Harnessing Orbital Debris to Sense the Space Environment

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ABSTRACT

A key requirement for accurate space situational awareness (SSA) is knowledge of the non-conservative forces that act on space objects. These effects vary temporally and spatially, driven by the dynamical behavior of space weather. Existing SSA algorithms adjust space weather models based on observations of calibration satellites. However, lack of sufficient data and mismodeling of non-conservative forces cause inaccuracies in space object motion prediction. The uncontrolled nature of debris makes it particularly sensitive to the variations in space weather. Our research takes advantage of this behavior by inverting observations of debris objects to infer the space environment parameters causing their motion. In addition, this research will produce more accurate predictions of the motion of debris objects.

The hypothesis of this research is that it is possible to utilize a "cluster" of debris objects, objects within relatively close proximity of each other, to sense their local environment. We focus on deriving parameters of an atmospheric density model to more precisely predict the drag force on LEO objects. An Ensemble Kalman Filter (EnKF) is used for assimilation; the prior ensemble to the posterior ensemble is transformed during the measurement update in a manner that does not require inversion of large matrices. A prior ensemble is utilized to empirically determine the nonlinear relationship between measurements and density parameters. The filter estimates an extended state that includes position and velocity of the debris object, and atmospheric density parameters. The density is parameterized as a grid of values, distributed by latitude and local sidereal time over a spherical shell encompassing Earth. This research focuses on LEO object motion, but it can also be extended to additional orbital regimes for observation and refinement of magnetic field and solar radiation models. An observability analysis of the proposed approach is presented in terms of the measurement cadence necessary to estimate the local space environment.

1. INTRODUCTION

Space plays an essential role in many aspects of daily life, such as phone calls, GPS navigation, and ATM transactions. Debris in Earth's orbit poses a significant and increasing threat to the critical space systems that make these capabilities possible. The US invests millions of dollars annually on SSA in an attempt to prevent loss, disruption, or degradation of space services and capabilities [1]. A key component of SSA is knowledge of the non-conservative forces acting on debris; these forces are driven by the dynamical behavior of space weather.

Existing systems adjust space weather models based on observations of well-understood, regularly tracked calibration satellites [2]. Of all catalogued space objects, 95% are rocket bodies, inactive satellites, or debris, yet their data are still not used for the benefit of updating and adjusting space weather models [3]. Moreover, the uncontrolled nature of debris makes it particularly sensitive to the variations of space weather. Our research takes advantage of this behavior by using observations of debris objects as an untapped source of information to infer the space environment conditions causing their motion.

The hypothesis of this research is that it is possible to utilize a "cluster" of debris objects (objects within relatively close proximity) to sense their local space environment, regardless of how much we know about them individually. Since it can be assumed that a "cluster" of debris encounter the same underlying space weather influences, a multi-object filter can be used to extract the underlying non-conservative forces without knowing precise details of the individual debris. The end goal is a data assimilation framework capable of tolerating high dimensional systems with highly nonlinear dynamics and sparse observations on specific objects, while also leveraging all available observations.

We focus on the low Earth orbital regime where mismodeling of atmospheric drag is the largest contributor to orbit prediction error. However, the methods developed are extendable to additional orbital regimes for refinement of magnetic field and solar radiation models. The initial work presented here explores a reduced version of the problem in which we use a single object filter to estimate atmospheric density parameters, as well as the position and velocity of a debris object.
1.1 DENSITY

An object in Earth orbit, particularly Low Earth Orbit (LEO), experiences atmospheric drag caused by particles in the atmosphere colliding with the surface of the object. Drag acts in the opposite direction of the velocity vector and effectively decreases the acceleration of an object. The magnitude of the force due to drag is directly dependent on neutral density (number of particles in the atmosphere). This is illustrated in the equation of acceleration due to drag (Eq. 1).

\[ \ddot{a}_{\text{drag}} = -\frac{1}{2} \rho \frac{C_D A}{m} v_{\text{rel}}^2 \left| \frac{\vec{v}_{\text{rel}}}{v_{\text{rel}}} \right| \]  

(1)

Atmospheric density is highly dynamic and depends on a number of things, including solar cycle, diurnal cycle, geomagnetic storms, altitude, and latitude. This dynamical behavior occasionally causes vast differences between the model and the true density, leading to relatively large errors in orbit prediction. Our research aims to estimate the atmospheric density and currently considers variability due to the diurnal cycle and latitude.

