Aperture Efficiency and Wide Field-of-View Optical Systems

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Abstract

Wide field-of-view optical systems are currently finding significant use for applications ranging from exoplanet search to space situational awareness. Systems ranging from small camera lenses to the 8.4-meter Large Synoptic Survey Telescope are designed to image large areas of the sky with increased search rate and scientific utility. An interesting issue with wide-field systems is the known compromises in aperture efficiency. They either use only a fraction of the available aperture or have optical elements with diameters larger than the optical aperture of the system. In either case, the complete aperture of the largest optical component is not fully utilized for any given field point within an image. System costs are driven by optical diameter (not aperture), focal length, optical complexity, and field-of-view. It is important to understand the optical design trade space and how cost, performance, and physical characteristics are influenced by various observing requirements. This paper examines the aperture efficiency of refracting and reflecting systems with one, two and three mirrors.

Introduction

Optical systems exhibit an apparent falloff of image intensity from the center to edge of the image field as seen in Figure 1. This phenomenon, known as *vignetting,* results when the entrance pupil is viewed from an off-axis point in either object or image space. The apparent area of the entrance pupil decreases with field angle and the amount of light passing through the aperture also decreases.

Figure 1. An Example of Vignetting [1]

For very simple optical systems—such as a camera with a single element lens, a two-element achromatic or apochromatic (APO) refracting telescope—the aperture stop and entrance pupil are the same and are collocated with the optical elements. As a result, the size of the entrance pupil is equal to the size of the optical elements. The system still experiences slight vignetting (based on geometry), but the ray bundle reaching the edge of the field is approximately the same size as that reaching the center of the field as illustrated in Figure 2.

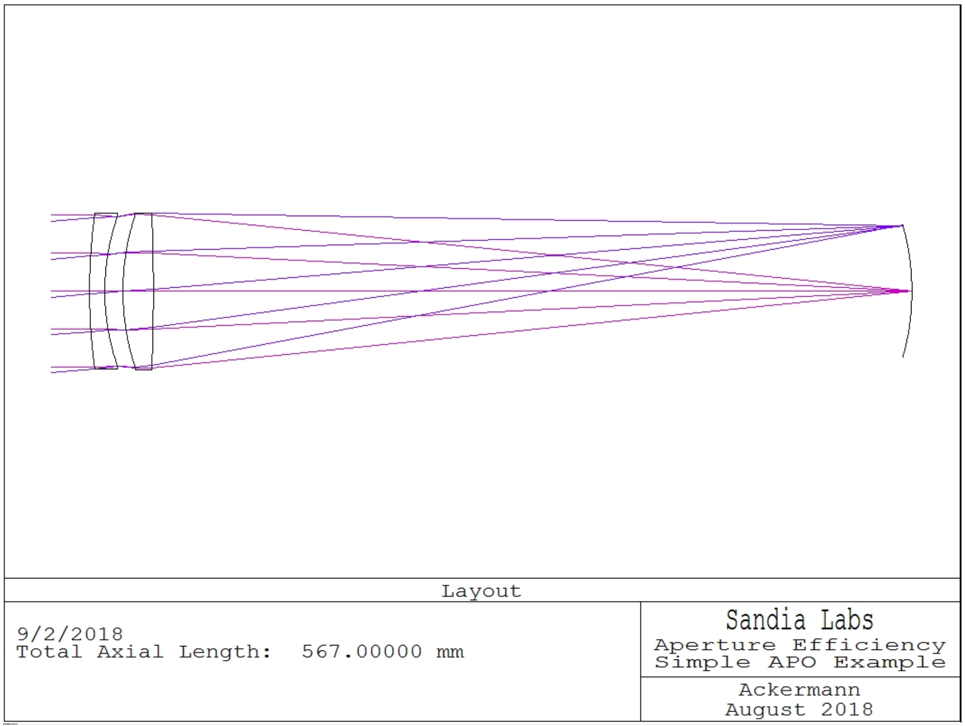

Figure 2. Simple APO Refractor Example Exhibiting Only Geometric Vignetting

The aperture stop can be buried deep *within* the system on more complex optical systems, such as a Cassegrain telescope or a double Gauss lens. Here, the relationship between the physical size of the aperture stop, the entrance pupil and the optical elements becomes more complicated. A generic double Gauss lens is shown in Figure 3. Note how the ray bundle for the field point at the edge of the image is smaller in size than the ray bundle reaching the center of the image. This is an indication of design-induced vignetting. While it is possible to design a double Gauss lens so that it does not exhibit off-axis vignetting, this practice can significantly complicate the design and reduce image quality. Vignetting is commonly used to reduce the field-dependent aberrations from off-axis points, thus improving image quality and producing a design that is more compact and lighter weight.

