Space Surveillance Network Scheduling Under Uncertainty: Models and Benefits

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ABSTRACT

Advances in space technologies continue to reduce the cost of placing satellites in orbit. With more entities operating space vehicles, the number of orbiting vehicles and debris has reached unprecedented levels and the number continues to grow. Sensor operators responsible for maintaining the space catalog and providing space situational awareness face an increasingly complex and demanding scheduling requirements. Despite these trends, a lack of advanced tools continues to prevent sensor planners and operators from fully utilizing space surveillance resources. One key challenge involves optimally selecting sensors from a network of varying capabilities for missions with differing requirements. Another open challenge, the primary focus of our work, is building robust schedules that effectively plan for uncertainties associated with weather, *ad hoc* collections, and other target uncertainties. Existing tools and techniques are not amenable to rigorous analysis of schedule optimality and do not adequately address the presented challenges.

Building on prior research, we have developed stochastic mixed-integer linear optimization models to address uncertainty due to weather's effect on collection quality. By making use of the open source Pyomo optimization modeling software, we have posed and solved sensor network scheduling models addressing both forms of uncertainty. We present herein models that allow for concurrent scheduling of collections with the same sensor configuration and for proactively scheduling against uncertain *ad hoc* collections. The suitability of stochastic mixed-integer linear optimization for building sensor network schedules under different run-time constraints will be discussed.

1. INTRODUCTION

Scheduling is an important part of the utilization and safe operation of remote sensing assets. Automated algorithms for the scheduling of sensors span many different applications including planning of Earth observing satellite collections [1, 2], routing of unmanned aerial systems under fuel constraints [3], and simulated operations of ground [4] and space-based sensors for space situational awareness. Algorithms designed using domain-specific scheduling heuristics are common across these different application spaces. The pervasive use of heuristics in these respective sensing domains has been in part due to the computational complexity and resultant runtimes required to produce feasible sensor schedules. These heuristics often convolve the scheduling model with the solution technique, requiring changes to both to accommodate updates to either the model or solution technique. Sensor development across satellites and ground telescopes continues to extend the capabilities of already very flexible sensors. The challenge of quickly producing optimal or near-optimal sensor schedules is further complicated when considering differing definitions of schedule optimality. Priority and orbit error covariance are naturally used as optimality criteria in many scheduling algorithms. Proper relative weighting of these criteria, amongst others such as timeliness of information or update and needed type of target characterization, necessitate a rigorous model definition separate from solution technique.

Inherent to the scheduling of sensors, perhaps even more so for space situational awareness, is the notion of schedule uncertainty. Certainly, one of the primary goals of the Space Surveillance Network (SSN) is to reduce orbital parameter errors. In this paper, we present a model that proactively schedules against uncertainty to produce a schedule with higher expected utility over scenarios of differing likelihood. For example, cloud cover at ground sensor sites may prevent or at least negatively effect the utility of scheduled collections at those sites. By making use of weather forecasts and historical weather data, multiple weather scenarios can be considered and depending on the likelihood of each, collections affected by cloud cover can be moved to sensors less sensitive to clouds or sensors without coincident weather conditions. Similarly, uncertainties in the timing of a target resident space object (RSO) passage through the sensor field-of-regard may negatively impact the utility of a scheduled collection and the sensor and constellation schedule as a whole. Schedule utility is reduced when a collection does not produce significant improvements in orbit uncertainty or object characterization. Despite pervasive uncertainty in the scheduling of remote sensor collections, existing literature, e.g. [5], does not present tractable and extensible models and algorithms to proactively reduce the impact of uncertainty on sensor schedules.