1.2 PRIOR WORK

Existing research efforts that estimate or model atmospheric density have taken a different approach to ours. For example, the High Accuracy Satellite Drag Model (HASDM) project aims to estimate a time-series of thirteen spherical harmonic global density correction coefficients \([4-5]\). This is achieved by using observations of 75-80 carefully selected calibration satellites (payloads and debris) in a batch fit. Intensive sensor tasking is made available for this effort, which allows for the collection of approximately 500 observations per day per calibration satellite. The batch fit solved for both temperature and density correction coefficients. HASDM decoupled the ballistic coefficient from the density parameter by first solving for the “true” ballistic coefficient of each satellite. This value was computed by averaging almost 3200 previously estimated ballistic coefficients of each calibration satellite.

The Direct Density Correction Method (DDCM) project took a slightly different approach by using TLEs of sixteen well-known objects \([6-7]\). DDCM estimated two time-series density correction coefficients for both the MSIS and GOST density models. This method included secondary data processing in which smoothed orbits of each object and smoothed ballistic coefficients were estimated. A drawback of the DDCM effort is that only long-period variations in the density were observable because of the availability of only daily ballistic coefficient estimates and a seven-day secondary data processing interval.

Overall, these methods estimate coefficients of density that are applied to a pre-defined model, not density directly. These approaches also do not allow for the resolution necessary to capture the dynamical behavior of density because of the averaging inherent to spherical harmonic coefficients. Combining information from objects in different regimes, or at different altitudes, in a single batch fit also decreases the spatial resolution of density corrections.

Unlike previous work, our research estimates density directly and updates density estimates with only local information from debris objects nearby. This work also aims to take advantage of all debris objects, instead of a small, handpicked portion of the RSO population. This approach allows for a higher resolution density estimate that has not been inherently averaged. The resolution of our density estimate is dependent on its spatial parameterization; details of the current parameterization scheme are found in Section 2.2.

It is recognized that if the physical characteristics of a debris object are unknown, the decoupling between the ballistic coefficient and density terms is not possible, and therefore the density, alone, cannot be estimated. Our research aims to utilize all debris objects, regardless of how much we know about them individually. Therefore, we are unable to use decoupling approaches similar to those described above, which use a priori knowledge of the objects. To resolve this, we plan to leverage a density calibration tool, such as HASDM, to provide initial density estimates. These estimates will be combined with tracking information of debris objects to solve for object ballistic coefficients, initially. Once this “bootstrapping” process is complete, our density estimation method can begin using the recently solved for ballistic coefficients.
2. METHOD

There are two primary aspects of our approach to density estimation that set our work apart from previous density estimation research: the type of filter and the parameterization of density. Both of these features are discussed in this section.

2.1 EnKF

For this work, we use the Ensemble Kalman Filter for data assimilation. The EnKF is an unconventional filter for this problem because it is typically used in high-dimensional non-linear geophysical applications, such as weather forecasting of atmosphere and ocean systems, where there is an abundance of observations [8]. Alternatively, this research applies the EnKF to a combination of a large-scale problem and a smaller scale problem. The latter is a typical orbit determination (OD) problem in which the position and velocity in Earth Centered Inertial coordinates are estimated. The large-scale portion is a high-dimensional geospatial estimation problem of the density field. This section introduces the details of a traditional EnKF and then the application of the EnKF to our unique problem is described in Section 3.2.