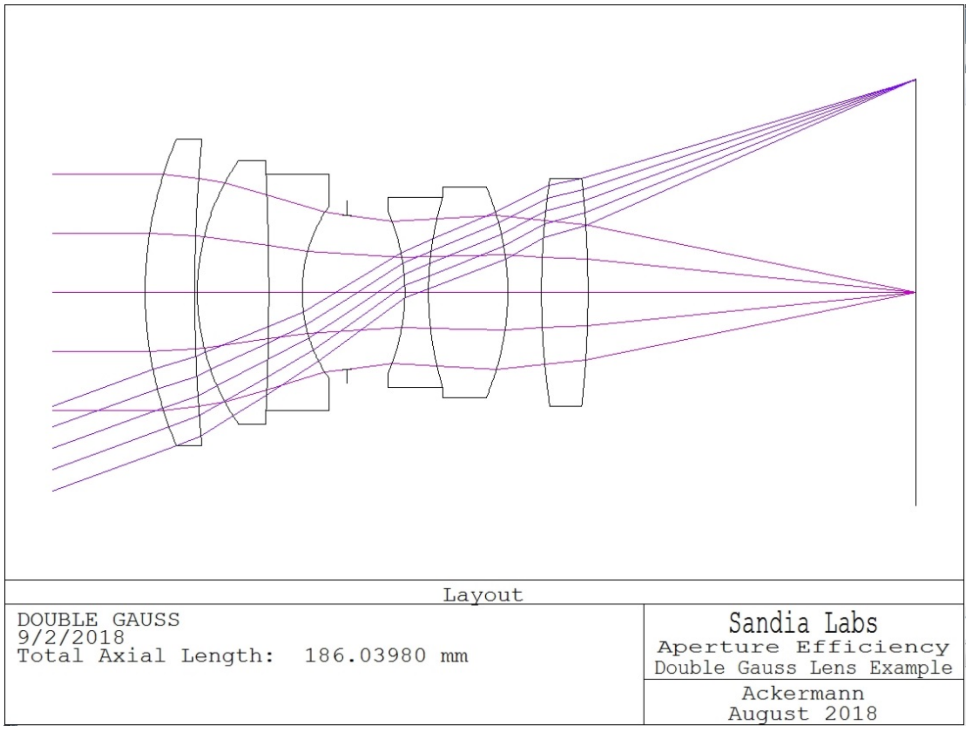

Figure 3. Double Gauss Type Lens Exhibiting Off-axis Vignetting

An optical system that is designed for single point photometry only needs to collect photons corresponding to an on-axis image location. In this case, the size of the entrance pupil and the size of the largest optical element are comparable for a well-designed system. When off-axis image points are required, then one of two conditions occurs. If the largest optical elements are of the same size as the on-axis entrance pupil, the system will experience vignetting in excess of the simple geometric light falloff described previously and seen in Figure 3. This is design-induced vignetting. If the system is designed to eliminate vignetting, then the size of the largest optical elements will exceed that of the entrance pupil. This condition is seen with the *unvignetted* double Gauss design shown in Figure 4. This is the exact same lens design as used in Figure 3, only with the optical elements sized to prevent vignetting. Note that the ray bundles for both the center of field and the edge of field in Figure 4 are essentially the same size. Also, note how for each field point, only a fraction of the front lens is used to form the image. In either case—for designs with or without vignetting—the optical system exhibits an aperture efficiency less than unity.

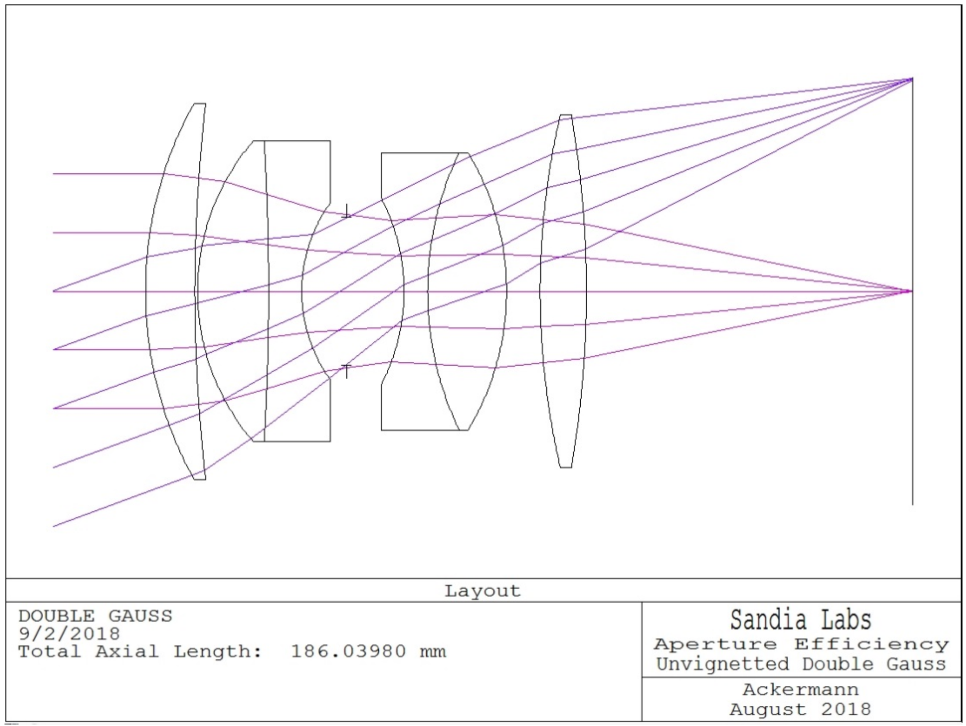

Figure 4. Double Gauss Type Lens Designed for No Off-axis Vignetting

Aperture Efficiency

The term *aperture efficiency* is not normally found in journals and texts on optical theory. Here aperture efficiency is defined as the product of the aperture diameter divided by the diameter of the largest optical element, squared, times the integrated field dependent vignetting function and obscuration function as seen in equation 1.

$$Aperture\ Efficiency = \left(\frac{aperture\ diameter}{largest\ optic\ diameter}\right)^2 * 2\pi \int_0^1 \rho V(\rho) O(\rho) d\rho \tag{1}$$

where ρ is the fractional field radius, $V(\rho)$ is the vignetting function, and $O(\rho)$ is the obscuration function.

