

Passive debris removal using orbital resonances

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ABSTRACT

Current advances in our understanding of the dynamical processes that act on satellites and space debris predict that they may undergo significant and perhaps chaotic drifts in circumterrestrial phase space throughout their existence. Here was the birth of a new ideology to remedy the space debris proliferation problem, based on a judicious use of the resulting instabilities to prescribe natural Earth re-entry itineraries and navigate the phase space. We present here recent results on the passive debris removal paradigm, and show how we can ‘kill’ satellites using resonances. We also show how such dynamical assessments are of considerable practical interest for the identification of long-term stability regimes, such that satellites (and their aging components) placed in graveyards will not interfere with active satellites or constellations. The underlying work uses tools and practices that are common in astrophysical and celestial dynamics, but have only just started to be applied to the dynamics of space debris in recent years, as did the need for extending our knowledge of these bodies on intervals longer than mission timescales.

1. INTRODUCTION

Recent efforts towards space debris remediation have explored passive means to curtail the growth rate of the debris population, by seeking to cleverly exploit the dynamical instabilities brought on by resonant perturbations to deliver retired Earth-orbiting satellites into the regions where atmospheric drag can start their decay [1, 2, 3]. Such orbital resonances, associated with commensurabilities among the frequencies of the perturbed motion, occur in profusion within the whole of the circumterrestrial phase space, in both dissipative and Hamiltonian settings. Recent work has uncovered a vast network of lunisolar and radiation pressure resonances, which can induce especially strong changes on the orbital eccentricity [1]. Many of these resonances can be exploited on decadal timescales to effectively remove satellites from crowded regions and their otherwise long lifetime orbits. A permanent debris-control scheme, based more on remediation than mitigation, will have to be developed well in advance of the critical orbit-clogging point predicted by Kessler. The proper definition of the end-of-life strategy that takes into account the long-term dynamics, in conjunction with relatively low-cost maneuvers or an area-to-mass-ratio-augmenting device (e.g., solar sail), from the early design phase can possibly yield a self-correcting mechanism, much needed to sustain the space environment. This paper will review recent results that link theoretical aspects of resonant and chaotic dynamics with the practical application of passive debris removal and mitigation, and lays an essential foundation for future developments.

2. ORBITAL RESONANCES

Among the various studies in astrophysical and celestial dynamics, the motion near *orbital resonances* occupies a special place [5]. Resonances profoundly affect the behavior of many physical systems, in both Hamiltonian and dissipative settings, as depicted in Fig. 1. These correspond to regions in the phase space in which the frequencies of certain angular variables become nearly commensurate. Because of this feature, many communications, navigation, scientific, and military satellite missions have employed these orbits.* Our knowledge of such phenomena in celestial mechanics comes mainly from studies on asteroid dynamics—attempts to describe the complicated dynamical structure of the asteroid belt, shown in Figure 2. The salient feature of a resonance (in the *pendulum model*) is the existence of an elliptic fixed point, with regular phase-space trajectories encircling it, and of hyperbolic fixed points, connected by a separatrix trajectory [5]. The principal effect of the interaction of two resonances is to produce qualitative changes in the separatrix of the perturbed resonance (Figure 1), causing a *stochastic layer* in its vicinity.

*For instance, Sun-synchronous orbits are particularly useful because the orbit plane follows the Sun, and orbits with mean motion commensurabilities to the Earth’s rotation rate, GPS and geostationary satellites, are important for ensuring repeating ground tracks.

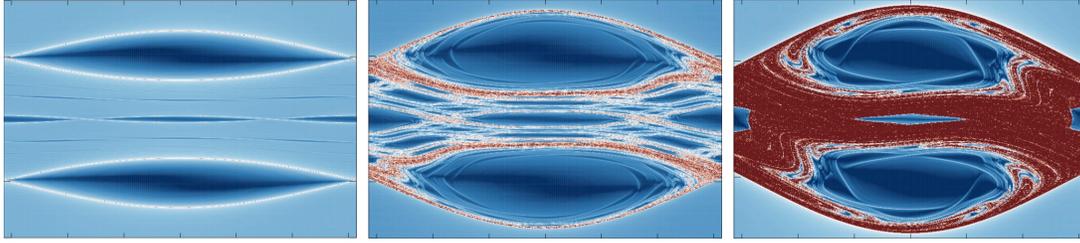


Fig. 1. Phase-space portraits (Φ_1, J_1) for the two DOF Hamiltonian (driven pendulum) $\mathcal{H} = -\frac{1}{2}J_1^2 + J_2 + \varepsilon \cos \Phi_1 + \varepsilon \cos(\Phi_1 + \Phi_2)$ for various parameters, showing the growth of the chaotic layer (red color) around the pendulum separatrix as the perturbation strength ε increases from left to right. Obtained using the fast-Lyapunov indicator [4]. Credit: J. Daquin, personal communication.