The EnKF employs a Monte Carlo method in the form of an ensemble representation of the probability distribution of the estimated state. The ensemble contains N members and each member is a sample realization of the state that is assumed to be normally distributed. Combined, the ensemble members represent an a priori distribution of the state, and hence, define the initial mean and variance of each state element. An ensemble representation of the initial state is generated using the a priori statistical information of the state X as shown in Eqs. 2 and 3.

\[
X_{0}^{(i)} = \tilde{X}_{0} + \eta
\]

\[
\eta \sim \mathcal{N}(0, P_{0})
\]

The time update of the state in the EnKF is the nonlinear propagation of the state ensemble (Eq. 4), where \(\mathcal{M}\) is the nonlinear operator. Superscript \(i\) denotes the \(i^{th}\) ensemble member and +/- denotes the posterior/a priori distribution. Subscript \(k\) indicates the \(k^{th}\) time step. The covariance time update is given by Eqs. 5 and 6.

\[
X_{k}^{(i)-} = \mathcal{M} (X_{k-1}^{(i)+})
\]

\[
\bar{x}_{k} = \frac{1}{N} \sum_{i=1}^{N} X_{k}^{(i)-}
\]

\[
P_{k}^{-} = \frac{1}{N-1} \sum_{i=1}^{N} (X_{k}^{(i)-} - \bar{x}_{k}) (X_{k}^{(i)-} - \bar{x}_{k})^{T} + Q
\]

The measurement update equations can be written as Eqs. 7-14. The function, G, is the measurement equation as a function of the state, \(\mathcal{Y}\) is the state in measurement space, and \(R\) is the measurement covariance matrix. Perturbed observations, \(y^{o}\), are used for the state measurement update in Eq. 14.

\[
y_{k}^{(i)} = G (X_{k}^{(i)+})
\]

\[
\bar{y}_{k} = \frac{1}{N} \sum_{i=1}^{N} y_{k}^{(i)}
\]

\[
P_{yy} = R + \frac{1}{N-1} \sum_{i=1}^{N} (y_{k}^{(i)} - \bar{y}_{k}) (y_{k}^{(i)} - \bar{y}_{k})^{T}
\]

\[
P_{xy} = \frac{1}{N-1} \sum_{i=1}^{N} (X_{k}^{(i)-} - \bar{x}_{k}) (y_{k}^{(i)} - \bar{y}_{k})^{T}
\]

\[
K = P_{xy} (P_{yy})^{-1}
\]

\[
y_{k}^{o} = \bar{y}_{k} + \varepsilon
\]
\[ \varepsilon \sim \mathcal{N}(0, R) \]  
\[ X^{(0)*}_k = X^{(0)*}_k + K (y^{o}_k - Y^{(0)}_k) \]

The EnKF extracts the posterior state covariance (Eq. 15) from the posterior distribution represented by the ensemble, similar to the Unscented Kalman Filter or a Monte Carlo approach. The inverse operation of \( G \) is implemented by linear regression using the prior ensemble, which transforms the posterior ensemble expressed in the measurement space to state space

\[ P^{+}_k = (I - KH) P^{-}_k (I - KH)^T + KRR^T \]  
\[ H = \frac{\partial y}{\partial x} \]

There are various formulations of the EnKF; for this work, the stochastic formulation is used (described above). The stochastic EnKF treats observations as random variables and applies perturbations to them in the measurement update (Eq. 12); this approach was first introduced by Houtekamer and Mitchell [9] and Burgers et al. [10]. Another popular flavor of the EnKF is the deterministic EnKF, also known as the Ensemble Square Root Filter (EnSRF). In the EnSRF, perturbations are not applied to the observations and instead, the Kalman gain is modified (Eqs. 17 and 18) so that effects of sampling errors associated with the perturbed observations are minimized [11].

\[ \hat{R} = \alpha K \]  
\[ \alpha = \left(1 + \frac{R}{\sqrt{HPP^T + \hat{R}}} \right)^{-1} \]

A more detailed description of the EnKF implementation for this application is discussed in Section 3.2 after the estimated state is introduced, below.

