

Early Blast Point Determination For Large GEO Fragmentation Events

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ABSTRACT

The rush to obtain space real estate has created an increase in planned satellite launches and with it an increase in potential fragmentation events. These events generate debris fragments that threaten the usability of space. Providing timely, accurate, and statistically significant Space Situational Awareness (SSA) data is crucial to protect space assets and operations. Determining the point of initial breakup, the blast point, is valuable in characterizing and modeling the event. A standard approximation of the blast point is to propagate the mean orbits of well-tracked debris fragments, backward in time, to the time of closest approach. At this time, fragment positions are averaged to determine the blast point location. The errors in this approach arise from the uncertainty in debris fragment state and the uncertainty in the time of the fragmentation event. Accounting for these uncertainties can increase blast point estimation accuracies. In this paper, the authors discuss the complexities involved in this approximation and discuss methods used to determine the blast point. Parameters such as the number of tracked debris fragments and the uncertainty in both time and fragment state are used to develop an understanding of method sensitivities. The authors present multiple approaches for blast point determination that take advantage of fragment state uncertainty. These techniques are tested using GEO fragmentation events simulated according to NASA's standard breakup model.

1. INTRODUCTION AND MOTIVATION

A fragmentation event occurs when a Resident Space Object (RSO) breaks apart, explodes, or collides with another object. If catastrophic, these events can create thousands of debris fragments. Tracking these fragments, in a timely and accurate fashion, is crucial to protect other assets from cascading collisions. The location at which the fragmentation occurred is called the blast point. Determining the blast point can aid in Space Situational Awareness (SSA) and allow for improved Space Traffic Management (STM).

According to the European Space Agency (ESA), there are about 4,300 satellites in space with less than a third still operational. Estimates predict 29,000 objects over 10 cm in characteristic length, of these objects 23,000 are actively monitored. The number of debris objects skyrockets to about 167,000,000 when acknowledging those between ranges 10 cm and 1 mm.^{1,2} Since the beginning of the space age in 1957 there have been more than 290 space object fragmentation events. These events are a testament to the importance of improving fragmentation tracking capabilities.

Determining the location and time of fragmentation is beneficial in many ways. It can aid in identifying the origin of the breakup and developing an initial understanding of the event. For instance, it can provide an idea of which space object(s) were involved, what type of fragmentation it was, and the potential size of the fragmentation. This information can then be used to model the event and provide early warnings to High Values Assets (HVAs) on possible debris intercept paths. Furthermore, these models can offer sensor systems information required for initial fragment tasking and allow for the collection of more fragments. Most importantly, it can aid in the more rapid characterization and tracking of fragments. Multiple object tracking and characterization algorithms consist of measurement and prediction models. Without accurate information about the fragmentation event, tracking algorithms will suffer from delays in converging to the correct estimates

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of location and cardinality.³⁻⁵ These delays depend on the size of the fragmentation but can take days even months to converge. Providing accurate models of the fragmentation to tracking algorithms will eliminate delays and allow for rapid characterization and tracking.

If the accuracy of the estimated blast point is poor then so will be the model of the fragmentation. The model errors will trickle down into all of the other tools depending on it. The model errors can lead to poor beliefs about the event, poor sensor tasking, and poor fragment state estimation. This paper discusses different approaches to determining the blast point and what influences inaccuracies in those approaches. Specifically, techniques from the fields of optimization, statistics, and conjunction analysis are used to interpret and develop methods for blast point determination. Solving for the time of fragmentation is a non-convex optimization problem that can be carefully solved using gradient descent or ascent. This paper expands previous work⁶ that considered many different objective functions including fragment squared distance, Bhattacharyya⁷ and squared Hellinger distance,⁸ Kullback-Leibler Divergence,⁹ and finally 2-D Probability of Collision.¹⁰ This paper considers modeling the fragmentation as a debris distribution and approximates a probability density function (PDF) using the fragment states as samples from that PDF. These methods are tested using GEO fragmentation events simulated using NASA's standard breakup model.¹¹ The main contributions of this paper are:

- Clearly outline techniques for determining the blast point.
- Determine the impact of estimation errors on blast point determination accuracy.
- Determine the impact of the number of tracked fragments on blast point determination accuracy.
- Determine the impact of the time of fragment state estimation on blast point determination accuracy.
- Develop "Rule of Thumb" guidelines for determining the blast point.
- Determine the benefits of modeling the entire fragmentation as a continuous PDF.