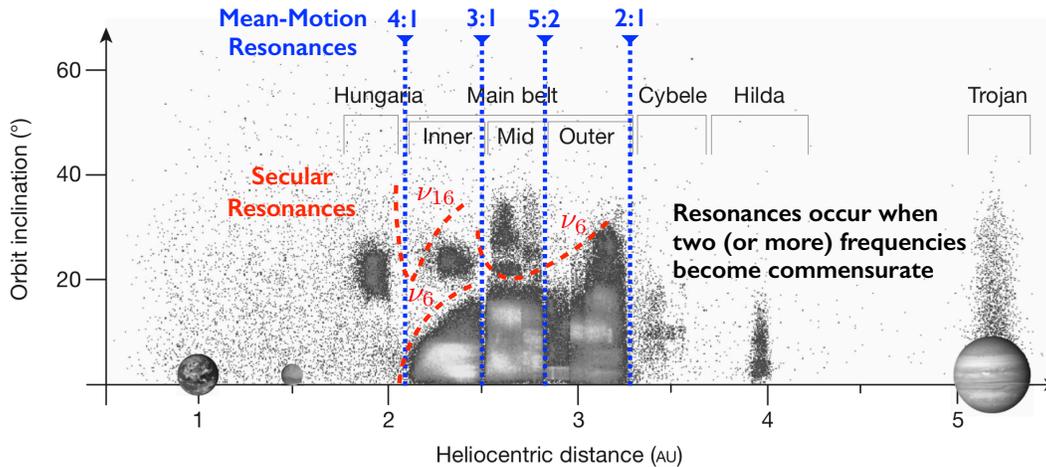
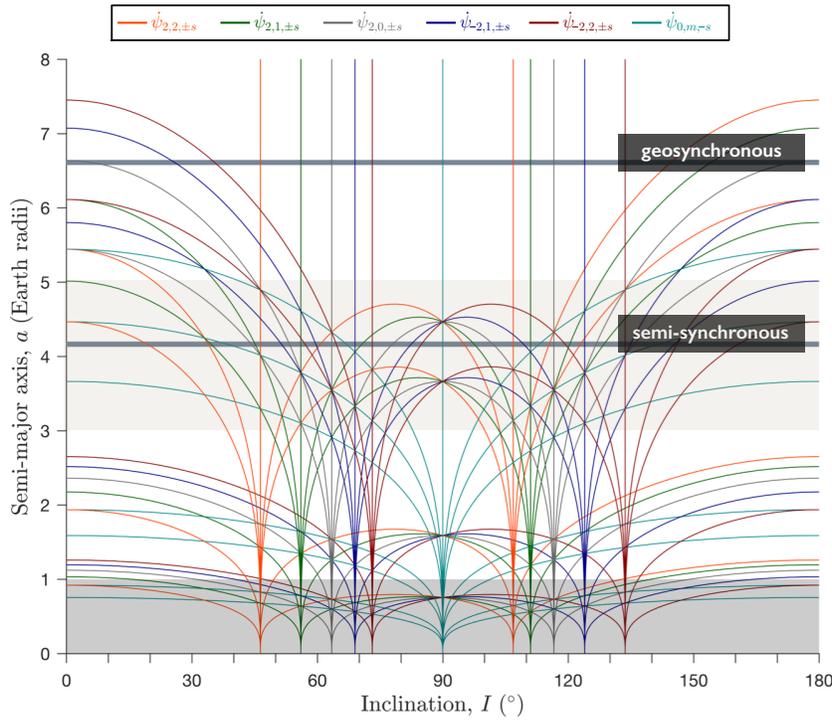


Fig. 2. A visualization of the main belt of asteroids and the predominant resonances, adapted from [6]. The process of dynamical clearing of resonant orbits is well illustrated by the paucity of asteroids observed in the Kirkwood gaps. The orbital structure of the main belt is divided by unstable regions, narrow gaps in the distribution of the asteroidal semimajor axes at 2.5 and 2.8 astronomical units (AU) from the Sun, where the orbital periods (or mean motions) of the asteroids are commensurable to that of Jupiter. The outer boundary is marked by the Jovian 2:1 mean-motion resonance at 3.3 AU. Linear secular resonances (dubbed ν_6 and ν_{16}) define the inner edge of the main belt near 2.1 AU and demarcate the recognized subpopulations (*asteroid families* and groups).