<u>Refractive Systems</u>

Using equation 1, the authors calculated the aperture efficiency of the designs used for Figures 2, 3 and 4. For the simple APO refractor as seen in Figure 2, the aperture efficiency is 1.0. For the double Gauss lens shown in Figure 3, the aperture efficiency is 0.75, which results entirely from the lens' vignetting characteristics as shown in Figure 5. For the unvignetted double Gauss lens design seen in Figure 4, the aperture efficiency is calculated to be 0.39.

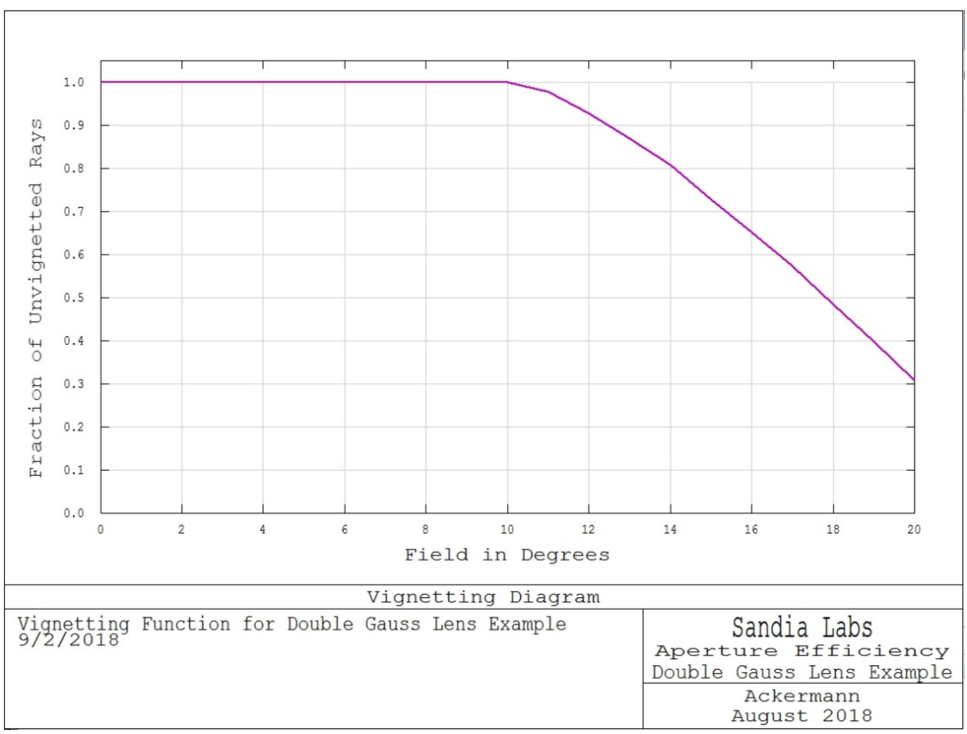

Figure 5. Vignetting Function for Double Gauss Lens Seen in Figure 3

Comparing the aperture efficiency for the designs seen in Figures 3 and 4 can be both enlightening and somewhat misleading. The image produced by the unvignetted design of Figure 4 will be brighter at the edges of the field than that produced by the vignetted design of Figure 3. However, the mass of glass required to realize Design 4 is more than 50% greater than for Design 3. Clearly, aperture efficiency impacts total system mass. However, the term *aperture efficiency* was chosen carefully. The aperture efficiency provides an indication of how photons striking the largest optical element contribute to the final image. For Design 4, only 40% of the photons that reach the front lens contribute to the final image.

All optical design challenges involve tradeoffs. Simply stated, the ideal optical system does not exist. Optical designers are required to trade one limitation against another to find the *optimum* combination of characteristics that provides the overall highest-level of performance within the requirements specified for the system. An issue that often arises is that project managers and optical designers tend to communicate concepts and requirements in different terms. If not carefully managed, this can lead to an important misunderstanding. Aperture efficiency is one of the areas where precise communication is essential.

Consider, for example, a real-world remote sensing project that requires a large relative aperture lens with a moderate field-of-view. Rather than design an expensive custom system, an optical designer might suggest the project manager use a commercial camera lens. An excellent example of relevance to space situational awareness is the Canon 135 mm focal length, f/2.0 lens as demonstrated by the JASON Advisory Group for detecting and tracking earth-orbiting satellites [2]. The Canon lens is relatively compact and lightweight. It is readily available from commercial sources. It is not too expensive and the image quality is more than acceptable for this application.

The optical diameter for the aperture of the Canon lens is 67.5 mm, the same as the diameter of its largest lens element. Project managers might think they are getting a high-speed, f/2.0 camera objective, but in reality, the lens only achieves that light throughput in the center of the field. Away from the center, light throughput—and consequently image brightness—fall off rapidly. As a result, the image will be significantly darker at the edges than at the center of the field. For certain applications, vignetting is unacceptable and the project manager might be dissatisfied with the overall results. On many modern photographic cameras, the camera body and lens communicate with one another, allowing the camera to recognize through what lens it is imaging. By knowing the vignetting characteristics of the lens in advance, the camera body can digitally brighten the edges of the image and make final image results appear as though they were obtained with a camera and lens that exhibited no vignetting. This approach works well for general photography, but for low-light remote sensing, the digital flat fielding only increases noise towards the edges of the image. Either way, the project manager might be dissatisfied with the results.

