 25	170 16	09 25	11.6558	12.5032
1801 12 01	172 15	09 48	11.7885	12.0665
1801 12 07	174 07	10 12	11.9141	11.6897
1801 12 13	175 51	10 37	12.0316	11.3805
1801 12 19	177 27	11 04	12.1417	11.1550
1801 12 25	178 53	11 32	12.2418	11.0116
1801 12 31	180 10	12 01	12.3331	10.9438

*Converted Gauss's Zodiac number, Z, to degrees, by multiplying by 30 degrees.

Given that astronomers, then as now, search to confirm the discovery of a celestial body by looking forward and backward along the expected trace in the sky, it is not surprising that von Zach found Ceres very close to where Gauss had predicted it would be. Gauss's predicted positions are indeed right on the trace, but a little ahead of Ceres, just as with our HDC-predicted positions using the 17 best actual Piazzi observations.

Because asteroids are typically dim, slowly-moving objects when visible in the night sky, it was a significant accomplishment on the parts of both Piazzi and von Zach to be the first and second observers, respectively, of the first-known asteroid, now called "1 Ceres." Indeed, without the patience, persistence, and determination of the keen-eyed astronomers Piazzi and von Zach, the much-deserved attention and acclaim that Gauss received as a dynamical astronomer and mathematician (at 24 years of age) might not have come so quickly.

In researching this paper, we of course consulted Gauss's *Theoria Motus* [19], wherein, in Article 159, Gauss computed the orbit of Ceres using observations by Olbers at Bremen (1805 September 5), by Harding at Lilienthal (1806 January 17), and by Bessel at Goettingen (1806 May 23). The question struck us, "why did not Gauss use the discovery observations of Piazzi from 1801 in *Theoria Motus?* The answer likely has to do simply with "length of observation span." That is, Piazzi's observations spanned only 40 days, whereas the three observations of Olbers, Harding, and Bessel spanned 260 days.

We make this note because it has been our experience, working with many observations of many deep-space Earth satellites made with modern telescopes, that the observation span can often be too short to arrive at a useful IOD solution when the observations are insufficiently accurate. We were not the first to make such an observation (no pun intended), but we think that Gauss may have been. We point to Gauss's observation selection in *Theoria Motus* Article 159 to support this notion.

In concluding the analyses of this paper, we refer back to Fig. 9 and note that the Piazzi observation span of forty days is quite short, considering that the orbital period of Ceres is about 4.6 years. So what might the geocentric ephemeris of Ceres, as viewed from Palermo, look like for a much longer period of time?

To answer this question, we relied again on the results of using HDC to fit the Ceres orbit to the 13 ORBIT2-predicted right ascensions and declinations, beginning 1801 January 1 and ending 1801 December 27, over twelve equal 30-day intervals of time. That orbital solution was propagated to1806 May 23 (1968 days of ephemeris) at equal 10-day intervals to smooth out the trace. The trace is plotted in Fig. 10.

Karl Friedrich Gauss is revered among mathematicians for his contributions to vector analysis, differential and non-Euclidean geometry, statistical theory, and the foundations of algebra [20]. We are mindful that (a) the key algorithm in HGM is a variant of Gauss's own Lambert solution, (b) Der IOD has a Gaussian framework, (c) Batch UPM DC, as a statistical method, has its theoretical basis in the Third Section, Articles 172-189 of *Theoria Motus,* and finally, (d) the gravity constant of orbital motion in our solar system, k = 0.01720209895 $AU^{3/2}$ per day, i.e., the fundamental dynamical constant in ORBIT2's solar system numerical integration, is called the *Gaussian* constant.