The dynamical environment occupied by resident space objects (e.g., Earth-orbiting satellites and space debris) is subject to motions that have widely disparate timescales: the earthly day, the lunar month, the solar year, and various precession frequencies ranging from a few years to nearly 26 000 years for the equinoxes. This provision of frequencies in the Earth-Moon-Sun system gives rise to a diverse range of complex resonant phenomena associated with orbital motions. While the orbits of most artificial satellites are too low to be affected by *mean-motion resonances* (MMRs), excepting the recent Interstellar Boundary Explorer (IBEX) and Transiting Exoplanet Survey Satellite (TESS) both placed in lunar MMRs [7], there exists many possibilities of lunisolar *semi-secular resonances* involving a commensurability of a secular precession frequency with the mean motion of the Sun or the Moon [8, 9, 10, 11]. Note that these solar commensurabilities are also sites of *solar radiation pressure resonances*, when the area-to-mass ratio becomes important. Among the more compelling and strongest of the semi-secular commensurabilities is the ejection resonance with the Sun, where the satellite's apsidal precession rate produced by Earth's oblateness equals the Sun's apparent mean motion, so that the line of apsides follows the Sun [8, 11]. *Tesseral resonances* arise from the slight longitude-dependence of the geopotential, where the satellite's mean motion is commensurable with the rotation of the Earth (e.g., GEO and GPS). Under such conditions, the longitudinal forces due to the tesseral harmonics continually perturb the orbit in the same sense and produce long-term changes in the semimajor axis and the mean motion [12]. Finally, there exists a class of resonances not of the mean-motion type, known as *lunisolar secular resonances* (i.e.,

those caused by the Moon and the Sun on long timescales), which permeate the highly inclined MEO region and beyond [12, 13]. These secular resonances, involving linear combinations of the satellite’s apsidal and nodal precession rates (dominated by Earth’s oblateness) and the rate of lunar node motion, occur in profusion so that the phase space is densely threaded by a complicated web-like structure. The center of each lunisolar and radiation pressure semi-secular and secular resonance (for both prograde and retrograde orbits) may be approximately defined in the inclination–semi-major axis phase space, as shown in Fig. 3.



The oblateness *apsidal* and *nodal* precession overshadows the lunisolar effects

$$\dot{\omega} \approx 4.98(R/a)^{7/2} \frac{5 \cos^2 I - 1}{(1 - e^2)^2} \text{ } ^\circ/\text{d}$$

$$\dot{\Omega} \approx -9.97(R/a)^{7/2} \frac{\cos I}{(1 - e^2)^2} \text{ } ^\circ/\text{d}$$

$$\dot{\Omega}_M \approx -0.053^\circ/\text{d}$$

$$(n_S = 0.986^\circ/\text{d}, n_M = 13.246^\circ/\text{d})$$

$$\alpha \dot{\omega} + \beta \dot{\Omega} + \gamma \dot{\Omega}_{\text{Moon}} \approx 0$$

(lunar secular res)

$$\alpha \dot{\omega} + \beta \dot{\Omega} + \gamma n_{\text{Sun}} \approx 0$$

(solar semi-secular res)

$$\alpha \dot{\omega} + \beta \dot{\Omega} + \gamma n_{\text{Moon}} \approx 0$$

(lunar semi-secular res)

Fig. 3. Location of all principal lunisolar resonances in the inclination–semi-major axis plane for circular satellite orbits, adapted from [1]. Semi-secular resonances occurring below three Earth radii are shown to give a complete picture of the circumterrestrial space [9, 10, 11]. The gray area corresponding to inside the Earth is unphysical, but shown to highlight the resonant skeleton structure. The beige box highlights the region of the navigation satellites.

3. IMPLICATIONS FOR PASSIVE STRATEGIES

Presented herein is a survey of recent results on the use of orbital resonances for passive debris removal and mitigation.

3.1. Chaos and Instability in Earth-Satellite Orbits

Many physical systems can be modeled as having an underlying *dynamical skeleton* that organizes and governs how all the possible behaviors are related. The global properties of multidimensional, nearly integrable Hamiltonian systems are determined by the relative location and size (widths) of the predominant resonances [5]. Multi-frequency systems with two or more resonances, involving two or more degrees of freedom, can undergo a transition from order to dynamical chaos in certain regions of phase space, as the system parameters are varied. Regular behavior is destroyed from the growth and overlap of resonances, and, as a result, the states of the system are free to explore in an apparently random manner throughout the phase-space region of influence of the interacting resonances. Using variational chaos indicators, we have shown that this order-to-chaos transition scenario occurs in highly inclined satellite orbits as the satellite’s semi-major axis recedes from the Earth [14, 16], as depicted in Fig. 4. An orbit that lies in the stochastic layer can eventually diffuse (in *Fick’s* sense) into different regions of the phase space by moving across or along the resonances [16].