The impact of vignetting is a loss of aperture efficiency. In the case of the 135 mm f/2.0 lens, the aperture efficiency is 0.66. If one designs a similar lens without vignetting, the largest optical element grows from 67.5 mm diameter to 108.5 mm diameter and the optical efficiency is then calculated as 0.39. The commercial Canon design requires 10 optical elements with a total mass of 0.421 kg (glass only) while the custom design requires 11 elements with a total mass of 3.615 kg (glass only).

For lens systems, vignetting has its origins in the need to balance aberrations on either side of the aperture stop. This results in placing the aperture stop approximately in the middle of the optical system. The benefits have been known for decades, and include smaller diameter optical elements, shorter optical systems and reduced total mass. These come at the expense of uniform illumination and an aperture efficiency lower than unity. To produce an image of uniform relative illumination, the costs appear to be greater optical complexity (in the form of increased optical element count) and greater mass. Sometimes these systems also require greater physical length. Consider for example, the Canon 50 mm aperture, 100 mm focal length, f/2 commercial camera lens and the Russian 50 mm aperture, 100 mm focal length, f/2 custom lens used for space surveillance. The Canon lens has seen application for university-level sky survey research while the Russian lens provides wide-field sensing for two satellite-tracking facilities. The Canon lens, seen in Figure 6, uses 9 optical elements, is approximately 125 mm in length from the front lens to the focal surface and weighs 0.417 kg for the complete lens. This lens, however, vignettes towards the edge of the field as shown in Figure 7. The Russian lens seen in Figure 8 requires 14 optical elements and is 385 mm from front lens to focal plane. The glass alone has a mass in excess of 4 kg, but it appears not to vignette. This example, as well as the examples seen in Figures 3 and 4, suggest systems that vignette are smaller, lighter, less complex, and have lower optical complexity when compared to systems designed not to vignette.

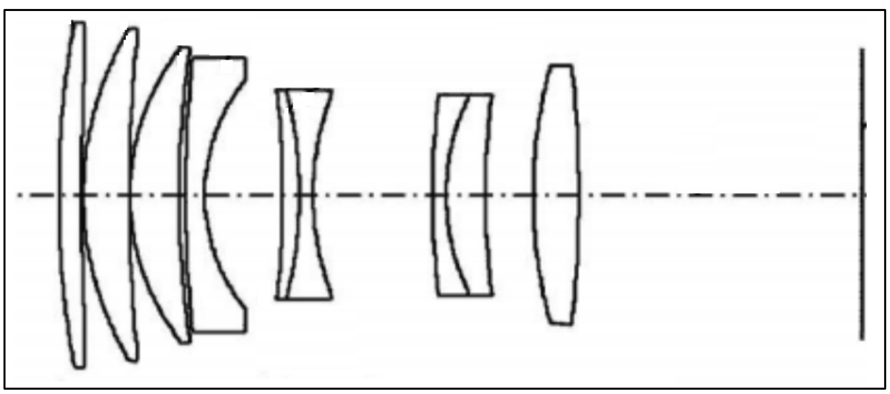

Figure 6. Optical Layout of Canon (50 mm) 100 mm f/2 Lens [3]