In this paper, we present a model for reducing the impact of *ad hoc* collections. This model makes use of a stochastic mixed-integer linear program [6] to proactively schedule against probability-weighted scenarios of different collection start times and durations. These scenarios differ across *ad hoc* collection start times and durations, yet aim to schedule a common set of collection windows whose timing and durations are known *a priori*. A model developed in [7] demonstrates the advantage of proactively scheduling against weather scenarios over deterministic models that assume a specific future weather outcome. We also build upon prior research to formalize a model that allows collections of the same configuration to run concurrently. We defer to [8] for collection window property and category definitions. Note that in the following we have adopted collection window instead of activity to describe time periods scheduled on sensors. In general, it is assumed that Category 4 collections, the category reserved for *ad hoc* collections, are of the highest priority and in general must be scheduled as soon as information establishes sufficient certainty of occurrence, barring conflicts with sensor safety constraints.

In Section 2, we present some preliminaries on the stochastic model. Section 2.1 presents the deterministic model that allows for concurrent scheduling of collections with same configuration. Section 2.2 presents details and definitions of the stochastic model. Section 3 discusses the model and directions for future research. Finally, Section 4 presents some concluding remarks.

2. A STOCHASTIC COVERAGE MODEL FOR *AD HOC* COLLECTIONS

In the following, a formulation is developed for a stochastic mixed-integer constellation scheduling problem. The problem is comprised of two decision stages. First, a deterministic model allowing for concurrent scheduling of collections is described. This model, representing the first stage of the stochastic mixed-integer program (SMIP), aims to schedule Category 1, 2, and 3 collection windows. In the second stage, scenario distributions capturing the variability in Category 4 collection window start times and durations are used to create a schedule that maximizes expected information gain and is resilient to *ad hoc* changes. In a two stage decision making setting, the first stage decision has to be made here-and-now without full information of future events. Second stage decisions are made later when the future is revealed. To formalize this decision making process, let x be the first stage decision variable vector and let $y(\xi)$ be the second stage (*recourse*) decision vector for outcome (scenario) ξ. Then a generic two-stage SMIP can be given as follows:

$$\begin{aligned} \max\ & f(x) + \mathbb{E}[g(x, \tilde{\xi})] \\ \text{s.t.}\ & Ax \leq b \\ & x \in X, \end{aligned} \tag{1}$$

where the set $X \subseteq \mathbb{R}^{n_1}$ imposes integer or binary restrictions on all or some components of x, $f(x)$ is the first stage objective function, $\mathbb{E}$ is the mathematical expectation operator and $\mathbb{E}[g(x, \tilde{\xi})]$ is the *expected recourse function*. The vector $\tilde{\xi}$ is a multivariate random variable defined on a probability space $(\Xi, \mathcal{F}, \mathcal{P})$ and represents the second stage uncertainty. So for a given outcome $\xi \in \Xi$ of $\tilde{\xi}$, the recourse function $g(x, \xi)$

is given by the following second stage problem:

$$g(x,\xi) = \max \ h(y(\xi)) \quad \text{s.t.} \ W(\xi)y(\xi) \leq r(\xi) - T(\xi)x, \quad y(\xi) \in Y \tag{2}$$

In problem (2), $Y \subseteq \mathbb{R}^{n_2}$ imposes integer or binary restrictions on all or some components of $y(\xi)$, $h(y(\xi))$ is the second stage objective function, $W(\xi) \in \mathbb{R}^{m_2 \times n_2}$ is the (rational) recourse matrix, $T(\xi) \in \mathbb{R}^{m_2 \times n_1}$ is the technology matrix, and $r(\xi) \in \mathbb{R}^{m_2}$ is the right hand side vector. A *scenario* ξ defines the realization of the stochastic problem data $\{W(\xi), T(\xi), r(\xi)\}$. If $W(\xi) = W$ for all $\xi \in \Xi$, the problem is said to have *fixed recourse*. Otherwise, the problem is said to have *random recourse*. Similarly, if $r(\xi) = r$ for all $\xi \in \Xi$, and $T(\xi) = T$ for all $\xi \in \Xi$, the problem is said to have *fixed right hand side* vector and *fixed technology* matrix, respectively. For more information about stochastic mixed-integer programming, see [6].