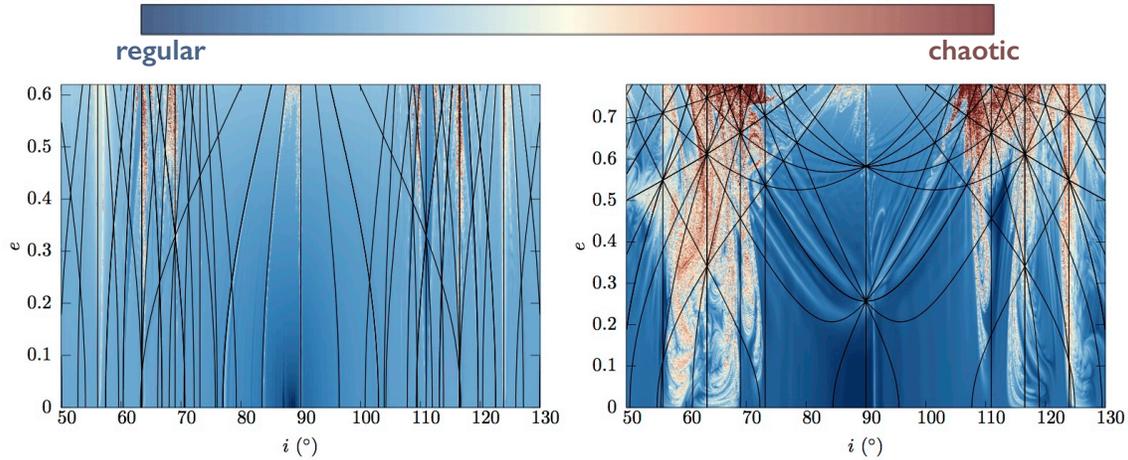


Fig. 4. Detailed views of the prograde and retrograde regions for two values of the semi-major axis: 2.92 Earth radii (*left*) and 4.65 Earth radii (*right*). Each map represents the result of 1000×500 initial conditions propagated over centennial timescales (greater than 10^5 orbital revolutions). The FLIs of all regular orbits appear with the same *dark blue* color. *Light* colors corresponds to invariant tori, and *red* to chaotic orbits. Despite the symmetry of the resonance centers (Fig. 3), superimposed on the FLI stability maps in *black*, the chaos in the prograde region is not mirrored in the retrograde region. To understand precisely how such chaotic structures occur is of remarkable practical application, and requires further study.

Fig. 5 shows three *canonical* planes for the system of Fig. 4, assembled into faces of a cube. The results presented in Figure 5 complement the global stability picture by ‘unrolling’ the dynamics according to one angle, highlighting an apparent bifurcation phenomenon. These considerations emphasize the importance of investigating the analytical character of the mechanisms that drive these complicated behaviors and of developing accurate models that describe the resonant interactions. Indeed, analytical and numerical techniques cannot be separated, and a complete, logically ordered picture is obtained only by the application of both methods jointly.

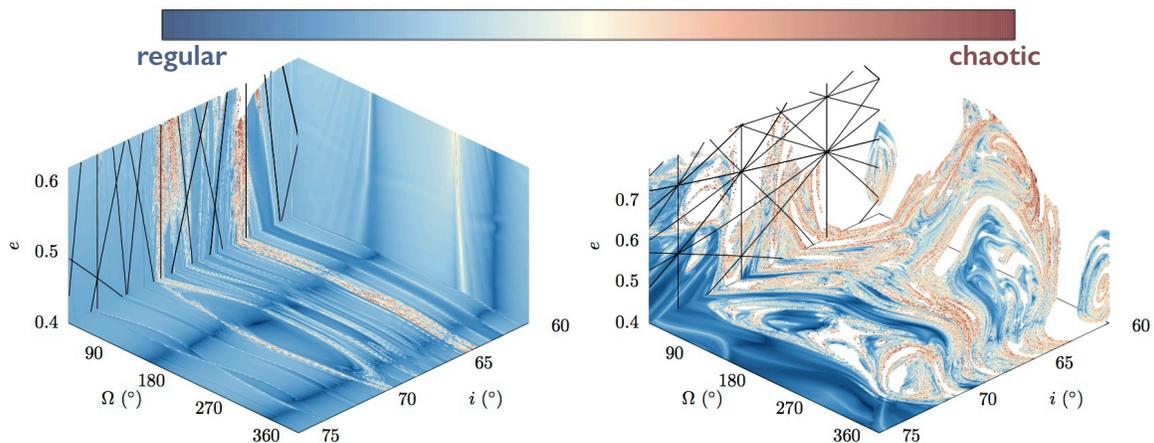


Fig. 5. In the regime of small perturbation (*left*), the pendulum-like structure of the *First Fundamental Model* of resonance is clearly identifiable. By increasing the size of the perturbation (*right*), a bifurcation-life phenomena seems to occur as the initial elliptic equilibrium point becomes hyperbolic where reentry orbits develop. The *left* vertical wall in each ‘cube’ represents a zoomed-in portion of the prograde region of Figure 3, but with reentry orbits being depicted in *white*.

3.2. Design of Disposal Orbits

We have made several important theoretical strides in understanding various aspects of the *secular chaos* phenomenon affecting the Earth’s navigation satellites. We have further probed the resonant dynamics of satellites in the complex *double resonance* environment from LEO to GEO, and rigorously work out their implications in the passive debris removal ideology, accounting also for non-gravitational perturbations. A sample from our extensive cartography is shown in Fig. 6, where the orbit lifetimes of satellites equipped with a large sail (within current capabilities) are presented. We refer the reader to [1] for more details and stability maps, but our main conclusions for each orbital region are summarized below.