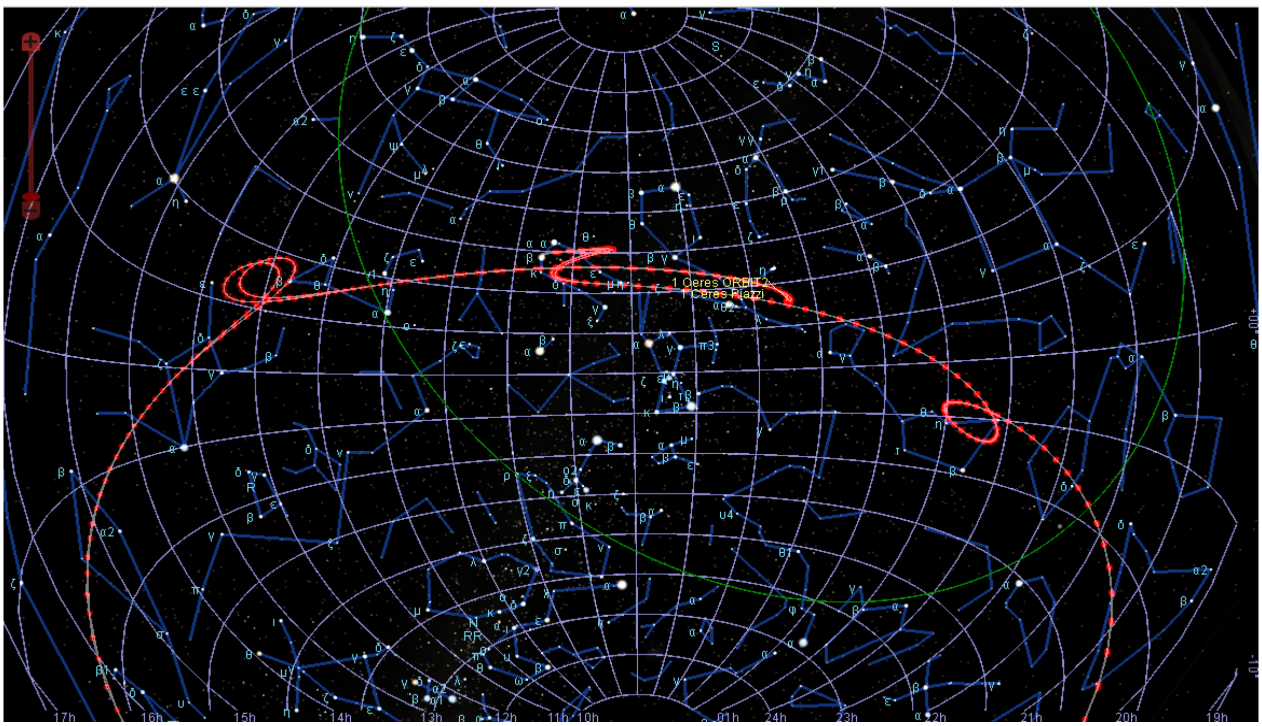


Fig. 10. Path of Ceres from 1801 January 1 to 1806 May 23. This figure shows what the path of Ceres, as a dwarf planet in the asteroid belt, looked like from the date of Ceres's discovery by Piazzi through its observation by Olbers, Harding, and Bessel in 1805-1806.

7. SPACE SITUATIONAL AWARENESS, THEN AND NOW

Space situational awareness is a relatively new term in the ever-growing lexicon of military space, but it is by no means a new concept in astronomy.

Then

Space situational awareness, as practiced in astronomy, took off with the invention of the telescope. The Palermo Circle was a specialized kind of telescope whose chief use was for measuring precisely and accurately the right ascensions and declinations of stars, for the purpose of star catalog compilation. Star catalogs were needed in order to surveil the skies for new comets, to determine more precisely the orbits of the known major planets, and to search for suspected new planets between the orbits of Mars and Jupiter.

Prior to Piazzi's 1801 January 1 discovery of Ceres, done with the aid of the Palermo Circle, telescopes were used mainly to observe the Moon, the major planets, and the moons around these major planets (the moons of Jupiter -- the four Galilean satellites -- being of especial interest). And of course, astronomers were keenly interested in the discovery of comets and in the determination of their orbits. Piazzi's discovery confirmed to astronomers that there were yet other classes of objects in in our solar system to be surveiled and studied.

So Piazzi's discovery of Ceres, Gauss's determination of that first-discovered asteroid's orbit, and von Zach's recovery of Ceres using Gauss's search ephemeris not only marked major milestones in observational and mathematical astronomy, but also major milestones in space situational awareness.

Now

We could not be faulted for thinking that modern space situational awareness had its beginning with the launch of the Russian Sputnik 1 satellite in 1957. But astronomers were even then quick to point out the applicability of Gauss's angles-only method to determination of the orbit of Sputnik 1 and the orbits of the many other artificial Earth satellites soon to follow.

The U.S. Air Force acknowledged the need to track and catalog man-made Earth satellites with the implementation of Project Space Track in 1957 and stood up a National Space Surveillance Control Center in 1959 at Hanscom Field in Massachusetts. That operations center evolved into the Space Defense Center that moved into the NORAD Cheyenne Mountain Complex (NCMC) in 1965 in Colorado.