2.1. STAGE ONE: DETERMINISTIC MODEL

Let δ_{ikt} be a binary decision variable where $\delta_{ikt} = 1$ if sensor i has scheduled collection window k starting at timestep t. We assume that the set of collection windows are organized into the categories given in [8]. Category 1 collection windows must be scheduled, but Category 2 and 3 collection windows may be deferred due to scheduling limitations imposed by scheduled Category 1 or Category 4 collection windows or other collection windows of higher priority or quality. We let $\mathcal{K}_1 \subseteq \mathcal{K}$ be the indices of the Category 1 collection windows. For convenience, we define the binary decision variable ω_k to represent whether or not collection window k has been scheduled at any time step, on any sensor:

$$\omega_k = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \delta_{ikt}, \forall k \in \mathcal{K}. \tag{3}$$

The following constraint ensures that all Category 1 collection windows are scheduled:

$$\omega_k = 1, \forall k \in \mathcal{K}_1. \tag{4}$$

We also introduce a set of binary indicator variables ϕ_{its} to signify that at least one collection with configuration $s \in \mathcal{S}$ is starting or active at timestep t on sensor i. As before, if a sensor is scheduled to begin observing a collection window at time $\bar{t}$, then it will continue collection for the next $d_k - 1$ timesteps, where d_k is the duration required to complete collection window k. Let $\mathcal{C}(k, \bar{t})$ be the set of feasible time steps where a collection window other than k could have been started prior to time step $\bar{t}$ and for which scheduling another sensor observation would conflict with starting a new sensor observation beginning at time step $\bar{t}$. Thus

$$\mathcal{C}(k, \bar{t}) = \max(e_k, \bar{t} - d_k + 1), \ldots, \min(l_k, \bar{t} - 1). \tag{5}$$

The constraint to allow for concurrent collections of a single configuration over sensor i's scheduling horizon is

$$\sum_{k \in \mathcal{K}} \delta_{ik\bar{t}} \leq (1 + M) \sum_{\bar{k} \in \mathcal{K} \setminus k} \sum_{t \in \mathcal{C}(k,\bar{t})} \phi_{its}, \forall \bar{t} \in \mathcal{T}, i \in \mathcal{I}, s \in \mathcal{S}, \tag{6}$$

where M is in general chosen to equal to number of *a priori* (as opposed to *ad hoc*) collection windows to be scheduled. With some preprocessing, this number can be chosen depending on the number of collection windows that can be scheduled at constraint index $\{i, t, s\}$ to provide a tighter constraint bound. When combined with the following constraint

$$\sum_{s \in \mathcal{S}} \phi_{i\bar{t}s} \leq 1, \forall i \in \mathcal{I}, \bar{t} \in \mathcal{T}, \tag{7}$$

these constraints prohibit scheduling a collection window k at time $\bar{t}$ if another collection window with a different configuration was scheduled "recently", at some time $t \in \mathcal{C}(k, \bar{t})$. When collections of the same configuration run concurrently sensors with fields-of-view large enough relative to collection requirements have the opportunity to schedule more collection windows and therefore raise overall schedule utility.

The first-stage objective function remains the same as described in our previous work with the mixed-integer linear program, changing slightly to accommodate concurrently scheduled collection windows. Let $\delta = \{\delta_{ikt}\}_{\forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}}$ and the objective function be defined as

$$f(\delta) = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} \sum_{t \in \mathcal{T}} \frac{\delta_{ikt} p_k d_k q_{ikt}}{\sigma}. \tag{8}$$

where σ is normalizing constant used for convenience and defined in our previous model. Then, the first-stage binary program is:

$$\begin{array}{lll} \max & f(\delta) & \\ \text{s.t.} & \omega_k = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \delta_{ikt} & \forall k \in \mathcal{K}, \\ & \omega_k = 1 & \forall k \in \mathcal{K}_1, \\ & \sum_{k \in \mathcal{K}} \delta_{ik\bar{t}} \leq (1+M) \sum_{\bar{k} \in \mathcal{K} \setminus k} \sum_{t \in \mathcal{C}(k,\bar{t})} \phi_{its} & \forall \bar{t} \in \mathcal{T}, i \in \mathcal{I}, s \in \mathcal{S}, \\ & \omega_k \in \{0,1\} & \forall k \in \mathcal{K}, \\ & \delta_{ikt} \in \{0,1\} & \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}, \\ & \phi_{its} \in \{0,1\} & \forall i \in \mathcal{I}, t \in \mathcal{T}, s \in \mathcal{S}. \end{array} \tag{9}$$

This model allows for alternate definitions of q_{ikt} according to sensor, time, and collection window so that a scheduling strategy depending on sensor platform (e.g., airborne and ground) and overarching mission (e.g., maintenance of the space catalog and maximum value of collected reconnaissance information) can be optimized. Figure 1 shows an example two-sensor schedule produced using the model described in (9). A legend depicting the colors of each sensor configuration is shown in the upper right. Configurations one and two are reserved to correspond to Category 1 and 2 collection windows. The other configurations depend on sensor type and capability. For both plots, time is given on the horizontal axis. In the top plot, scheduled collections of each of two sensors are depicted on distinct timelines. Collection windows are displayed as rectangles with height corresponding to priority, width corresponding to duration, and color corresponding to configuration. The opacity of the rectangles allow for clear indication of concurrently scheduled collections. The bottom plot depicts collection windows that were not scheduled. As in the top plot, rectangles are used to describe the priority, duration, and configuration of collection windows, had they been scheduled. Dotted lines are used to denote the earliest and latest potential start time of the unscheduled collection windows. Numbers to the right of each collection window indicate which sensor(s) the collection window can be scheduled on.

2.2. STAGE TWO

The second stage of the SMIP aims to optimize the schedule in light of a known distribution $\mathcal{P}$ of Category 4 (*ad hoc*) collection windows. As previously defined, all Category 4 collection windows have priority equal to one. From a remote sensing applications standpoint, Category 4 collection windows are modeled to describe important events that have uncertain time windows and durations that fall within a forecasted range. An example of such a collection window is an observation of a newly identified piece of space debris or collections of a wildfire crossing a boundary of interest such as a state line or a fence near critical infrastructure. When and how long it takes the wildfire to cross the boundary can be forecasted, but is uncertain.

Each scenario ξ represents an instance of a set of Category 4 collection windows and their time window and duration realizations, i.e., for each $\xi \in \Xi$ we have the realization of the data $\{(e_k^\xi, l_k^\xi), d_k^\xi, q_{ikt}^\xi\}$. An important nuance to the second stage model is that it is assumed Category 4 collection windows cannot preempt Category 1 collection windows. The second stage carefully considers which collection windows would be interrupted by analyzing each Category 4 scenario and computing the resulting changes to the stage one objective function. An important assumption in the following model is that Category 4 collections require a specific configuration and accordingly, no concurrent collections are allowed when a Category 4 collection is scheduled.