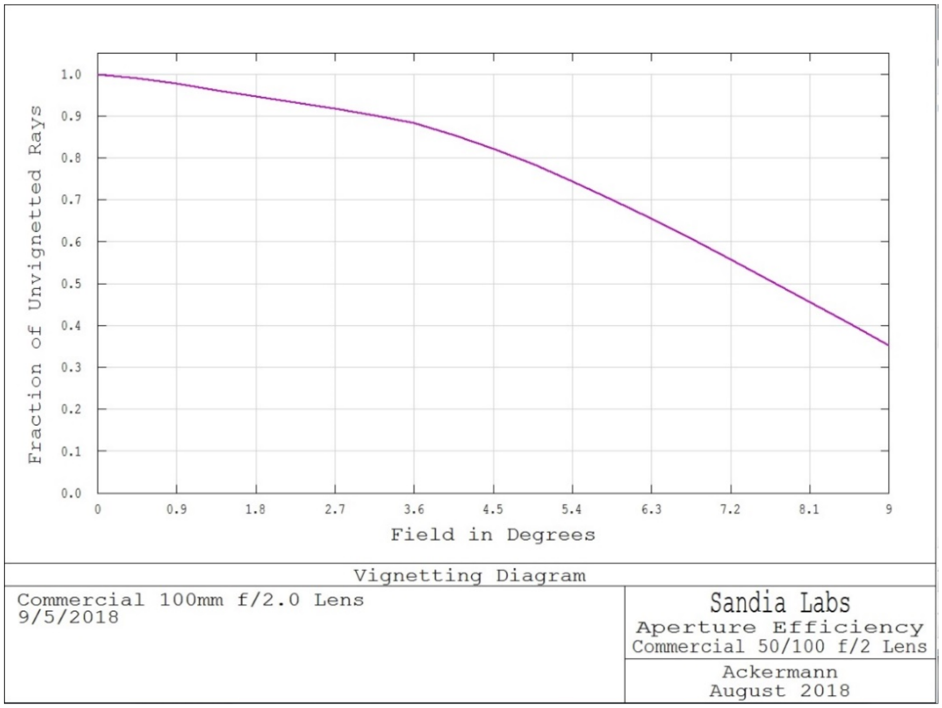

Figure 7. Approximate Vignetting Characteristics for Commercial 50 mm f/2 Lens

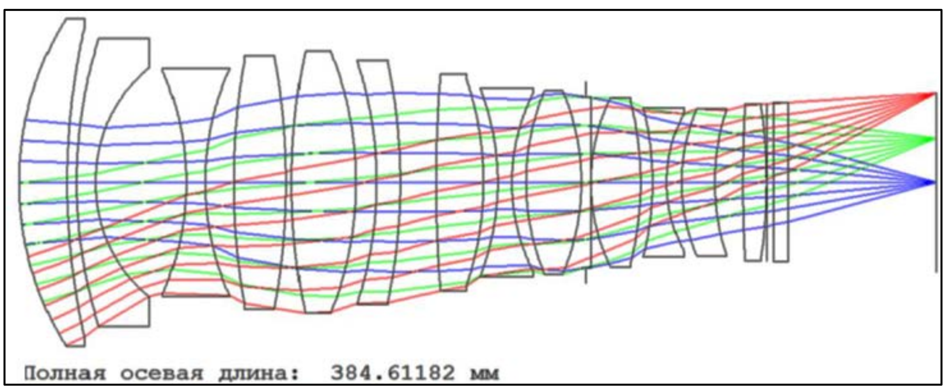

Figure 8. Optical Layout of Russian 50 mm (100 mm) f/2 Lens [4]

Reflective Systems

Only refractive systems have been examined in this paper at this point. For reflective systems, the story is similar, but with the added complication of obscuration. Traditionally for reflective systems, the aperture stop would be located on the largest optical element as seen in systems such as the classical Cassegrain, Newtonian and Gregorian reflectors. However, these were narrow field-of-view instruments with vignetting resulting only from inherent geometrical considerations. As early telescopes were used only for visual observation, the requirement to place the observer in a convenient position necessitated the use of secondary mirrors that partially obscured the primary mirror from incoming light, thereby reducing aperture efficiency.

A ray-trace diagram for a simple Cassegrain with a one-degree field-of-view is shown in Figure 9. The ray bundles reaching the center and edge of the field are almost identical indicating very little geometric vignetting, but the figure also shows a significant loss of light in the center caused by the secondary mirror obscuring a portion of the primary mirror. Using equation 1, the aperture efficiency for this Cassegrain telescope is calculated to be 0.86, which is good for this type of instrument. The fraction of incoming light obscured by the secondary mirror is a complex function of design parameters including focal ratio and field-of-view with no closed form equation known to the authors. In general, faster systems have greater obscuration than slower systems and those with wider fields-of-view have greater obscuration than those with a narrow field-of-view.

What is missing from the aperture efficiency value cited previously is the loss of photons resulting from the reflection of two mirrors. With each mirror having a reflectance of approximately 0.9, only 81% of the photons reaching the primary mirror will contribute to image intensity. While this is not technically a part of aperture efficiency, overall radiometric efficiency for reflecting telescopes is still important when evaluating radiometric performance.

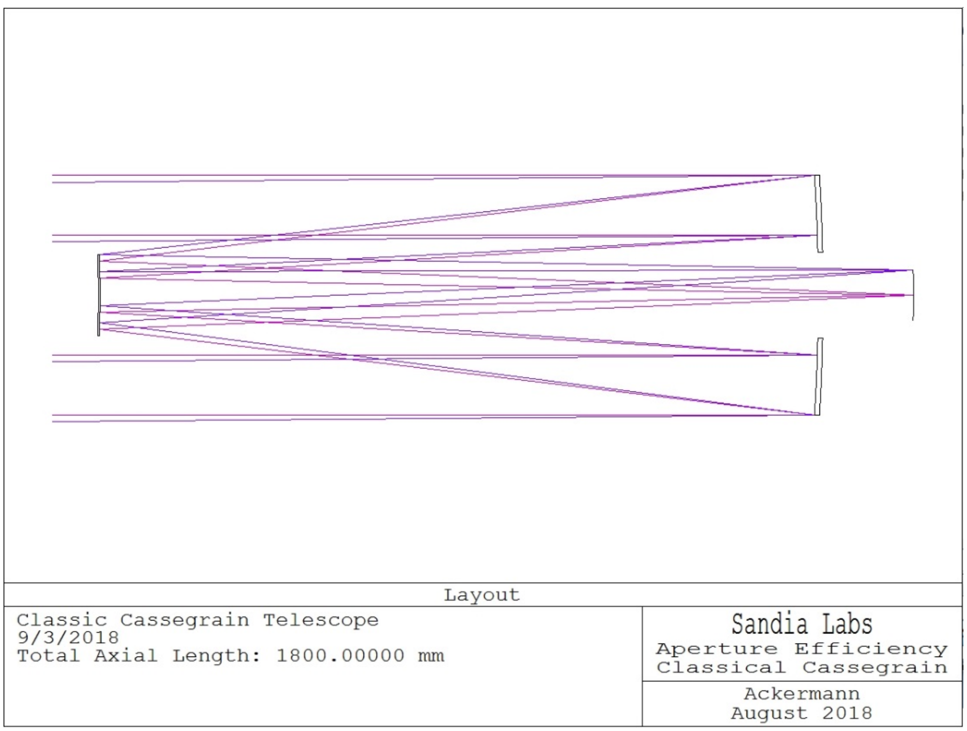

Figure 9. Classic Cassegrain Ray-trace Diagram

While the classic Cassegrain example is an interesting academic exercise, it is instructive to consider additional complex examples to understand better the importance of aperture efficiency. For this purpose, the authors chose to examine the Russian VT-78a camera, the DARPA-developed Space Surveillance Telescope (SST), a notional 2.7 m wide-field telescope, and the Baker-Nunn camera.