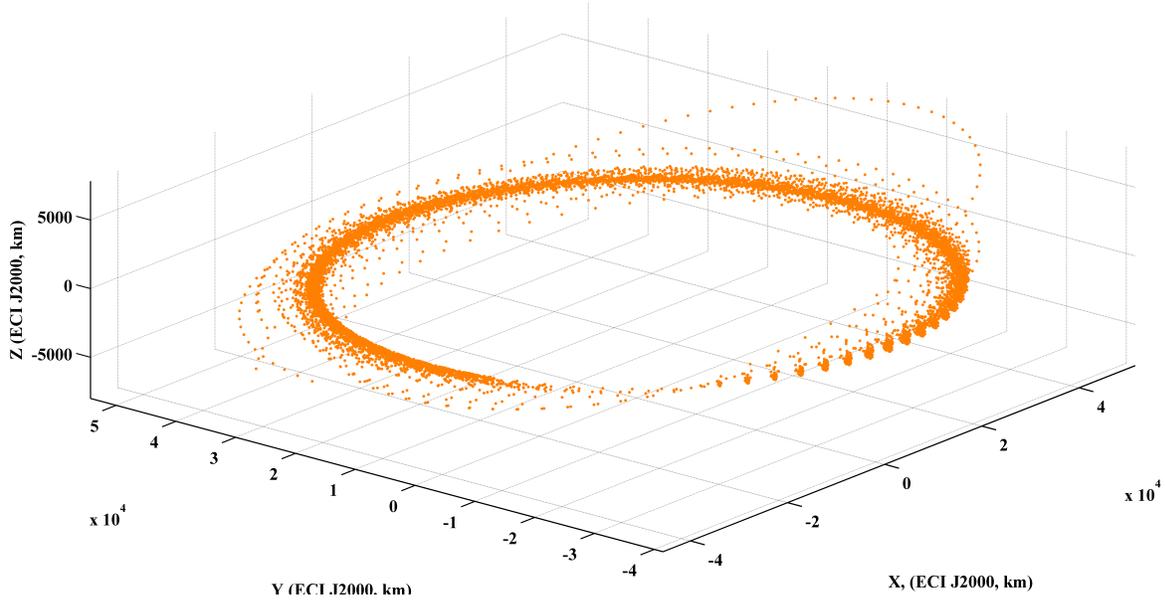
In the next section the details about how the fragmentations were simulated is The next section lays out the details about how the fragmentations were simulated. The section following discusses the different blast point determination techniques. In the applications section, contains the technical analysis and a discussion about performance and sensitivities. The last section provides an overview of the results and proposes future extensions.

2. SIMULATION DETAILS

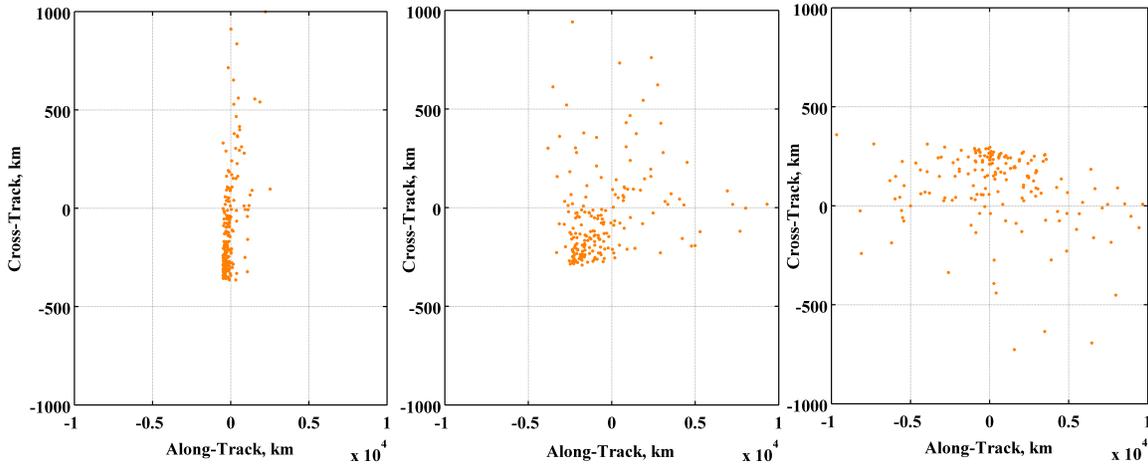
To accurately simulate the breakup events, a breakup model was developed using a version of the NASA standard breakup model.¹¹ This model characterizes the size distributions, area-to-mass distributions, impact velocity assignments, and delta-velocity distributions, where relevant, for both colliding or exploding rocket body and spacecraft. The simulations are of a GEO spacecraft explosion using a NASA scaling factor of $s = 1$. Figure 1 illustrates an example simulation by showing the fragment positions over time. All spacecraft and fragment dynamics were modeled using Orekit analytical propagation.

Multiple Monte Carlo simulations were performed for each of the three scenarios discussed in the applications section. The three scenarios were generated by conducting a sensitivity analysis on three different parameters. The first of which is the number of tracked fragments, n . This parameter refers to the number of known fragment PDFs during the time at which the blast point is to be computed. Varying this parameter addresses the question of how many fragments are needed to estimate the blast point and what impact does it have on the estimation accuracy. The second parameter considered is the time, t . In reality, fragment PDFs are provided by tracking algorithms. Some tracking algorithms take longer to return beliefs than others.¹² The parameter t refers to the duration between the fragmentation and the time an algorithm would return estimates of the n fragment PDFs. Varying t provides an understanding of how quickly the fragments need to be tracked to get reasonable estimates of the blast point. The last parameter is the error and uncertainty in fragment PDFs. In the applications section, simulations do not involve fragment tracking. The fragment PDFs are instead developed by sampling a Gaussian with mean equal to the true state and assuming Gaussian PDF with mean equal to that sample. Selecting

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(a) GEO breakup over a one day period shown every thirty-minutes in J2000 ECI.