\section*{2.2 ESTIMATED STATE}

As mentioned previously, there are essentially two estimated states in this scenario that are combined into one: the orbital debris position and velocity and the atmospheric density. It is noted that the trajectory of this research includes utilizing numerous debris objects in a multi-object filter; however, currently we present a single debris object scenario. The second portion of the estimated state is a spherical shell of neutral density (encompassing Earth) parameterized by Local Sidereal Time (LST) and latitude; these parameters define a spatial grid in the sun fixed coordinate frame. Both LST and latitude have a spatial resolution of five degrees. The resulting vector of densities, one at each grid point, contains 2701 elements, yielding a total of 2707 estimated elements when combined with the OD state.

The grid of density elements is defined with respect to the sun, and therefore rotates with the sun and not with the Earth. Hence, one of the major contributors of density variability, the diurnal cycle, is inherently accounted for.

\section*{2.3 INITIAL ENSEMBLE GENERATION}

The method for generating a randomly distributed ensemble is different for the two portions of our estimated state. They are described separately in Sections 2.3.3 and 2.3.4. The number of ensemble members, \( N \), was determined via a Monte Carlo study of the density state distribution as a function of \( N \). It was determined that a 450-member ensemble is sufficient for the density state representation; the OD portion of the problem replicates this and also has 450 members per ensemble.

\subsection*{2.3.3 ORBIT DETERMINATION STATE ENSEMBLE}

In order to generate the ensemble members for the position and velocity portion of the state, we first perturb the true initial state (Eq. 19). This is done so that the filter does not begin with perfect information about the state.
\[
\hat{x}_{0,OD} = [\hat{R}_0] = [\hat{R}_{0,\text{truth}}] + \sqrt{\sigma_0}
\]

For this scenario, we applied a perturbation of 100 meters and .1 meters per second in position and velocity, respectively. The applied perturbations are equal to the standard deviations of the corresponding elements in the covariance matrix. Thus, the initial covariance matrix is defined as follows in Eq. 20.

\[
P_0 = \text{diag}(\{\sigma_x, \sigma_y, \sigma_z, \sigma_x, \sigma_y, \sigma_z\}) =
\]

\[
\text{diag}(\{100m^2, 100m^2, 100m^2, (1m/s)^2, (1m/s)^2, (1m/s)^2\})
\]

Next, a distribution of the initial OD state \((\hat{x}_{0,0})\) is randomly sampled 450 times using Eqs. 2 and 3.

### 2.3.4 DENSITY STATE ENSEMBLE

The density ensemble is generated with a different approach because an appropriate covariance of the density random variable is unknown. In this case, a model is used to generate density values; we use the MSIS Atmosphere Model [12]. The MSIS model generates a density value when provided with various input parameters. These parameters include latitude, LST, altitude, day of year (DOY), F10.7 index, and the Ap index, amongst others. Latitude and LST are inherently defined for each value of the density state due to the parameterization scheme. Orbit altitude is a constant in this scenario because we simulate/estimate a circular orbit. June 24th is chosen as the date of the simulation (DOY = 175), while 80 and four are used for the F10.7 index and Ap index, respectively. A single member of the ensemble can be generated with these inputs. In order to generate multiple members, we begin by defining an array of normally distributed values for the latter three variables (DOY, F10.7 index, and Ap index).

DOY = [169, 171, 173, 175, 177, 179, 181]
F10.7 = [74, 76, 78, 80, 82, 84, 86]
Ap index = [3.6, 3.8, 4, 4.2, 4.4]

Each parameter array is randomly sampled 450 times to generate 450 combinations of input to MSIS; each combination produces a density ensemble member. Varying these parameters represents a range of possible density behavior, similar to how an ensemble of debris position should encompass the majority of possible true positions.

### 3. SIMULATION

This section describes the synthetic measurements that are assimilated and the details of their generation. The intricacies involved in the implementation of the EnKF for this scenario are also discussed.