The Russian VT-78a is the workhorse of the International Scientific Optical Network (ISON) [5]. The telescope features an aperture of 192 mm, a focal length of 300 mm and it images a field of 10 degrees diameter. All optical surfaces are spherical and the design requires no exotic or expensive glass materials. The instrument is called a *camera,* as there are no provisions for visual observation. The VT-78a has been produced in large numbers for use at ISON observing facilities around the world.

While the design for the VT-78a has not been published, the optical layout is well known and an approximate optical design can be developed. One author suggested the VT-78a is based on the old Shenker patent [6], but the design is clearly different—possibly being an advancement on the already novel Richter-Slevogt-Terebizh optical system. The optical layout for an approximate VT-78a is shown in Figure 10. To achieve extremely high image quality, Terebizh modified the front corrector lens group to include three lenses in a positive/negative/negative configuration, with the positive lens spaced well apart from the two negative elements. He also located the aperture stop on the primary mirror, which helps to control off-axis aberrations by making the design more symmetric about the stop. While the VT-78a generates high image quality over a very wide field for a reflecting system, this performance comes at a cost. Overall obscuration is high and aperture efficiency is reduced. The aperture efficiency for the approximated VT-78a design is only 0.33 and there is the additional photon loss resulting from two reflections, reducing the overall radiometric efficiency to 0.27. The major factor impacting aperture efficiency of the VT-78a system is the diameter of the front optic compared to that of the design aperture. Still, the VT-78a design is extremely impressive with very high image quality over an enormous 10 degree field-of-view.

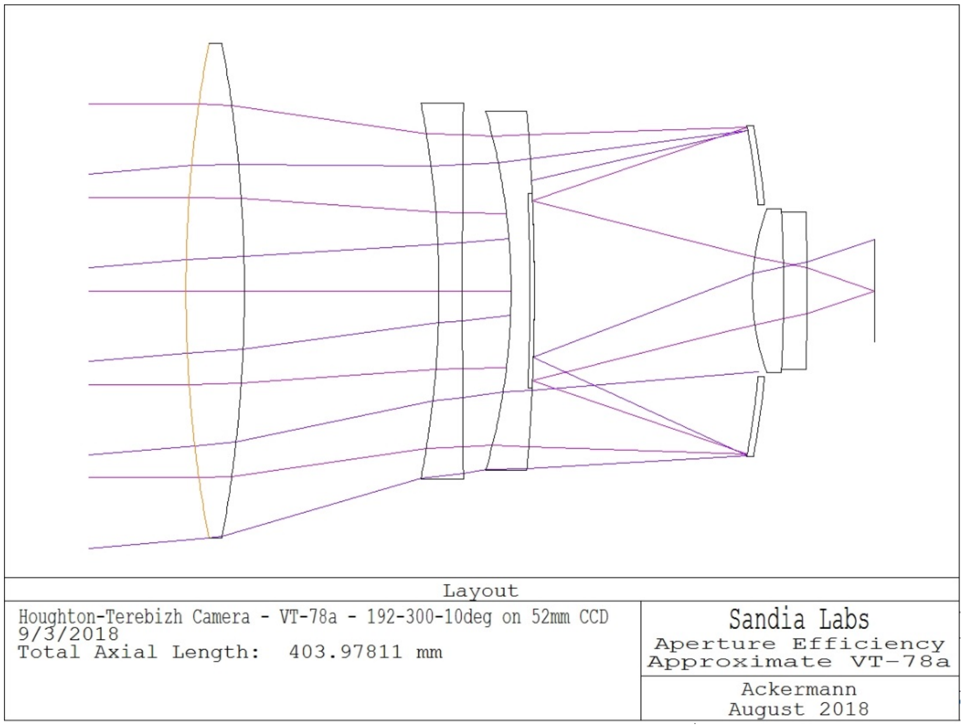

Figure 10. Optical Layout of VT-78a Camera (approximate)

The SST developed by DARPA features a 3.5 m aperture with a very impressive 3.5 degree diameter field of view. Much has been written about this instrument. In summary, the SST is an amazing achievement in optics and opto-mechanical engineering. The system is not without its limitations, but as an optical instrument, it currently stands alone for its ability to survey rapidly large portions of the sky to very faint limiting magnitudes. Like the VT-78a, the design has never been published, but the optical layout has appeared in the literature, allowing an approximate design to be developed. The SST features a three-mirror anastigmatic (TMA) design loosely based on the original TMA by Paul [7], but more closely resembling the Mersenne-Schmidt variant as explored by Willstrop [8]. The optical layout is seen in Figure 11. Even with the large obscuration of the secondary mirror, the aperture efficiency of the approximate SST design is calculated to be 0.65, but this should be reduced slightly as the system requires a baffle around the secondary mirror, which increases obscuration by approximately five percent. One problem with the TMA design is that it requires three reflections, each of which contributes to a loss of image intensity. With all reflections accounted for, the overall radiometric efficiency—again without the additional losses due to the required baffles—is approximately 0.48.