(b) In track and cross-track positions for GEO breakup fragments at six hours (left), twelve hours (center), and one day (right) after the breakup.

Figure 1: An example GEO fragmentation simulated using NASA’s standard breakup model.

the covariance values for the PDFs controls the initial mean error and uncertainty. For simplicity, the chosen covariance values for both the sampled Gaussian and the fragment PDFs are the same. The number displayed on the x axis of figure 5 corresponds to the Cartesian x -position variance. All position variances are the same, and the velocity variances are a tenth of the position variances.

3. BLAST POINT DETERMINATION

This section describes the methods used to determine the blast point. Techniques from the fields of optimization, statistics, and conjunction analysis are leveraged to find both the time and location of the fragmentation. This

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paper solves the corresponding optimization problem using numerical approaches rather than analytical. Each technique is discussed in detail first the standard total squared distance approach than those that account for uncertainties.

3.1 Problem Description

Before describing the different techniques to determine the blast point, it is important to define the problem we are trying to solve clearly. Consider a scenario where at some time, t , a number of fragments, n , are being tracked. The fragment states, $x \in \mathbb{R}^m$ are known with some uncertainty and their PDFs are represented, $f_i(x)$ where $i = 1, 2, \dots, n$. Each fragment state is a function of time and varies according to orbital dynamics of which there are a variety of models that vary in accuracy. The goal is to estimate the time, t_0 , and location, ℓ of the blast point. The level of accuracy of this estimation is a function of the number of tracked fragments and the level of state and time uncertainty. The applications section contains a discussion on the sensitivity to these parameters.

3.2 Standard Approach Without Utilizing Covariance

The standard approach to solving this problem attempts to determine the blast point by finding the time, t_0 , at which the fragments are at their closest approach. In the case where fragment dynamics are known, and there is no uncertainty in initial fragment state, the time of closest approach is the time when the distance between fragment positions is zero. Under certain conditions, this problem can be solved analytically, which is a topic of future research. In this paper, we consider framing this problem as an optimization problem and solve it using a gradient descent approach. To solve this problem, we assume the following:

- n fragments are known at time t with PDFs $f_i(x)$ where $i = 1, 2, \dots, n$.
- All fragment PDFs are Gaussian.
- The states, x , consist of a six-dimensional representation of the fragments position and velocity in J2000 Earth Centered Inertial (ECI) frame, i.e., $x \in \mathbb{R}^m$ where $m = 6$.
- There underlying fragment dynamics are known and fragment PDFs can be found numerically at any time using Unscented Transform, i.e., $x(t) = F(x(t), t)$.
- There were no subsequent explosions or collision after initial fragmentation.

Under these assumptions, determining the blast point requires finding a time t at which the missed distance between the fragment positions is minimized. The distance between two fragments a and b at time, t , is defined using the mean position:

$$D_{a,b}(t) = \sqrt{(\mu_a(t) - \mu_b(t))^T (\mu_a(t) - \mu_b(t))}, \quad (1)$$

where $\mu_a(t)$ and $\mu_b(t)$ are the means of the fragment PDFs a and b at time t . The optimization to determine t_0 can then be written:

$$\underset{t}{\text{minimize}} \quad \sum_{i=1}^n \sum_{j=1}^n D_{i,j}^2(t) \quad (2)$$

The minimization was performed using a gradient descent approach. In this approach the following policy was iterated until convergence:

$$t_{i+1} := t_i - \alpha \frac{d}{dt} \sum_{i=1}^n \sum_{j=1}^n D_{i,j}^2(t), \quad (3)$$

where

$$\frac{d}{dt} \sum_{i=1}^n \sum_{j=1}^n D_{i,j}^2(t) \approx \frac{\sum_{i=1}^n \sum_{j=1}^n D_{i,j}^2(t_i) - \sum_{i=1}^n \sum_{j=1}^n D_{i,j}^2(t_{i-1})}{t_i - t_{i-1}}. \quad (4)$$

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In Eq. 4, the values t_{i-1} and t_i are initialized such that $t_i - t_{i-1} = h$ where h is a small value. The value α is a user defined value that allows the user to have some control over the rate of convergence.