- **LEO:** There exists possible disposal routes at high-altitude LEO, related to solar gravity and radiation pressure resonances. It is important to note that the escape times comply with the IADC 25-year rule.
- **MEO:** For the GNSS region, intricate escape hatches are carved by lunisolar secular resonances and widened by SRP. Although direct reentry from very low eccentricities is very unlikely in most cases of interest, we find that a modest “delta-V” budget would be enough for satellites to be steered into a relatively short-lived resonance and achieve reentry into the Earth’s atmosphere within realistic timescales. A detailed study of the GTO region is given in [2].
- **GEO:** There is a natural deficiency of re-entry solutions for geosynchronous orbit, except for high- I , which can be exploited for the Beidou and other highly inclined GEO birds (with dwell times in LEO and MEO on the order of only days). See also [3] for a more detailed study of GEO.

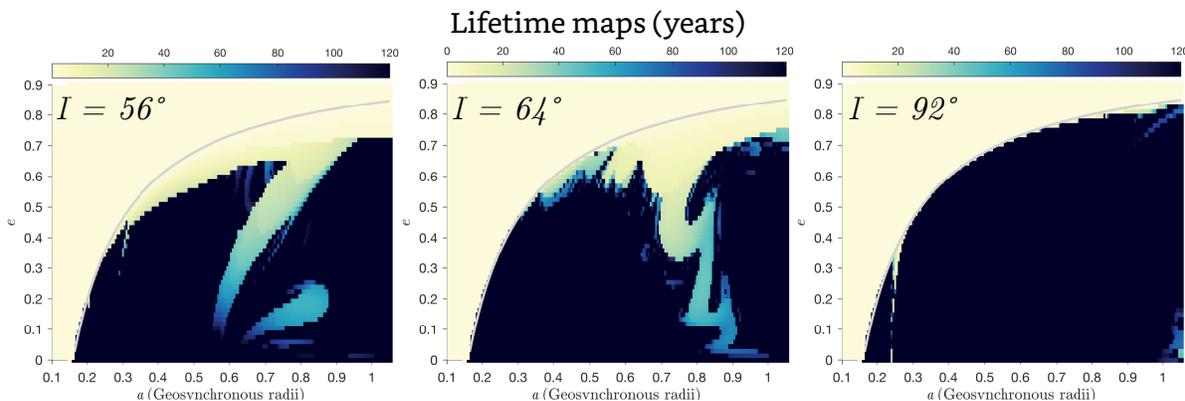


Fig. 6. Lifetime maps of the global LEO-to-GEO phase space in $a - e$ as a function of i , for a particular epoch and satellite orientation angles, and $C_R(A/m) = 1 \text{ m}^2/\text{kg}$. The colorbar is from lifetime 0 to 120 years. Adapted from [1].

In [17], we provided a case study on the European Galileo system that can be used as a reference for the other constellations, and to serve as a springboard for investigating new dynamical situations that may arise. There are two principal resonances affecting Galileo-like orbits and their disposal regions, as depicted in Fig. 7. We noted in [17] how the phase-portrait topology induced by these resonances leads to simple criteria for the definition of the initial parameters of the *graveyard* disposal orbits, which does not require the strict and seemingly arbitrary perigee-targeting scheme advocated by others.

3.3. Launch Window Maps and Designing Orbits for Demise

For the investigation of the Earth’s magnetosphere and the interplanetary space outside of it, satellites with orbits of large semi-major axis and large eccentricity are often used [7]. While end-of-life (EOL) disposal options are well established for missions in LEO (atmospheric decay) and GEO (near circular graveyards), existing mitigation guidelines do not fully regulate the whole, usable circumterrestrial orbital space, such as these highly eccentricity orbit (HEO) science missions; e.g., NASA’s Magnetospheric Multiscale Mission (MMS) and ESA’s INTERNATIONAL Gamma-Ray Astrophysics Laboratory (INTEGRAL). The non-negligible collision risks posed by these LEO-GEO transiting spacecraft has motivated both theoretical study and practical implementation [18].