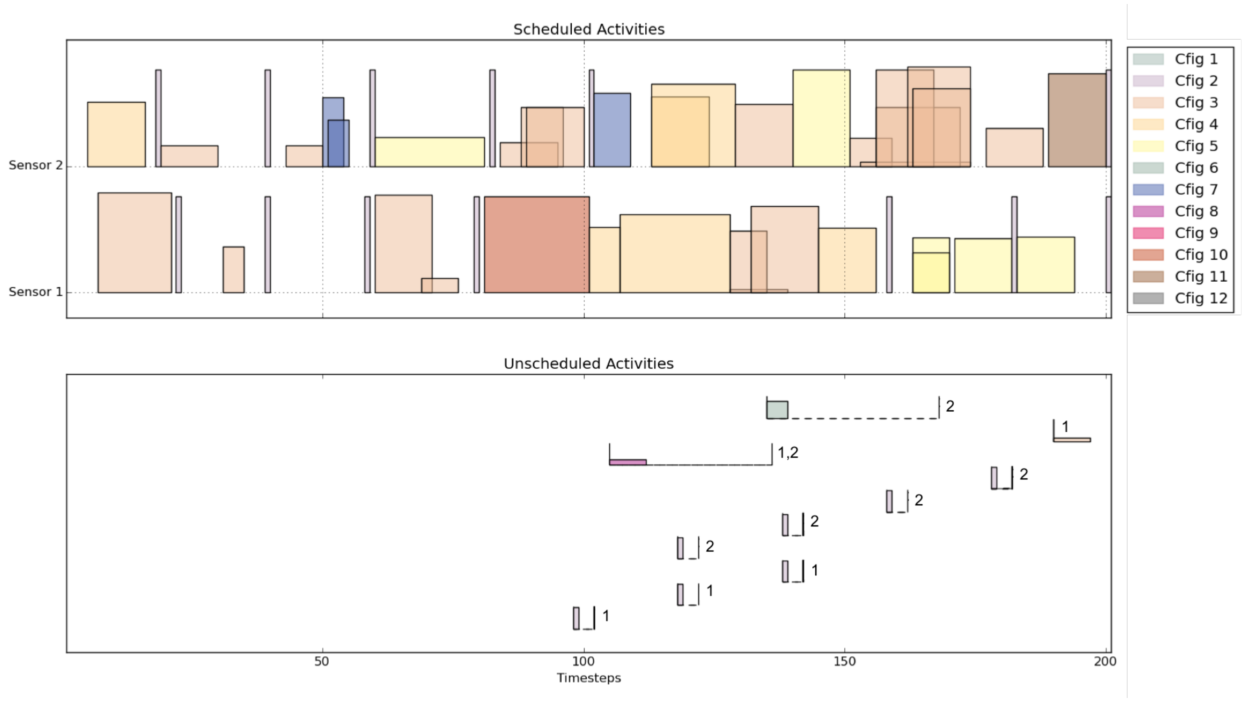

Fig. 1: Example sensor schedule allowing concurrent collections of a single configuration.

2.2.1 DEFINITIONS

Let $\mathcal{K}_4^\xi \subseteq \mathcal{K}$ be the set of scenario dependent Category 4 collection windows. Let $\mathcal{K}_4^{p\xi}$ be the set of scenario dependent collection window pairs (each pair including exactly one Category 4 collection window) that could possibly conflict. Let Δ_{ikt}^ξ be a binary decision variable where $\Delta_{ikt}^\xi = 1$ if sensor i is scheduled to observe collection window k starting at time period t under scenario ξ, where $i \in \mathcal{I}$, $t \in \mathcal{T}$ and $k \in \mathcal{K}_4^\xi$. The variable Ω_k^ξ represents whether or not a specific Category 4 collection window is scheduled under scenario ξ:

$$\Omega_k^\xi = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \Delta_{ikt}^\xi, \forall k \in \mathcal{K}_4^\xi \tag{10}$$

Let $y_{ikk_zt}^\xi$ be a binary variable defined over the collection window pair $k, k_z \in \mathcal{K}_4^{p\xi}$ under scenario ξ. The decision variable $y_{ikk_zt}^\xi$ is equal to one when collection windows k and k_z conflict when scheduled at time t on sensor i under scenario ξ. Let γ_{ikt}^ξ be a binary variable defined over $i \in \mathcal{I}$, $t \in \mathcal{T}$ and $k \in \mathcal{K}_4^\xi$ under scenario ξ that equals one if the corresponding Category 4 collection window will interrupt at least one overlapping collection window scheduled in the first stage. Let Γ_{ikt}^ξ be a binary variable indexed over $i \in \mathcal{I}$, $k \in \mathcal{K} \setminus \mathcal{K}_4^\xi$ and $t \in \mathcal{T}$ that is equal to one if a Category 1, 2 or 3 collection window will be interrupted by a Category 4 collection window under scenario ξ.