The NCMC sheltered a succession of operations-supporting space surveillance computer systems, e.g., the Delta System, the 427M Space Computational Center, and the Space Defense Operations Center (SPADOC), the last of which, after several more incremental upgrades, is today the Joint Space Operations Center at Vandenberg Air Force Base in California. Advances in computers, astrodynamic algorithms, and computer programs have continued over the years along with advances in radar tracking and electro-optical tracking/imaging technology.

For deep-space objects (those with orbital periods greater than 225 minutes), electro-optical tracking is still to be preferred in the general case for reasons of cost and safety. But for these kinds of objects, angles-only measurements are still required for catalog building with UCT discovery, confirmation, and follow-up observations; and for catalog maintenance.

Preliminary orbit determination (e.g., Der IOD and HGM) and the differential correction of the initial orbital state estimates (Batch UPM DC) apply during the discovery, confirmation, and follow-up phases at electro-optical sites. But these methods belong just as much in the codes of a space situational awareness operations center, where the observations from all electro-optical sensors are combined in order to maintain a catalog of man-made, deep-space Earth satellites.

So Piazzi's discovery of Ceres during 1801 January-February and von Zach's recovery of that first-known asteroid on the night of 1801 December 31 - 1802 January 1 are noted in support of our assertion that modern space situational awareness really had its beginning with Piazzi, Gauss, and von Zach. Of course, what has changed up to the present is that we now have more sophisticated computer technology and telescopes. But what has not changed is that modern orbit determination and differential correction techniques are still valuable, powerful, and worthy of continued attention and improvement.

And the need for ever-more accurate star catalogs is still with us, because the more precisely and accurately we know the positions of our reference stars, the more precise and accurate will be our right ascension and declination measurements of space objects of interest. So Piazzi and his fellow astronomers were space situational awareness pioneers for their work in compiling star catalogs, too.

8. SUMMARY AND CONCLUSIONS

The analyses described in this paper are summarized as follows.

1. ORBIT2 was used to numerically integrate the orbit of the asteroid now designated as “1 Ceres” back to the times of Piazzi’s observations, and ORBIT2-computed observations were generated at Piazzi’s observation times. The results were placed in Table 1 for comparison.

2. HGM and Der IOD were used to generate preliminary osculating orbital elements for Ceres using both the ORBIT2-computed observations and the actual Piazzi observations. HGM placed epoch at the time of the first observation on 1801 January 1, and input 19 observations, whereas Der IOD placed epoch at the time of the second observation on 1801 January 22, and input three observations.

3. Batch UPM DC was used to generate improved osculating elements for Ceres using all 19 of the complete Piazzi observations. Through examination of the residuals plots for the final iteration of the DC, it became evident that the third and sixth observations spoiled the fit. So Batch UPM DC was run again with the 17 best Piazzi observations (the two spoilers being excluded) in order to arrive at a better solution (lower RMS of residuals).

4. The Batch UPM DC solution for the best 17 Piazzi observations was plotted in the night sky of 1801, as depicted by Software Bisque’s *TheSkyX Professional,* together with ephemeris points from the highly accurate ORBIT2 numerically-integrated orbit with epoch on 1801 January 1. It was seen that although the 17-best-Piazzi-observations solution led the ORBIT2-integrated solution in time, both solutions resulted in the same trace in the sky during most of 1801.

We note that planetary aberration and light-time displacement were investigated, but not found to be significant factors in the analyses. Also, there was little change in the solutions when Piazzi's observations were treated as geocentric rather than topocentric.

All of the P and Q residuals with HGM were smaller than 40 arc-seconds, approximately 2/3 of an arc-minute (quite good). It is likely that Piazzi's estimates of the local mean times were quite good, and that the measurements of right ascension and declination using the Palermo Circle were quite good.

But given that European astronomers were still in the process of compiling, refining, and standardizing their reference star catalogs in the early 1800s [1, p. 23, point 3], it seems likely that the greatest errors in the measurements were in right ascension (see again Fig. 5 in Section 5).

We conclude that by applying contemporary algorithms and models to an example of solar system orbit determination requiring highly-precise numerical integration of the orbit going back more than two centuries into the past, we have demonstrated the following.

1. HGM, Der IOD, and Batch UPM DC are powerful contemporary tools for gaining and maintaining space situational awareness, the principal application being rapid determination of the osculating orbits of high-interest objects, both geocentric and heliocentric, when time is of the essence.