Once the time of the blast point is found, the location ℓ can be estimated by averaging the mean position of all the fragment PDFs,

$$\ell = \frac{1}{n} \sum_{i=1}^n \mu_i(t_0) \quad (5)$$

3.3 Utilizing Fragment Uncertainty

This section describes three techniques that utilize the knowledge of fragment state uncertainty when determining the blast point. Similarly to the previous method these techniques consist of a minimization problem, however, in each case the objective function uses a different measure. The three objective functions that considered originate from probability and statistics. They consist of the Bhattacharyya Distance and the Hellinger Distance, which measure the similarity between probability distributions, and the Kullback-Leibler Divergence that determines the relative entropy between distributions. The Bhattacharyya Distance for multivariate normal distributions a and b is,

$$B_{a,b}(t) = \frac{1}{8}(\mu_a(t) - \mu_b(t))^T \Sigma^{-1}(\mu_a(t) - \mu_b(t)) + \frac{1}{2} \ln\left(\frac{|\Sigma|}{\sqrt{|\Sigma_a||\Sigma_b|}}\right), \quad (6)$$

where Σ_a and Σ_b are the determinant of the a and b covariances and $\Sigma = (\Sigma_a + \Sigma_b)/2$ (time dependencies are dropped for convenience). The Hellinger distance is related to the Bhattacharyya Distance but is not the same. The Hellinger distance for multivariate normal distributions a and b is,

$$H_{a,b}(t) = \sqrt{1 - \frac{|\Sigma_a|^{\frac{1}{4}} |\Sigma_b|^{\frac{1}{4}}}{\sqrt{|\Sigma|}} \exp\left(-\frac{1}{8}(\mu_a(t) - \mu_b(t))^T \Sigma^{-1}(\mu_a(t) - \mu_b(t))\right)}. \quad (7)$$

Lastly, the Kullback-Leibler Divergence between distributions a and b at a particular time t is,

$$K_{a,b}(t) = \frac{1}{2}(tr(\Sigma_b^{-1}\Sigma_a) + (\mu_b(t) - \mu_a(t))^T \Sigma^{-1}(\mu_b(t) - \mu_a(t)) - k + \ln\left(\frac{|\Sigma_b|}{|\Sigma_a|}\right)), \quad (8)$$

where the k is the dimensionality of the distributions.

The Hellinger distance and Kullback-Leibler divergence take values from $0 \rightarrow 1$ while the Bhattacharyya distance takes values from $0 \rightarrow \text{inf}$. In any of the three cases, solving for the blast point involves minimizing the particular distance or divergence. The assumptions are:

- n fragments are known at time t with PDFs $f_i(x)$ where $i = 1, 2, \dots, n$.
- All fragment PDFs are Gaussian.
- The states, x , consist of a six dimensional representation of the fragments position and velocity in J2000 Earth Centered Inertial (ECI) frame, i.e., $x \in \mathbb{R}^m$ where $m = 6$. However, only the elements corresponding to position are considered, i.e., $\mu_i(t) \in \mathbb{R}^3$ and Σ_a, Σ_B and $\Sigma \in \mathbb{R}^{[3 \times 3]}$
- There underlying fragment dynamics are known and fragment PDFs can be found numerically at any time using Unscented Transform, i.e., $x(t) = F(x(t), t)$.
- There were no subsequent explosions or collision after initial fragmentation.

The objective functions for these three techniques are, *Case 1: Bhattacharyya*

$$J_1(t) = \sum_{i=1}^n \sum_{j=1}^n B_{i,j}(t) \quad (9)$$

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Case 2: Hellinger

$$J_2(t) = \sum_{i=1}^n \sum_{j=1}^n H_{i,j}^2(t) \quad (10)$$

Case 3: Kullback-Leibler

$$J_3(t) = \sum_{i=1}^n \sum_{j=1}^n K_{i,j}(t) \quad (11)$$

Furthermore, the optimization to determine t_0 can then be written:

$$\underset{t}{\text{minimize}} \quad J_d(t), \quad (12)$$

where $d = 1, 2$ and 3 . The minimization can then be performed using a gradient descent approach. In this approach the following policy was iterated until convergence:

$$t_{i+1} := t_i - \alpha \frac{d}{dt} J_d(t), \quad (13)$$

where

$$\frac{d}{dt} J_d(t) \approx \frac{J_d(t_i) - J_d(t_{i-1})}{t_i - t_{i-1}}. \quad (14)$$

In Eq. 14, the values t_i and t_{i-1} correspond to user defined initial time values and α is a user defined value that allows the user to have some control over the rate of convergence.

Once the time of the blast point is found, the location ℓ can be estimated by averaging the mean position of all the fragment PDFs,

$$\ell = \frac{1}{n} \sum_{i=1}^n \mu_i(t_0) \quad (15)$$

3.4 Utilizing Probability of Collision

The last technique utilizes conjunction analysis, in particular 2-D Probability of Collision (PC). The two dimensional PC between fragments a and b can be determined by converting to a relative encounter frame by first defining,

$$r_{rel}^{\vec{}} = r_a^{\vec{}} - r_b^{\vec{}} \quad (16)$$

$$v_{rel}^{\vec{}} = v_a^{\vec{}} - v_b^{\vec{}} \quad (17)$$

where r_i and v_i are the mean position and velocity in the J2000 ECI frame, respectively, for the distributions $i = a, b$. Using these values we can define the relative Radial, In-track, and Cross-track (RIC) frame as,

$$\vec{R} = r_{rel}^{\vec{}} / \|r_{rel}^{\vec{}}\|, \quad (18)$$

$$\vec{C} = \frac{r_{rel}^{\vec{}} \times v_{rel}^{\vec{}}}{\|r_{rel}^{\vec{}} \times v_{rel}^{\vec{}}\|} = \frac{h_{rel}^{\vec{}}}{\|h_{rel}^{\vec{}}\|}, \quad (19)$$

$$\vec{I} = \vec{C} \times \vec{R}. \quad (20)$$

The rotation matrix to the RIC frame from the J2000 frame can then be defined,

$$\mathbf{W} = [\vec{R} \ \vec{I} \ \vec{C}]. \quad (21)$$

The combined covariance of PDFs a and b represented in the relative frame and reducing to the appropriate 2D representation is then,