As before, collections concurrent with Category 4 collection windows are prevented from occurring on each sensor i:

$$\sum_{k \in \mathcal{K}_4} \Delta_{ik\bar{t}}^\xi \leq 1 - \sum_{k \in \mathcal{K}_4} \sum_{t \in \mathcal{C}(k,\bar{t})} \Delta_{ik\bar{t}}^\xi, \forall \bar{t} \in \mathcal{T}, i \in \mathcal{I}. \tag{11}$$

All Category 4 collection windows are forced to be scheduled:

$$\Omega_k^\xi = 1, \forall k \in \mathcal{K}_4^\xi. \tag{12}$$

Using the following three constraints, $y_{ikk_zt}^\xi$ is defined as:

$$y_{ikk_zt}^\xi + (1 - \Delta_{ik_jt}^\xi) \geq \delta_{ik_zt} \ \forall i \in \mathcal{I}, k, k_z \in \mathcal{K}_4^{p\xi}, t \in \mathcal{T}, \tag{13}$$

which sets $y^{\xi}_{ikk_zt}$ to one when either a Category 1, 2 or 3 collection window in the first stage is scheduled and a Category 4 collection window in the second stage overlaps. Notice that we put δ_{ik_zt} on the right hand side because it is determined in the first stage and is known at this time;

$$(1 - y^{\xi}_{ikk_zt}) + \Delta^{\xi}_{ikt} \geq 1 \ \forall i \in \mathcal{I}, k, k_z \in \mathcal{K}^{p\xi}_4, t \in \mathcal{T}, \tag{14}$$

which prohibits $y^{\xi}_{ikk_zt}$ from being one if the Category 4 collection window is not overlapping; and

$$(1 - y^{\xi}_{ikk_zt}) \geq 1 - \delta_{ik_zt} \forall i \in \mathcal{I}, k, k_z \in \mathcal{K}^{p\xi}_4, t \in \mathcal{T}, \tag{15}$$

which prohibits $y^{\xi}_{ikk_zt}$ from being one if the Category 1, 2 or 3 collection window is not scheduled. In order to define γ^{ξ}_{ikt}, the next two constraints are used:

$$\gamma^{\xi}_{ikt} + \sum_{k_z \in \mathcal{K}_1} \sum_{t \in \mathcal{T}} y^{\xi}_{ikk_zt} \geq 1 \ \forall i \in \mathcal{I}, k, k_z \in \mathcal{K}^{p\xi}_4, t \in \mathcal{T}, \tag{16}$$

which enforces $\gamma^{\xi}_{ikt} = 1$ every time there is no actual conflict with a Category 1 collection window and if $k_z \in \mathcal{K}_1$,

$$(1 - \gamma^{\xi}_{ikt}) + (1 - y^{\xi}_{ikk_zt}) \geq 1 \ \forall i \in \mathcal{I}, k, k_z \in \mathcal{K}^{p\xi}_4, t \in \mathcal{T}, \tag{17}$$

which prohibits a Category 4 collection window from interrupting a Category 1 collection window. Finally, to define $\Gamma^{\xi}_{ik_zt}$ we strictly enforce an if and only if relationship between interrupted collection windows and Category 4 collection windows scheduled in the second stage:

$$\Gamma^{\xi}_{ik_zt} + (1 - y^{\xi}_{ikk_zt}) + (1 - \gamma^{\xi}_{ikt}) \geq 1 \ , \tag{18}$$

$$(1 - \Gamma^{\xi}_{ik_zt}) + \sum_{k \in \mathcal{K}^{\xi}_4} \sum_{t \in \mathcal{T}} y^{\xi}_{ikk_zt} \geq 1 \ , \tag{19}$$

$$(1 - \Gamma^{\xi}_{ik_zt}) + \sum_{k \in \mathcal{K}^{\xi}_4} \sum_{t \in \mathcal{T}} \gamma^{\xi}_{ikt} \geq 1 \ \forall i \in \mathcal{I}, k, k_z, \in \mathcal{K}^{p\xi}_4, t \in \mathcal{T}. \tag{20}$$