2. ORBIT2 is a powerful tool for potentially modeling cislunar and interplanetary trajectories, e.g., lunar close approaches and flybys, and interplanetary trajectories that approach more distant solar system objects.

9. REFERENCES

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The following figure, excerpted from this Reference 1, p. 18, illustrates the Palermo Circle that Piazzi used to discover Ceres.

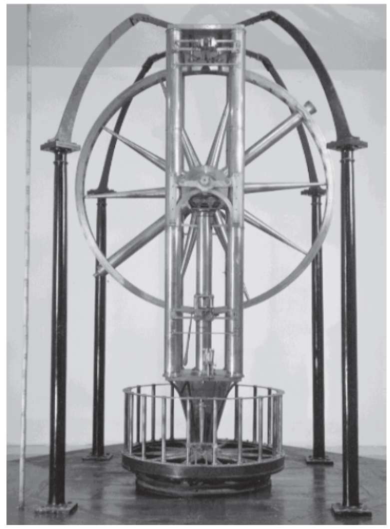

Fig. 2. The Palermo Circle. This meridian circle was constructed by Jesse Ramsden (1730–1800), the greatest of the eighteenth-century instrument makers. It was completed in 1789 after almost two years of intense work. The telescope has a 7.5-cm objective lens; the altitude scale (5 feet in diameter) was read with the aid of two diametrically-opposed micrometer microscopes while the azimuth scale (3 feet in diameter) was read by means of a micrometer microscope.

(The mount for the Palermo Circle was actually an early version of what we now call an altazimuth mount. That is, the telescope was capable of pointing in azimuth as well as in altitude. But orienting the zenith-angle-measuring circle to a fixed azimuth of 180 degrees afforded the best precision for transit timing and therefore for star catalog compilation.)

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12. Mansfield, Roger L. *Topics in Astrodynamics,* Astronomical Data Service, Colorado Springs, Colorado, February 28, 2003.

Chapter 14 presents the UPM equations of universal-variables-based, two-body orbital motion. Chapter 15 presents the author's treatment of batch least squares UPM differential correction ("Batch UPM DC") for Earth-orbital motion.

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10. ADDENDUM: AN UNEXPECTED FINDING

With Gauss's search ephemeris newly in hand, it was possible to convert the geocentric ecliptic longitudes and latitudes to right ascensions and declinations (see again Table 4, Section 6) and to plot the resulting angles-only (right ascension, declination) sky trace vs. the sky traces predicted by ORBIT2 and the Batch UPM DC solution with the 17 best Piazzi observations. That is done in Fig. A1.