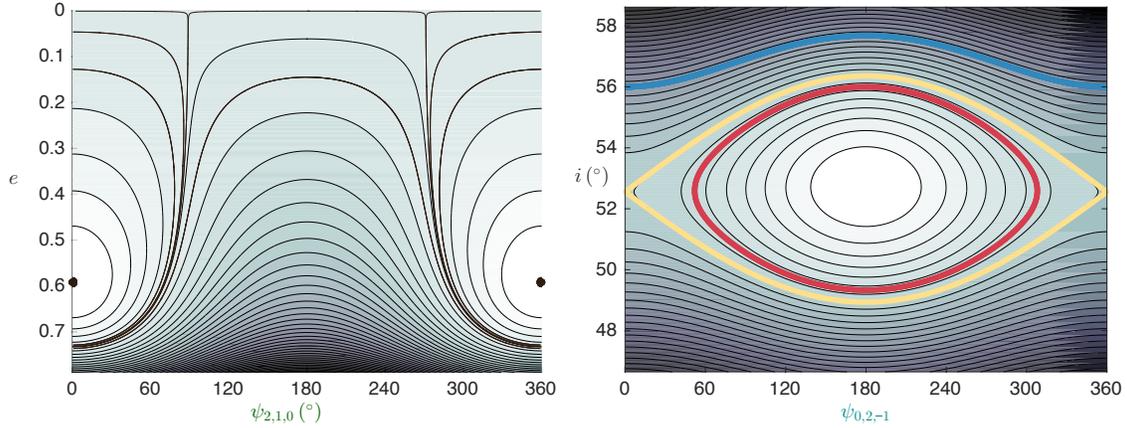


Fig. 7. Phase portraits of the two primary resonances affecting Galileo disposal orbits. The $2\dot{\omega} + \dot{\Omega} \approx 0$ apsidal resonance (*left*) is responsible for large scale secular growth of the eccentricity. The $2\dot{\Omega} - \dot{\Omega}_M \approx 0$ nodal resonance (*right*) does not directly perturb the eccentricity, but there is a significant indirect perturbation because it can shift the orbits inside or outside of the domain of the $\psi_{2,1,0} = 0$ resonance, depending on the initial value of the resonant argument. See [17] for more details.

Explorer VI (1959 Delta 1), with an apogee of 48 700 km, perigee of 6649 km, and equatorial inclination of 47° , represented the first satellite to be deliberately launched into a highly eccentric orbit, prompting the first detailed studies of the perturbing gravitational forces of the Sun and Moon acting on near-Earth satellites. The effects of these distant third bodies were often assumed negligible in comparison to the perturbing forces due to the Earth's oblateness, but in the case of HEOs, were found to change the elements of the orbit to a large degree over extended periods of time. Kozai found that lunisolar perturbations shortened the orbital lifetime of Explorer VI by a factor of ten, and Musen et al. found that it could be as little as a month, depending on the time of day of launch [8]. These results became significant in deciding the launch programs for future satellites with highly eccentric orbits, and the term 'launch window', originally applied to lunar and interplanetary probes to designate minimum-energy missions, was adapted to Earth-satellite missions to indicate when all orbital constraints will be satisfied [19]. Explorer XII (1961 Upsilon) was the first satellite to be launched at a time that was preselected for a minimum orbital lifetime of a year, utilizing lunisolar perturbations to ensure relative stability [20].

The lifetime of a satellite in a highly eccentric orbit depends critically on the initial conditions [8], and a realistic lifetime prediction must consider the atmospheric drag and the gravitational perturbations as acting simultaneously throughout the orbital history [2]. The Orbiting Geophysical Observatory (OGO-1; 1964-054A) is particularly interesting as the launch orbit selected happened to be near the minimum of the long-period oscillation of perigee height on the rising side, thus displaying a very strong perigee rise early in the orbital history. This satellite, launched in September of 1964, still orbits the Earth today, in contrast to earlier predictions that indicated a lifetime of several decades [19]. The INTEGRAL satellite (2002-048A), despite the longevity of its original operational orbit (lifetime greater than two centuries), will now come down in 2029 as its orbit was modified recently via a series of four thruster burns [18].

For all of these science missions described above, the time of launch, and therefore the location of the orbit in space, must be chosen to satisfy a number of limiting constraints. The orbital launch window restraints, imposed by the satellite and experiments, generally entail that the perigee height will not drop below a certain value to minimize the radiation dose and that the orbital lifetime exceeds the nominal mission time; that the eclipse duration is limited and to spend several hours in sunlight between injection and the first eclipse; that the angle between the line of apsides and Earth-Sun line be fixed or other experimental constraints; and to maximize the science collection time and maintain communications.

In all of these launch window studies, the orbital lifetime is a factor, but its reduction or control is hardly a consideration. For the purposes of space debris mitigation and remediation, we proposed incorporating lifetime predictions as a fundamental mission constraint [21]. Doing so will not only ensure that the missions will have predictable behaviors over both the nominal (and possibly extended) mission, thus avoiding an IBEX-like situation, but that the satellites will eventually meet their demise through atmospheric reentry (without the need to make future significant

orbital adjustments, à la INTEGRAL).

Fig. 8 shows the two-line element sets of OGO-1 (formerly called EGO) taken from the ‘Norad’ Resident Space Object Catalog, converted to osculating elements in the J2000 reference frame, and overlaid onto the perigee height evolution as predicted by a highly accurate non-averaged and regularized orbit propagation code [22]. Note the space observations during the operational lifetime of OGO-1, missing data points in the catalog for the next ~ 30 years, and very dense TLEs starting in 2001. While OGO-1 is now predicted to demise in July 2020, had the launch been delayed or advanced by just ten minutes, it would have re-entered decades ago, without compromising the mission.