#### 3.1 MEASUREMENTS

The debris object simulated for this project is in a circular orbit with an inclination of 40° and an altitude of 400 kilometers. The object trajectory is simulated for five orbital periods for the measurement generation portion of this project. The ground-track of the simulated debris object orbit, that the measurements are generated from, is shown in Fig. 1. Yellow stars represent the two stations from which azimuth (β), elevation (el), and range (r) measurements are collected. The stations are located in California, USA and Madrid, Spain, signified by station 1 and station 2, respectively hereafter.
A cadence of 10 seconds is used for the measurements. Random noise is added to the synthetic measurements with a standard deviation of five arcseconds for the azimuth and elevation, and .1 meters for range.

\[
\vec{y} = [\beta \ e \ l \ r]^T \\
\vec{y} = \vec{y}_{true} + \varepsilon \\
\varepsilon \sim \mathcal{N}(0,R)
\]

\[
R = diag([\sigma_{\beta} \ \sigma_{e} \ \sigma_{r}]) = diag([\text{5 arcseconds}^2 \ \text{(5 arcseconds)}^2 \ \text{(0.1m)}^2])
\]

The synthetic measurements are then passed to the EnKF for data assimilation.

### 3.2 DATA ASSIMILATION

The formulation of the EnKF for this scenario varies slightly from that which was described in Section 2.1. This section will begin by describing assumptions of this scenario, and then the overall application of the EnKF to this scenario is described, as well as any departures from the nominal EnKF and why they are valid.

The ballistic coefficient is assumed known in the filter in order to allow for decoupled estimation of density. An explanation for why this is realistic is found in Section 1.2. At any given epoch/time, the estimated state only includes the debris object position and velocity, and the density for the latitude and LST corresponding to the current position of the debris object. Therefore, the filter operates on an ensemble of seven-element states at each epoch.

The time update step for the two portions of the estimated state is approached differently. The OD portion of the state undergoes a nonlinear propagation; alternatively, the density portion does not due to its stationary nature. The estimated density, corresponding to the current position of the debris object, is used in the OD state propagation dynamics. It is noted that the density used is from the location of the debris object before propagation to the time of the measurement. Essentially, at each measurement time, the density being estimated is from the beginning of the trajectory (previous measurement time); whereas, the position and velocity being estimated is at the current observation time. This process is necessary because the density that influences the debris object trajectory from point A to B is revealed by the measurement at point B. This is illustrated in Fig. 2.
The true dynamical model is used to propagate the OD portion of the state, aside from the density used in the acceleration due to drag (the estimated density); likewise, the initial density ensemble is also generated by its true model, MSIS. Hence, process noise is not necessary for this application of the EnKF, and Eq. 6 can be truncated to form Eq. 25. The measurement update follows Eqs. 7-16.

\[
P_k^+ = \frac{1}{N-1} \sum_{i=1}^N (\chi_k^{(i)-} - \bar{x}_k) (\chi_k^{(i)-} - \bar{x}_k)^T
\]

(25)

In the measurement update, a correction to the density estimate is computed. This correction is applied to the density estimate corresponding to a single location on the spatial grid (defined by LST and latitude) in Eq. 26. However, this density correction is reflective of not only a single point on the spatial grid, but also, the density at nearby locations. Therefore, we apply the density correction to all density estimates within a radius of 15 degrees to the primary density estimate location. A ratio of the correction that is applied is defined by an exponential function (C), dependent on distance from the primary density; this is illustrated in Eq. 27.

\[
\chi_k^{(i)+} = \chi_k^{(i)-} + K (y_k^o - \chi_k^{(i)}) = \chi_k^{(i)-} + K \Delta y
\]

(26)

\[
\chi_k^{(i)+} = \chi_k^{(i)-} + C K \Delta y
\]

(27)

It is noted that the correlation between density points is not considered in Eq. 27. Future work includes extending the state to include all density elements; this will enable such information to be utilized in the measurement update of surrounding density estimates. Section 5 expands upon this approach.