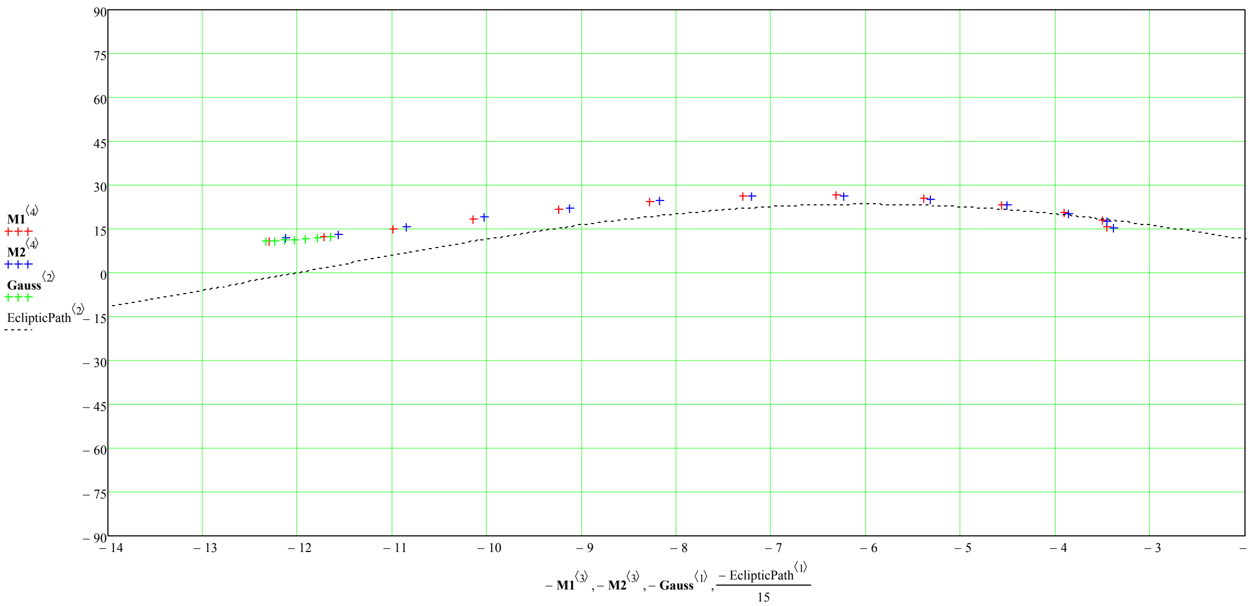

Fig. A1. Sky Plot of Gauss Search Ephemeris Points (Green) vs. ORBIT2 (Blue) vs. Batch UPM DC with 17 Best Piazzi Observations (Red). Declination is on the vertical axis and goes from -90 degrees to +90 degrees. Right ascension is on the horizontal axis and goes from 2 hours to 14 hours (ignore the minus signs preceding the right ascension labels, which are a consequence of the fact that Mathcad assumes that the horizontal axis scale values ascend from left to right, whereas on a star chart, right ascension ascends from right to left). Ecliptic path of Sun in 1801 is dotted line. Stars are omitted to emphasize calculated positions of Ceres.

It is clear from the figure that on the seven dates 1801 November 25 through December 31 spaced six days apart, with the night of 1801 December 31 - 1802 January 1 being the night that von Zach recovered Ceres, all of the plotted positions agree well with each other.

But is it possible to tell which ephemeris, (a) the Gauss search ephemeris or (b) the Batch UPM DC ephemeris (i.e., the ephemeris obtained by propagating the Batch UPM DC solution with 17-Piazzi observations), is closer to the ORBIT2-predicted ephemeris?

To answer this question, we replotted the part of the sky trace that contains the seven ephemeris points from 1801 November 25 through 1801 December 31 and extended the ORBIT2 sky trace farther into 1802. Fig. A2 shows the resulting plot.

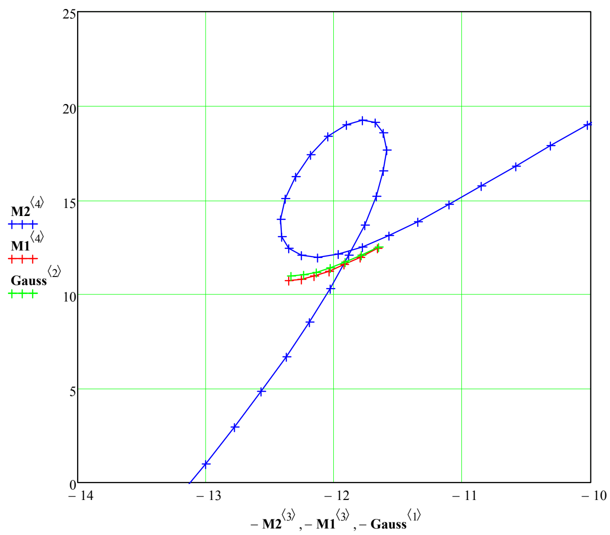

Fig. A2. Gauss Search Ephemeris Points (Green) vs. Ephemeris Points Derived from Batch UPM DC Solution with 17 Best Piazzi Observations (Red) vs. ORBIT2 Ephemeris. ORBIT2 ephemeris points are plotted at 10-day intervals in this plot.

We see in Fig. A2 that there are seven green plusses (Gauss) and seven red plusses (Batch UPM DC with 17 Piazzi observations), and that the corresponding ephemeris points are close to each other. But we can also see that the green plusses seem to be closer to their corresponding points on the ORBIT2 sky trace than the red plusses are to their same corresponding ORBIT2 sky trace points (see Note* below). The implication is that the Gauss search ephemeris is slightly better than the ephemeris derived from the Batch UPM DC solution with the 17 best Piazzi observations. (*Note: Due to 10-day spacing of ORBIT2 ephemeris points in Fig. A2, the tick marks for the ORBIT2 ephemeris points that correspond to the time points of the Gauss search ephemeris are not all shown. But they are indeed shown in black in Fig. A3, below.)

So this was the unexpected finding: that the Gauss search ephemeris, which resulted from Gauss's determining the orbit of Ceres in 1801 from just three Piazzi observations, is better than the ephemeris we get when we propagate the Batch UPM DC solution with the 17 best Piazzi observations. This situation suggested that we do a Batch UPM DC with just the three observations that Gauss used. Fig. A3 illustrates the results.