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{W}(\Sigma_a + \Sigma_b) \mathbf{W}^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}. \quad (22)$$

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Using Eq. 16 - Eq. 22 one can write expression for PC as,

$$C_{a,b}(t) = \frac{1}{2\pi\sqrt{|\mathbf{C}|}} \iint_A \exp(-\frac{1}{2}r^T \mathbf{C}^{-1}r) dA, \quad (23)$$

where A is the user defined area estimate and $r = [xz]$.

In order to determine the blast point it is necessary to determine the time at which Eq. 23 is maximized. For the purposes of this paper, a gradient ascent approach is used to determine t_0 . The optimization takes the form,

$$\underset{t}{\text{maximize}} \quad P_C(t), \quad (24)$$

where $d = 1, 2$ and 3 . The minimization can then be performed using a gradient descent approach. In this approach the following policy was iterated until convergence:

$$t_{i+1} := t_i + \alpha \frac{d}{dt} P_C(t), \quad (25)$$

where

$$\frac{d}{dt} P_C(t) \approx \frac{P_C(t_i) - P_C(t_{i-1})}{t_i - t_{i-1}}. \quad (26)$$

In Eq. 26, the values t_i and t_{i-1} correspond to user-defined initial time values, and α is a user defined value that allows the user to have some control over the rate of convergence.

Once the time of the blast point is found, the location ℓ can be estimated by averaging the mean position of all the fragment PDFs,

$$\ell = \frac{1}{n} \sum_{i=1}^n \mu_i(t_0) \quad (27)$$

3.5 Modeling The Fragmentation Using A Multi-Variate Gaussian PDF

This section describes a new blast point determination technique that utilizes fragment states to approximate a continuous PDF that represents the entire fragmentation. Once modeled as a PDF, one can then minimize entropy over time to determine the blast point.

The PDF can be modeled by making the assumption that the fragment dispersion is Gaussian. The PDF can then be expressed,

$$p(x) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} \exp(-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)) \quad (28)$$

where μ and Σ are the sample mean and covariance computed using the mean fragment states with dimensionality k .

The objective function used in the gradient descent approach is a measure of the entropy,

$$J(t) = E(t) = \frac{1}{2} \log(|\Sigma|(2\pi e)^k) \quad (29)$$

The minimization can then be performed using a gradient descent approach. In this approach the following policy was iterated until convergence:

$$t_{i+1} := t_i - \alpha \frac{d}{dt} J(t), \quad (30)$$

where

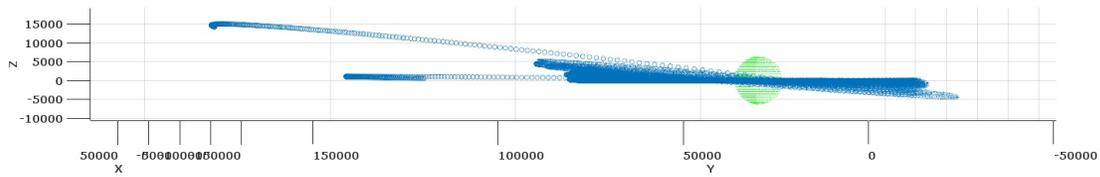
$$\frac{d}{dt} J(t) \approx \frac{J(t_i) - J(t_{i-1})}{t_i - t_{i-1}}. \quad (31)$$

In Eq. 31, the values t_i and t_{i-1} correspond to user defined initial time values and α is a user defined value that allows the user to have some control over the rate of convergence.

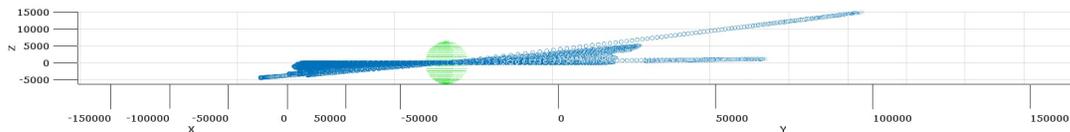
Once the time of the blast point is found, the location ℓ can be estimated by averaging the mean position of all the fragment PDFs,

$$\ell = \frac{1}{n} \sum_{i=1}^n \mu_i(t_0) \quad (32)$$

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(a) Simulated GEO space object fragmentation. View of a pinch point associated with blast point.



(b) Simulated GEO space object fragmentation. View of another pinch point opposite the blast point.

Figure 2: Pinch point views of a GEO fragmentation

3.6 Points Of Caution When Optimizing

Although the gradient descent performs well in simulation for non-convex optimization, there are a few things that should be considered before attempting to solve for the blast point.