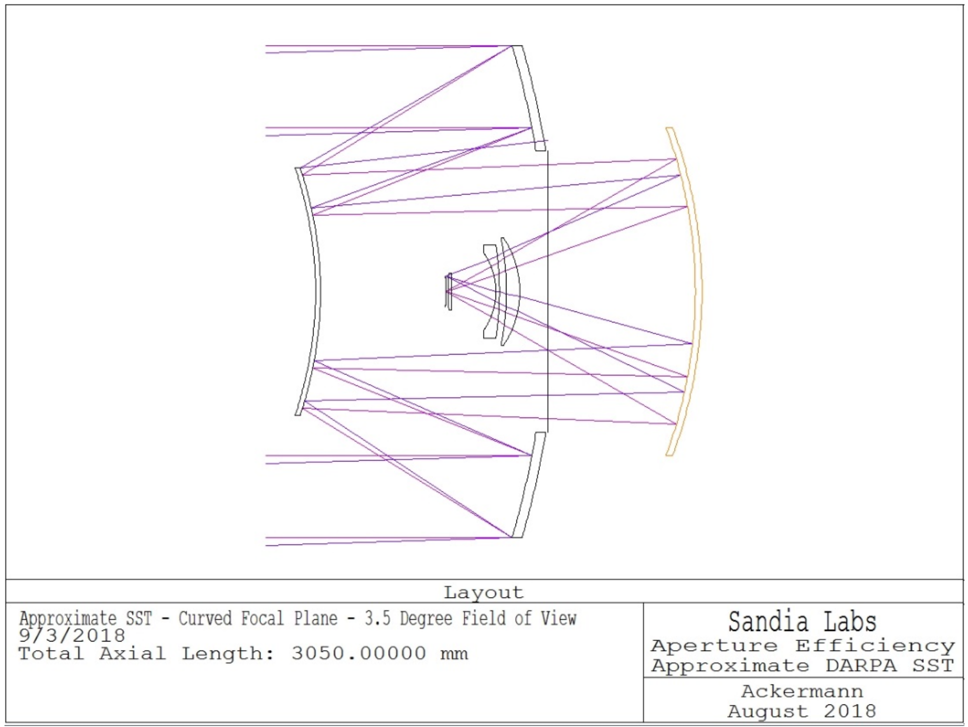

Figure 11. Optical Layout of the DARPA SST (approximate)

To further illustrate the importance of both obscuration and reflection losses on the overall performance of a wide-field imaging system, the authors compare the approximate SST design with that of a smaller, prime focus corrected telescope shown in Figure 12. The prime focus instrument has a 3.5 degree field-of-view and a 3.5 m focal length. The aperture is only 2.7 m, but the telescope requires a 3.05 m diameter primary mirror in a configuration sometimes known as a one mirror anastigmat (OMA) [9]. Note the aperture stop located between the primary mirror and the first corrector lens. With the OMA design, aperture efficiency will be lower than for a true prime focus instrument, but shifting the aperture stop improves image quality and reduces the diameter of the corrector lenses. The 2.7 m telescope design has an aperture efficiency of 0.70 with an overall radiometric efficiency of 0.63. Since the design falls within the family of prime focus instruments, baffling is not an issue and there will be no additional loss of aperture efficiency. The 2.7 m OMA was designed to equal the radiometric performance of the SST and will thus produce images of equal intensity—but being smaller the cost will be significantly lower. The SST and OMA comparison clearly illustrates the impacts of aperture efficiency and light loss upon multiple mirror reflections.