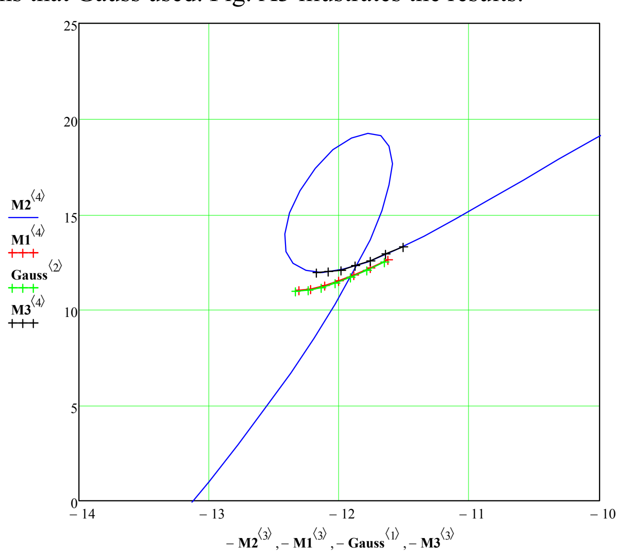

Fig. A3. Gauss Search Ephemeris Points (Green) vs. Batch UPM DC Ephemeris Points As Derived from the Three Piazzi Observations that Gauss Chose (Red) vs. ORBIT2 Ephemeris Points (Black). ORBIT2 ephemeris points are plotted at same 6-day intervals as the other points in this plot.

We see in Fig. A3 that the red plusses are closer to the corresponding black plusses than the green plusses are to these same black plusses. So in this case, the ephemeris derived from the Batch UPM DC solution, using precisely the three observations that Gauss chose for his own IOD, is (slightly) better than Gauss's search ephemeris. We attribute this slight improvement to our having a better solar ephemeris model to work with in 2016 than Gauss had available to him in 1801.

To sum up, we are inclined to think that a modern, nonlinear least-squares orbit determination with 17 observations should be better than a preliminary orbit determination with just the minimum number of three observations.

But Gauss has shown us, using the example of the 1801 discovery orbit of Ceres, that sometimes one can get a better solution by carefully picking just the minimum number of three observations, even when many more good observations are available.

FINAL NOTE

When the paper to which this Addendum was added was originally conceived and outlined, we did not have in hand any of von Zach's *Monatliche Correspondenz* relating to the discovery and recovery of Ceres in 1801. We had simply planned to apply our contemporary algorithms to Piazzi's observations and to report the results, on the assumption that with our contemporary models, we could vet each of Piazzi's 19 usable observations individually in a way that was not possible in 1801. And we did that.

But when we found out, in March 2016, that von Zach's 1801 correspondence with Gauss and Piazzi has recently become available as a Nabu Public Domain reprint, we immediately obtained a copy and translated from German to English the most important and relevant article, the one dated December 1801 [3, pp. 638-649], since this article contained Gauss's search ephemeris for the recovery of Ceres just as Gauss had communicated it to von Zach.

With Gauss's search ephemeris in hand, we could now do much more analysis, but the results did not neatly fit with the outline that we had originally conceived for the paper. Hence this Addendum.

GLOSSARY OF ACRONYMS

Acronym	Meaning
AAS	American Astronautical Society *or* American Astronomical Society
AIAA	American Institute of Aeronautics and Astronautics
DC	Differential Correction
DDA	Division on Dynamical Astronomy
DE	Development Ephemeris of JPL
DEC	Declination
ECI	Earth-Centered Inertial
HDC	Batch UPM DC of a Heliocentric Orbit
HGM	Herget/UPM IOD
IOD	Initial Orbit Determination
JPL	Jet Propulsion Laboratory
LMT	Local Mean Time
MOD	Mean of Date
NASA	National Aeronautics and Space Administration
NCMC	NORAD Cheyenne Mountain Complex
NORAD	North American Aerospace Defense Command
ORBIT2	Der's Version of Lear's Solar System Numerical Integration Program
RA	Right Ascension
RMS	Root Mean Square
SPADOC	Space Defense Operations Center
SSA	Space Situational Awareness
TOD	True of Date
UCT	Uncorrelated Target
UPM	Uniform Path Mechanics
USNO	U.S. Naval Observatory