2.2.2 SECOND STAGE MODEL

The objective function for stage two is computed first by taking the additional value of scheduled Category 4 collection windows that interrupt all other collection windows and then subtracting value of interrupted Category 2 and 3 collection windows. Again, the value is computed prioritizing long duration, high priority collection windows, and high quality observations. Let $\gamma(\xi)$, and $\Gamma(\xi)$ be the second stage decision variable vectors whose components are the γ^{ξ}_{ikt}'s and Γ^{ξ}_{ikt}'s, respectively. Then the second stage objective function, $h(\gamma(\xi), \Gamma(\xi))$, can be given as follows:

$$h(\gamma(\xi), \Gamma(\xi)) = \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}^{\xi}_4} \sum_{t \in \mathcal{T}} \frac{(\gamma^{\xi}_{ikt})(p_k)(d_k)(q_{ikt})}{\sigma'} - \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K} \setminus \mathcal{K}^{\xi}_4} \sum_{t \in \mathcal{T}} \frac{(\Gamma^{\xi}_{ikt})(p_k)(d_k)(q_{ikt})}{\sigma'}.$$

Again, σ' denotes a normalization constant so that an objective function value of 100 will denote a set of sensor schedules in which all collection windows are scheduled, modified to incorporate the value of scheduling Category 4 collection windows. Using the constraints defined above, together with the objective function in Equation (8), then for given first stage decisions δ and scenario $\xi \in \Xi$ the second stage recourse binary program is given as follows:

$$
\begin{aligned}
g(\delta,\xi) = \quad & \max \; h(\gamma(\xi),\Gamma(\xi)) & & \\
& \text{s.t.} \; \textstyle\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} \Delta^{\xi}_{ikt} - \Omega^{\xi}_{k} = 0 & & \forall k \in \mathcal{K}^{\xi}_{4}, \\
& \textstyle\sum_{k\in\mathcal{K}^{\xi}_{4}} \Delta^{\xi}_{ik\bar{t}} \le 1 - \sum_{k\in\mathcal{K}^{\xi}_{4}}\sum_{t\in\mathcal{C}(k,\bar{t})} \Delta^{\xi}_{ik\bar{t}} & & \forall \bar{t} \in \mathcal{T}, i \in \mathcal{I} \\
& \Omega^{\xi}_{k} = 1 & & \forall k \in \mathcal{K}^{\xi}_{4}, \\
& y^{\xi}_{ikk_zt} + (1 - \Delta^{\xi}_{ikt}) \ge \delta_{ik_zt} & & \forall i \in \mathcal{I}, k, k_z, \in \mathcal{K}^{p\xi}_{4}, t \in \mathcal{T}, \\
& (1 - y^{\xi}_{ikk_zt}) + \Delta^{\xi}_{ik_jt_j} \ge 1 & & \forall i, k_j, k_z, t_j, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& (1 - y^{\xi}_{ik_jt_jk_zt_z}) \ge 1 - \delta_{ik_zt_z} & & \forall i, k_j, k_z, t_j, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& \gamma^{\xi}_{ikt} + \textstyle\sum_{k_z\in\mathcal{K}_1}\sum_{t_z\in\mathcal{T}} y^{\xi}_{iktk_zt_z} \ge 1 & & \forall i, k, k_z, t, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& (1 - \gamma^{\xi}_{ikt}) + (1 - y^{\xi}_{iktk_zt_z}) \ge 1 & & \forall i, k, t, k_z, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& \Gamma^{\xi}_{ik_zt_z} + (1 - y^{\xi}_{iktk_zt_z}) + (1 - \gamma^{\xi}_{ikt}) \ge 1 & & \forall i, k, t, k_z, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& (1 - \Gamma^{\xi}_{ik_zt}) + \textstyle\sum_{k\in\mathcal{K}^{\xi}_{4}}\sum_{t\in\mathcal{T}} y^{\xi}_{ikk_zt} \ge 1 & & \forall i, k_z, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& (1 - \Gamma^{\xi}_{ik_zt}) + \textstyle\sum_{k\in\mathcal{K}^{\xi}_{4}}\sum_{t\in\mathcal{T}} \gamma^{\xi}_{ikt} \ge 1 & & \forall i, k_z, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& \Omega^{\xi}_{k} \in \{0,1\} & & \forall k \in \mathcal{K}^{\xi}_{4} \\
& \Delta^{\xi}_{ikt} \in \{0,1\} & & \forall k \in \mathcal{K}^{\xi}_{4}, i \in \mathcal{I}, t \in \mathcal{T} \\
& y^{\xi}_{ikk_zt} \in \{0,1\} & & \forall i, k_j, k_z, t_j, t_z \in \mathcal{K}^{p\xi}_{4}, \\
& \gamma^{\xi}_{ikt} \in \{0,1\} & & \forall i \in \mathcal{I}, k \in \mathcal{K}^{\xi}_{4}, t \in \mathcal{T} \\
& \Gamma^{\xi}_{ikt} \in \{0,1\} & & \forall i \in \mathcal{I}, k \in \mathcal{K}, t \in \mathcal{T}
\end{aligned} \tag{21}
$$