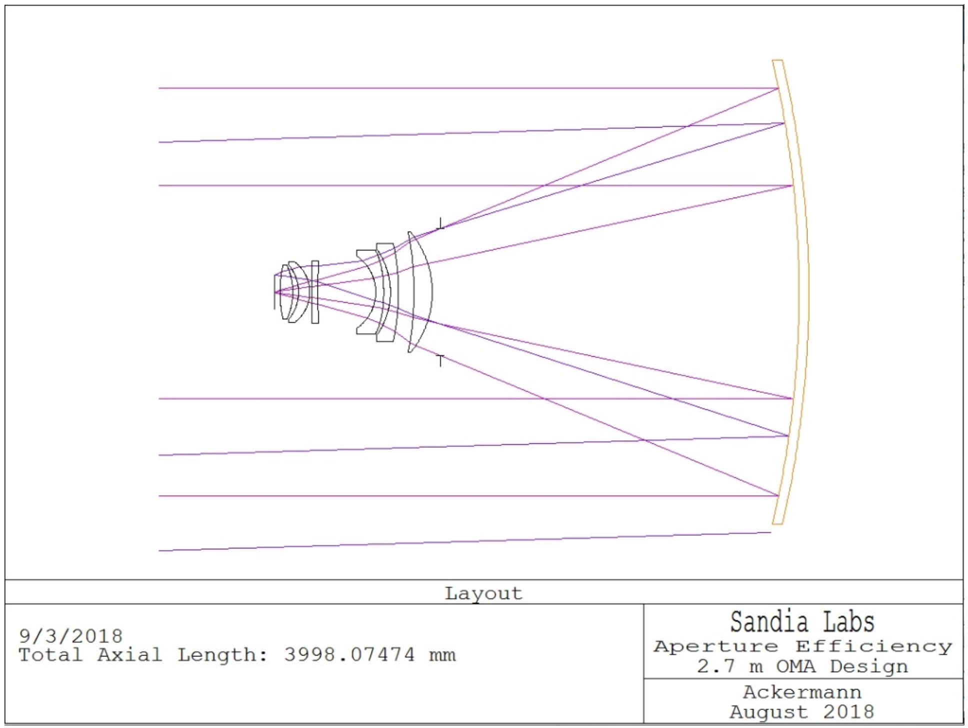

Figure 12. Layout of 2.7 m Aperture OMA Telescope

The final reflective system considered is the Baker-Nunn camera used for satellite detection and tracking from the late 1950s through the 1970s [10]. The camera featured a 500 mm aperture with three corrector lenses and a single spherical mirror. It operated at a focal ratio of f/1 and produced an image covering 5 degrees by 30 degrees on a curved strip of film. While an impressive optical instrument, the corrector lens assembly required four aspheric surfaces and two pieces of exotic glass that tended to degrade during routine use, resulting from exposure to the environment. The optical layout of the Baker-Nunn design is seen in Figure 13. Most obvious in the layout is the diameter of the mirror relative to that of the corrector lenses. With a 20 inch aperture and a 31 inch diameter primary mirror, the aperture efficiency is only 0.35, which is acceptable for an instrument with this field-of-view. When including light loss from the mirror, overall radiometric efficiency is 0.315. Another feature seen in the design is that the primary mirror was actually undersized a bit, most likely to reduce size, mass and cost. The reduced mirror diameter resulted in a slight loss of image intensity near the edges of the field.

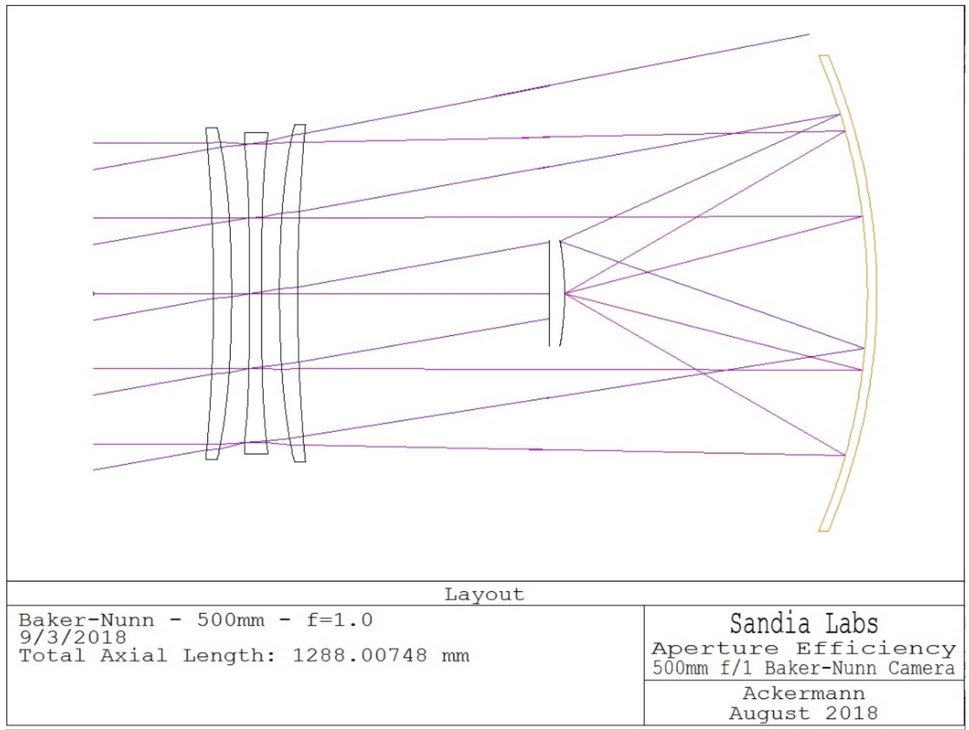

Figure 13. Optical Layout of Baker-Nunn Camera

Summary and Conclusions

Table 1 presents a summary for most of the aperture efficiency examples presented in the preceding sections.

Table 1. Summary of Aperture Efficiency Examples

System Type	Aperture Efficiency	Example	FOV (deg)
Simple Refractor	1.00	Commercial Achromatic Refractor	1.0
Commercial Wide-Field Lens	0.66	Canon 135mm f/2 Lens	18.0
Custom Lens w/o Vignetting	0.39	Custom 135mm f/2 Lens	18.0
Wide-Field Catadioptric Camera	0.33	Russian VT-78a	10.0
Wide-Field Catadioptric Camera	0.35	Baker-Nunn	30.0
Wide-Field, Large-Aperture Three-Mirror Telescope	0.60	SST or LSST	3.5
Wide-Field Prime Focus Telescope	0.70	Notional 2.7m SSA Telescope	3.5

What can be learned from the exploration of aperture efficiency presented in this paper? First, wide-field systems will have a lower aperture efficiency than narrow-field systems of similar size. Second, intentional vignetting is a useful design tool for controlling off-axis aberrations and improving image quality, but it also results in a higher aperture efficiency than designs that do not vignette. This higher aperture efficiency, in turn results in smaller and more compact optical systems that have lower overall mass and lower cost.

From a systems perspective, aperture efficiency helps one to understand that wide-field refractive systems will generally have optical elements that are larger than the system aperture. This feature will impact cost and can result in packaging difficulties when trying to fit the optical assembly into a higher-level system. For reflective systems, the largest optic and system aperture are often on the same surface, but obscuration from secondary and tertiary mirrors tend to reduce aperture efficiency significantly. Finally, for catadioptric systems, such as the Schmidt camera and VT-78a camera, both obscuration and vignetting become significant.

For low-light remote sensing applications, where photon noise is a significant consideration, optical systems that do not vignette are the best solution, but with their lower aperture efficiency, they are larger and heavier than they otherwise might be. For imaging of brighter scenes where darkening of the image towards the edge of the field is acceptable, or can be easily corrected, optical systems that vignette offer a higher aperture efficiency and a lower cost.

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