Non-Convex Objective Functions Even the most intuitive objective function used in the blast point determination techniques are non-convex. For example, consider the method that uses the total squared distance between fragments as the objective function. Fragments will converge to pinch points throughout their orbits,^{13,14} and illustrated in figure 2. These pinch points cause a sinusoidal effect when plotting the total distance between fragments over time. Also, figure 3 shows a course illustration of the object functions over time. Similar behavior is expected at every pinch point.

Gradient Descent Convergence The gradient descent approach, mainly when used in optimization of non-convex functions, is not guaranteed to converge to a global minimum or a blast point. To avoid convergence to a point that does not represent the blast point minimum/maximum, we assume that the fragments time of capture is within an orbital period and that the global minimum or maximum over that orbital period range is the blast point. We also perform a course search over time to seed the algorithm near the minimum/maximum of interest. The gradient descent approach, primarily when used in optimization of non-convex functions, is not guaranteed to converge to a global minimum or a blast point. To avoid convergence to a point that does not represent the blast point minimum/maximum, we assume that the fragments time of capture is within an orbital period and that the global minimum or maximum over that orbital period range is the blast point. We also perform a course search over time to seed the algorithm near the minimum/maximum of interest. Also, from figure 3, it can be seen that as the duration from blast point increase the behavior of the evaluated objective function becomes less well behaved. This results in longer convergence times and even instances where the solution converges to the wrong local minimum. It is considered a disadvantage of the particular technique if, under the given conditions, the gradient descent technique converges to the wrong minimum.

Adaptive Step Size Choosing the initial time and step size values is crucial to keep the gradient descent technique from straying away from the global minimum. Incorporating an adaptive step size can control the time to convergence while keeping the algorithm from diverging to a neighboring minimum. However, use the adaptive step size with caution. In some instances where the step size α is too small, the gradient may seem to suggest premature convergence.

Non-Gaussian PDFs All object functions used in this paper are formulated under the assumption that the

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fragment PDFs are Gaussian. This assumption simplifies the computation of the measures. However, in cases where uncertainty is propagated for long periods of time, a Gaussian assumption may be invalid.

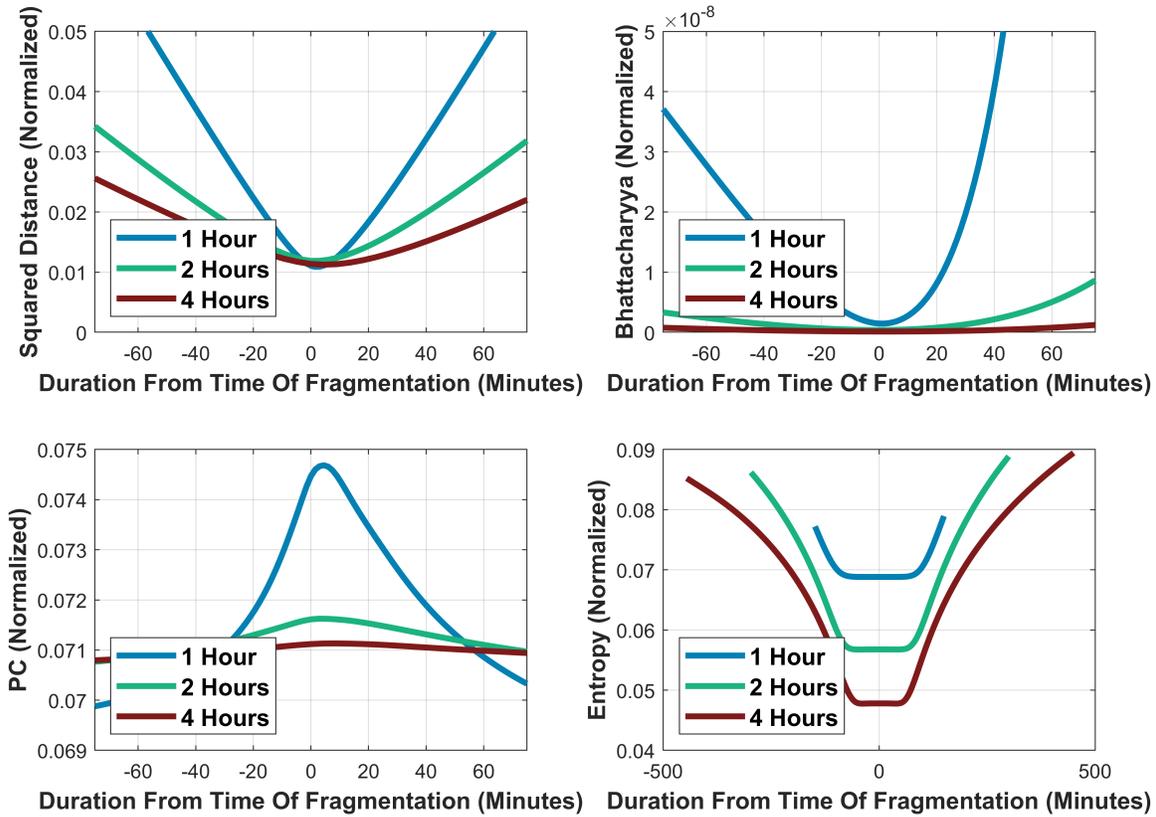


Figure 3: Normalized objective functions evaluated near the time of the blast point for capture times $t = 1, 2,$ and 4 hours from blast point.