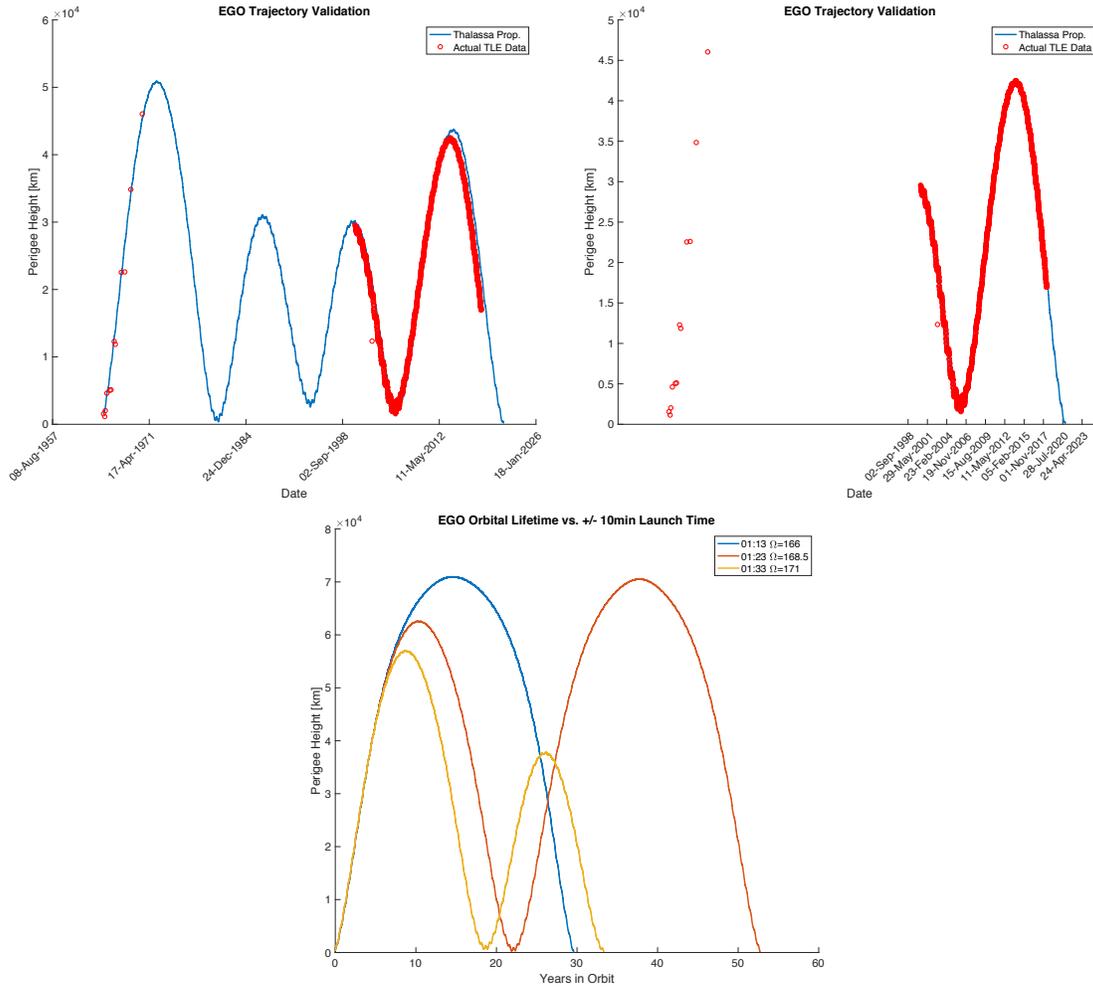


Fig. 8. The cataloged observations of OGO-1 (EGO) and a prediction of its lifetime (*top panels*), and the implications of constraining its lifetime during the launch (*bottom*).

3.4. Satellite Mega-Constellations

An emerging concern for the Earth’s orbital environment is the robustness of the current debris mitigation guidelines, which were based on the continuation of space traffic at the rates observed in the 1990s. Largely governed by geopolitical and economic factors, space traffic has always been subject to considerable fluctuations, however recent trends point to a significant increase of traffic in the low-Earth orbit (LEO) region. Companies such as SpaceX, OneWeb, and Boeing have each submitted ambitious satellite mega-constellation proposals to the FCC; the design of which is shown in Fig. 9 for OneWeb. The implementation of these mega-constellations involves the introduction of thousands of satellites into the LEO environment to provide low-latency broadband internet to the entire world.