4. RESULTS

In this Section, we will review the results of the simulation described in Section 3. To begin, the results for the first orbital period of the simulation are presented in order to analyze the smaller scale features of the filter performance. Then, overall results of the full five orbit period simulation are discussed.

4.1 SINGLE ORBITAL PERIOD RESULTS

Fig. 3 and Fig. 4 show the post-fit residuals of the azimuth and range measurements. The post-fit residuals represent the error between the realized measurement, computed from the updated state, and the observed measurement. These figures also indicate a covariance bound for the measurement error. This bound is a 3-sigma depiction of the measurement noise added to the synthetic measurements within the generation. As desired, the post-fit residuals stay within these bounds for the duration of the simulation, aside from the occasional outlier.
The OD state errors are calculated with the true trajectory, obtained during the measurement generation, and the density errors are calculated with the true density generated by MSIS. First we compare the x-direction position errors and its 3-sigma covariance bounds (Fig. 5). The covariance bounds are computed from the posterior covariance at each time step.

The x-direction position error is relatively small with an overall root mean squared (RMS) error of just .0058 kilometers; the error also stays within the covariance bounds for the duration of the orbital period. However, the
error does approach the covariance bound just after 60 minutes into the simulation. This corresponds to the point in the orbit where the density is greatest and, as discussed below, the density is also underestimated in the filter during this time. This combination is likely the cause of the deviation in the x-direction position estimate.

We will now focus on the density estimation results, beginning with the density estimate error. Fig. 6 shows the error between the density estimate and the true density, produced by MSIS. The density estimate deviates from the truth and approaches the covariance bounds near 50 minutes into the simulation. To investigate this deviation, we consider density's dependence on latitude and LST. Fig. 7 and Fig. 8 show the latitude and LST of the density element estimated at each time.

Fig. 6. Density Error & 3σ Covariance Envelope

Fig. 7. Orbital Debris Latitude

Fig. 8. Orbital Debris LST
It is evident that the deviations increase as the debris object passes over the equator, which also happens to be around the same time that the object approaches noon (LST = 12 hours). Independently, both of these situations produce enhanced local atmospheric density. Therefore, as the debris object approaches these conditions, the local density is growing at an increased rate. This dynamical behavior may cause lag in the filter, which may account for some of the error in this region. The percent error in the estimated density shown in Fig. 9 also demonstrates the relatively large error in this region, but also shows the return to lower percentage errors, afterwards.

![Percent Error of Estimated Density](image)

**Fig. 9. Percent Error of Estimated Density**

A rank histogram is used to demonstrate the ensemble behavior and to determine if the ensemble distribution includes the truth. A uniform distribution is desired in a rank histogram; such a distribution means there is consistency between the ensemble and truth distribution, and therefore the ensemble is unbiased. Fig. 10 reveals that our ensemble is biased, i.e., the truth is outside of the ensemble roughly 45 times during the simulation. It is discovered that the majority of the biased occurrences take place between 40 and 60 minutes where there is also the greatest error in the density estimate.

![Density Ensemble Rank Histogram](image)

**Fig. 10. Density Ensemble Rank Histogram**

Because the initial density ensemble has noise added to it, the filter encounters density elements that are not representative of the truth at each new location of the grid. It is desirable to let the filter update the initial grid of density estimates, along the debris object trajectory, and then assess the performance of the filter. However, it is noted that if the debris object does not travel through the same spatial grid locations, it will again encounter density elements that have not yet experienced a measurement update. This demonstrates the necessity to utilize multiple debris objects in order to have objects traveling through the majority of the spatial grid; which enables a shorter spin-up time. This is discussed further in Section 5.

### 4.2 FULL SIMULATION RESULTS
A longer simulation of five orbital periods demonstrates the ability of the filter to improve upon density estimates with additional passes. Fig. 11 and Fig. 12 show that the orbital debris trajectory is not perturbed by density so much that it doesn’t pass through the same spatial grid points. There are no evident deviations from the original trajectory, so we can expect to see improvement in the density estimate with additional orbits.