The end result of the model and constraints in Equation (21) is a single schedule that maximizes the expected information gain across all potential scenarios. While the first stage ensures that a feasible schedule is constructed, the second stage computes schedule performance once an *ad hoc* collection scenario is realized. The second stage assumes a fixed first-stage schedule. More specifically, once an *ad hoc* collection scenario is realized, two things occur. Conflicting Category 2 or 3 collections, i.e. overlapping in time, are removed from the schedule. Schedule performance is updated to account for the addition of a Category 4 collection and the subtraction of overlapping collections. At the same time, Category 4 collections are prevented from being scheduled when a Category 1 collection is scheduled. To prevent infeasible scenarios, *a priori* information about Category 1 collections is used in the scenario generation step to prevent conflicting scenarios.

3. DISCUSSION

Work investigating the suitability of the proposed model continues. Preliminary results for a related SMIP suggest that exact branch-and-bound solution methods scheduling 24-hours of two- and three-sensor constellations can produce optimal or near-optimal schedules within operational timeframes (i.e., on the order of minutes and generally less than 20 minutes.) In these experiments, tens and hundreds of scenarios have been considered and schedules consistently outperform those generated without proactively scheduling for uncertainty. Ongoing research continues to focus on careful definition of *ad hoc* collection scenarios through mining of historical event data and conversations with sensor operators and planners to incorporate subject matter expertise. To expedite exploration of model constraint and objective changes as well as to elicit planner feedback, an existing prototype schedule visualization application remains under development.

Future work will investigate SMIP heuristics to reduce the time required to produce near-optimal schedules proactive to uncertainty. This work will be critical for producing schedules that maximize information gained for larger constellations of sensors. The proposed models push much of the complexity of generating collection windows and establishing the quality of individual collections into pre-processing steps. Model extensions are being investigated that will reduce this complexity by allowing collections to be accomplished using multiple configurations. The perceived reduction in complexity may come at the cost of additional preprocessing steps and modeling requirements.

4. CONCLUSION

We have presented a model for reducing the impact of ad hoc collections on sensor and constellations schedule utility. Related models utilizing stochastic mixed-integer linear programs and associated solution techniques have consistently produced schedules that gain significantly more information than deterministic models and algorithms. This model also allows collections of the same configuration to run concurrently, a higher-fidelity model for many sensors. Future work will include acquiring realistic scenario data to perform experiments investigating performance of the presented model over current, deterministic techniques.

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