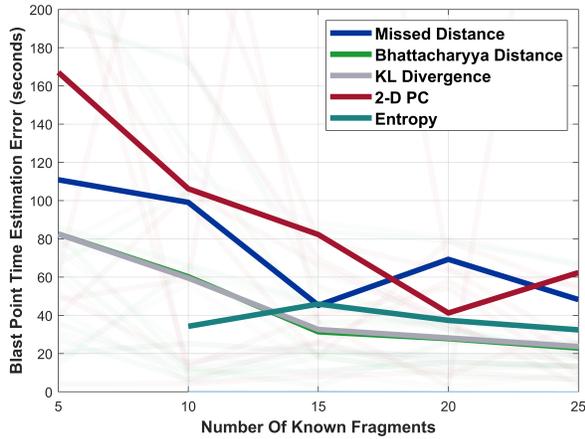
4. APPLICATIONS

In this section, the above techniques were tested using simulated fragmentations according to NASA’s standard breakup model. Parameters such as the number of known fragments, the time from blast point, and the fragment state uncertainty were controlled and varied to highlight the advantages and disadvantages of each technique. The performance of each technique was evaluated by comparing the estimated time and location of the blast point to those of the true blast point. For each experiment, we performed multiple Monte Carlo simulations of an exploding GEO space object with characteristic length of 5 meters.

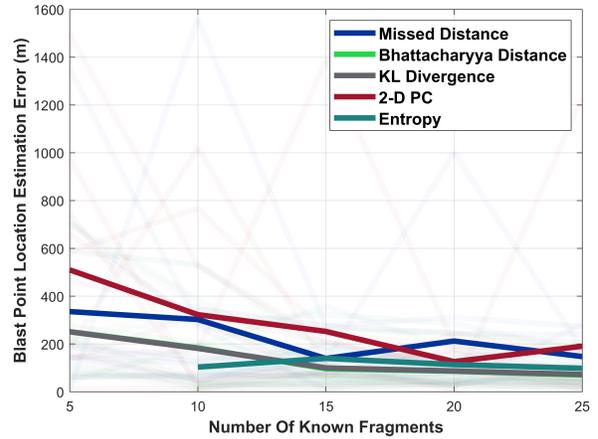
In the first scenario, the blast point was determined using each technique for a varying number of tracked fragments n . Monte Carlo simulations were averaged to determine mean performance. During these simulations, we kept the time at which we obtain custody of the objects fixed, as well as, the levels of uncertainty associated with the estimated state. The goal of this simulation is to understand how the number of tracked fragments impacts the overall performance of each technique.

In general, as the number of tracked fragments increases the blast point estimation error decreases. For all techniques, except for the technique using entropy, an increase in computational burden accompanies the number of fragments. Figure 4 (a) and (b) illustrate the error in blast point time and location approximation against the number of tracked fragments. The techniques that take advantage of fragment uncertainty and compare total position distribution similarity tend to perform better than the others. Including the Bhattacharyya,

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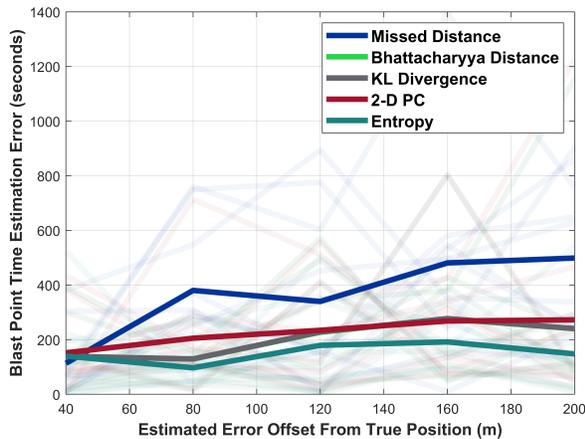
(a) Blast point time estimation error against the number of tracked fragments



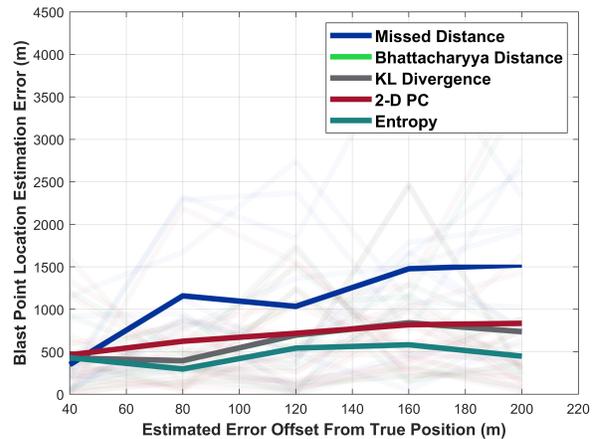
(b) Localization error against the number of tracked fragments

Figure 4: A study of how the number of tracked fragments effects the accuracy of blast point determination

Kulback-Leibler, and Hellinger (not shown) based techniques. The technique titled entropy corresponds to the technique that models the fragmentation as a single continuous PDF. It can be seen in figure 4 (a) and (b) that the results for the entropy technique do not occur until the number of known fragments increases enough to accurately calculate entropy. However, once the number of fragments increases as does the accuracy of the entropy technique.