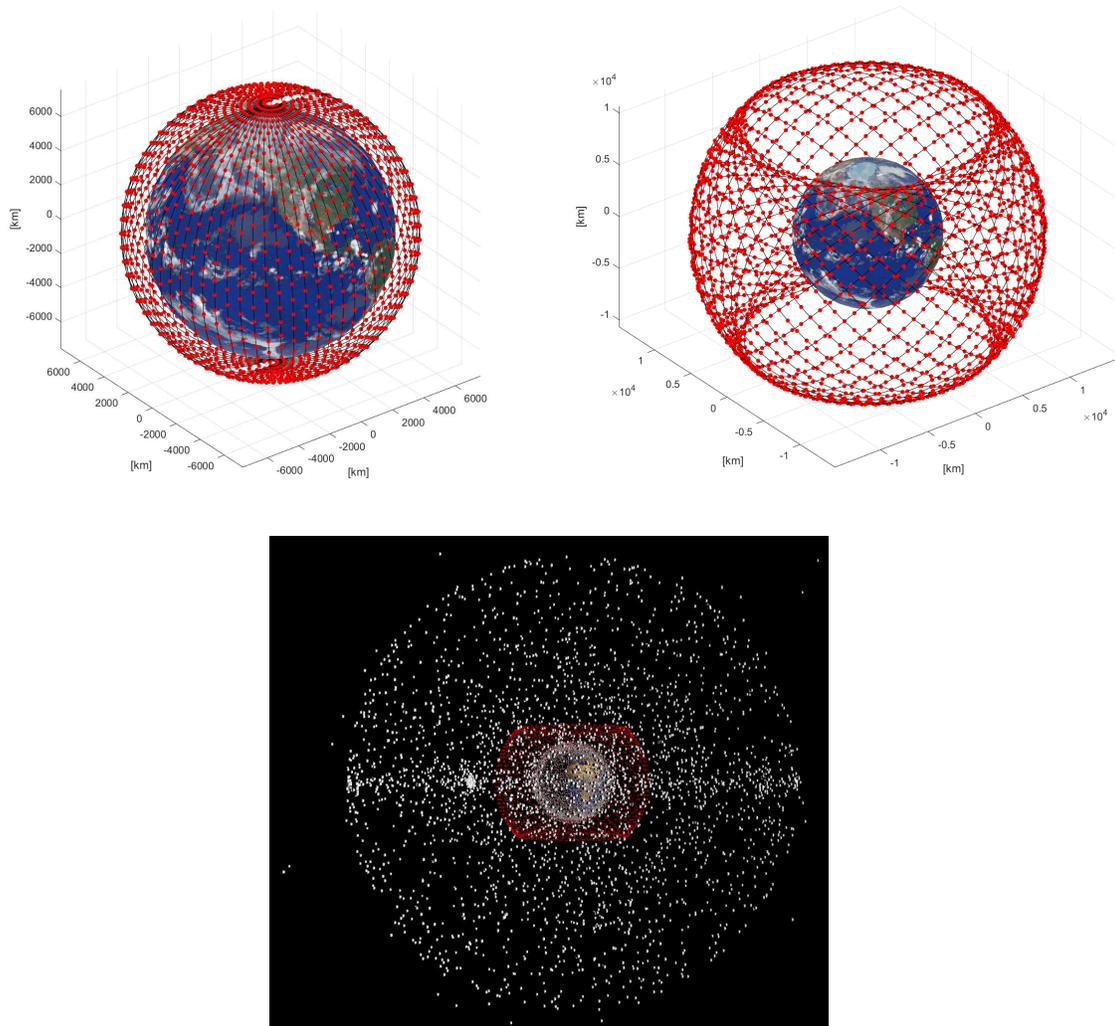


Fig. 9. A depiction of the nominal constellation design of OneWeb’s LEO and MEO components (*top panels*), and the entire OneWeb constellation embedded in the current three-dimensional complex of resident space objects (*bottom*).

We have investigated the effects of the proposed SpaceX and OneWeb constellations on the sustainability of LEO [23, 24, 25], as well as the efficacy of possible passive satellite removal techniques; particularly, the exploitation of dynamical instabilities caused by resonant perturbations. The results of these studies are intended to serve as a baseline for the selection of orbital parameters for the proposed LEO and MEO mega-constellation satellites, which satisfy mission requirements and perhaps employ a passive decommission strategy.

Results are ongoing and will be reported in a future publication. We highlight, however, a new concept known as the Minimum Space Occupancy (MiSO), introduced by Bombardelli [25] as a generalization of the classical *frozen orbits* [26], which is showing great merit in reducing the possible conjunctions of endogenous mega-constellation satellites. The MiSO is also deeply connected with the numerical calculation of RSO *proper elements* [27], which could be used as a dynamical classification of debris into families, among other things.

4. CONCLUSIONS AND OUTLOOK

The solution to the debris proliferation problem, brought on by decades of unfettered space activities, can only be found by coupling mitigation and remediation methods with a deeper understanding of the dynamical environments in which these objects reside. This more holistic vision parallels that of ESA’s Clean Space initiative, fostering innovative techniques and tools in orbital dynamics to novel spacecraft design and manufacturing to reduce the space

industry's environmental impact. We emphasized here the new paradigm of self removal of satellites through natural perturbations (passive disposal), and discussed how lifetime estimates can be incorporated into launch window constraints to ensure the future demise of the satellites. We have developed an extensive set of computational tools (regularized formulations of special perturbation theory and semi-analytical averaged models) for the exploration and simulation of the dynamics of Earth-satellite orbits, accounting for gravitational and non-gravitational perturbations. We are currently applying these capabilities towards the mega-constellations. Such dynamical assessments could have a profound and tangible influence on constellation design, perhaps attacking the debris problem at its source.

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