The estimated density is compared to the true density in Fig. 13. The diurnal affect is evident in this figure; the debris object passes through local midnight and local noon with every revolution (roughly 90 minutes). Fig. 14 is the percentage error between the density estimate and the truth as a function of time. The percentage error in the density estimate decreases with each orbital debris pass, as expected. Fig. 15 shows the corresponding density estimate error and its 3-sigma covariance bounds. There is not a noticeable decrease in the covariance bounds because of the formulation of the filter, at this time. Because the state contains only seven elements at any one time, each time a new density element is approached, this new density element replaces the previous density element in the state vector. When this occurs, the covariance information of the previous density element is lost, and thus, this formulation does not allow for the covariance to decrease over time. The only period of time for which the density covariance can decrease is while the debris object is spatially near the corresponding density, which is typically for only a few time steps. This causes the slight fluctuations seen in the covariance, but allows for no overall reduction. Extending the state vector to include all density elements should resolve this.
Fig. 13. Estimated v. MSIS/True Density

Fig. 14. Percent Error of Estimated Density

Fig. 15. Density Error & 3σ Covariance Envelope

Fig. 16 reveals that the position estimate still has a relatively large deviation from the truth during the times when the density estimate also has a high percent error. Regardless, the x-direction position error still decreases as a result of multiple passes. It is noted that the error does exceed the covariance bounds, making the covariance an inaccurate representation of the true error. The cause of this is still being investigated.
5. CONCLUSIONS & FUTURE WORK

Overall, the single orbit and multi-orbit sets of results demonstrate the plausibility of this particular density estimation method. Obviously, the entire density state is not observable because only the information from one debris object is being used. Other weaknesses of the current implementation and formulation were also discussed, but regardless of those disadvantages, the EnKF is still able to maintain a density estimate relatively close to the truth. Further investigation into the causes of particular features of the results must be done, but there is a clear idea of the steps that will be taken moving forward with this research.

The next step with this work is to extend the state vector to include all density elements so that there is a 2707-element state instead of just a seven-element state. This change allows for the information about the correlations between density elements at different locations to be considered in the measurement update. Also, an observability analysis can be performed with the complete state because the information matrix can be accumulated and analyzed; this feature, alone, will be very useful for this research.

A different form of localization will have to be implemented with the augmentation of the state so that the affect of an update is limited to a certain region spatially close to an observation; we plan to implement Gaspari and Cohn’s formulation of localization [13]. This is necessary because the covariance estimated from the ensemble is likely subject to sampling errors; this causes spurious correlations of the density portion of the state that are located in areas far from the debris object observation. Localization of the covariance is a widely used approach to rectify detrimental effects of sampling errors influencing an EnKF measurement update in this way.

After extending the state, we plan to implement a multi-object EnKF and will most likely chose the Labeled Multi-Bernoulli filter. The primary advantage of a multi-object filter is having multiple debris objects in different types of orbits so that more of the density elements on the spatial grid are observable. It also enables a debris object to take advantage of updated density estimates from other debris objects that previously passed through the same area. A high measurement cadence was required for the results shared above, but an addition of other objects should alleviate the necessity of such a high cadence. The idea being that the more objects made available to gather information from, the more sporadic the measurements can be. Therefore making the scenario more realistic and similar to current SSN tasking of debris objects.

Another change we wish to make is to use a different model (i.e., Jacchia Reference Atmosphere) to generate the initial density ensemble than the one used in the truth and measurement generation. This is similar to perturbing the initial position and velocity state; the EnKF will begin with an a priori distribution that does not have a mean of the truth. Such a change will create a more challenging problem and will therefore be implemented after the shift to a multi-object filter so that knowledge from multiple objects is available to increase the observability of the state.

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7. REFERENCES