(a) Blast point time estimation error against initial fragment estimation error.



(b) Localization error against initial fragment estimation error.

Figure 5: A study of how the inaccuracies in fragment state estimation impact the accuracy of blast point determination.

The next experiment implemented each blast point determination technique for a range of initial fragment uncertainty and estimation error. Monte Carlo simulations were averaged to determine mean performance. During these simulations we fixed the number of tracked fragments, n , and the time, t to $n = 10$ and $t = 3600s$. The goal of this simulation is to understand how accurately we must track fragments to have an accurate approximation of the blast point.

As initial fragment beliefs move further away from the truth and more uncertain the ability to determine the blast point deteriorates. The x axes of figure 5 corresponds to the Cartesian x -position variance of both the sampled Gaussian centered at the true fragment position and the covariance of the fragment PDF. The

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level of fragment state estimation accuracy is often at best in the 10 of meters range. This goes to show that very accurate estimates of fragment states are required to have a precise blast point estimate. In any case, the techniques that utilize fragment position uncertainty outperform those that do not. Furthermore, the new technique (titled Entropy) out performs all other techniques.

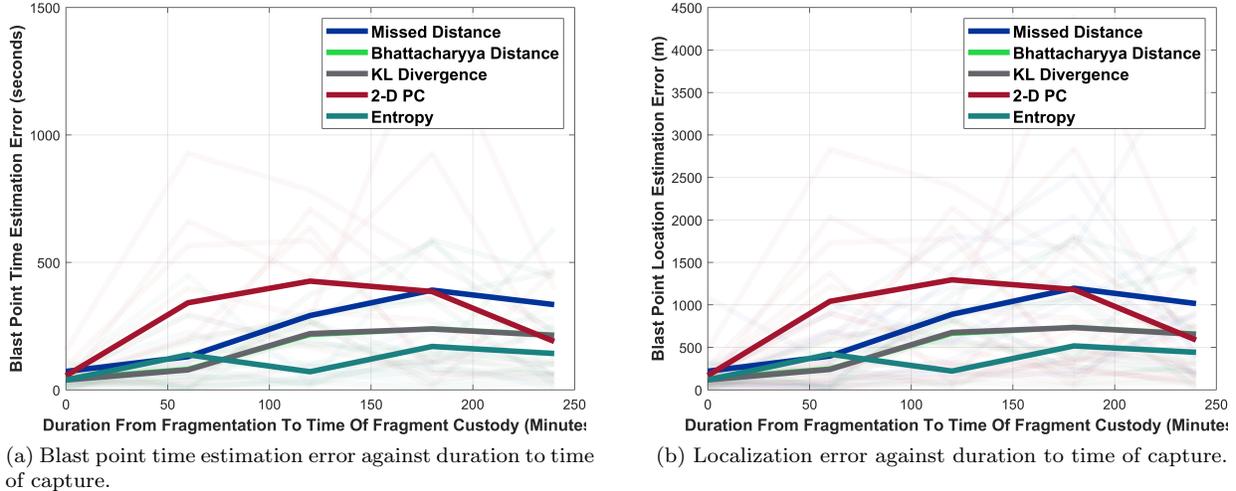


Figure 6: A study of how the time to capture impacts the accuracy of blast point determination.

In the last experiment, the time of fragment capture t was the varying parameter. The number of tracked fragments was fixed at $n = 10$ and the levels of uncertainty associated with the estimated state to $100m$ position variance. The goal of this simulation is to understand how quickly we must capture fragments to determine an accurate estimate of the blast point.

As seen in previous cases, the techniques that utilize knowledge of fragment uncertainty perform better than those that do not. The proposed PDF entropy technique performs better than all other techniques.

5. SUMMARY, CONCLUSION, AND FUTURE WORK

This paper highlights the motivation and importance of accurately determining the blast point of space object fragmentations. The authors introduce many blast point determination techniques that combine research from optimization, statistics, and conjunction analysis. The techniques were applied to multiple GEO fragmentation events and analyzed for their sensitivities to different parameters. In general, for a more accurate approximation of the blast point it behooves to estimate and track as many fragments as accurately and quickly as possible. Use techniques that utilize position uncertainty in scenarios where fragment uncertainties and durations from breakup are significant. Furthermore, if the number of known fragments is significant enough to accurately approximate the fragmentation with a single PDF, then the use of the newly proposed technique will tend to provide more accurate blast point estimation.

From this work, many research extensions are being or will be explored. These include the extensions of the determination techniques to relax some assumptions such as Gaussianity and subsequent collisions, the incorporation of a 3-D probability of collision. The development of methods that take advantage of velocity and velocity uncertainties. Fragment debris cloud modeling and expansions of this research to the field of fragment re-entry.

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