Statistical Covariance Realism Assessment of LeoLabs' Orbit Determination System

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ABSTRACT

We investigate the covariance realism of LeoLabs' orbital ephemeris data and an applicable metric for consistency monitoring to improve reliability. An overlap analysis has been developed for this work. In this, the Mahalanobis distance between two propagated states is computed over many orbits. We then perform a containment analysis to characterize the Mahalanobis distance distribution. We show that 95.2% of Mahalanobis distances are less than or equal to 4.0, and 97.4% are less than 5.0. Throughout this research, we verify that LeoLabs' covariance keeps its realism through a 7-day of propagation window. We also demonstrate that the Mahalanobis distance distribution follows a log-normal distribution. Lastly, we define two applicable consistency monitoring metrics from the containment percentage and fitted parameters of the log-normal probability density function.

1. INTRODUCTION

As low-Earth orbit (LEO) becomes increasingly congested, more accurate and reliable information is required to protect space objects from potential threats, such as satellite-satellite or satellite-debris collisions. State covariance information is an important part of the information that satellite operators need in order to make decisions regarding whether to maneuver space assets to avoid potential collisions. Therefore, the accuracy of the covariance as it is propagated, called covariance realism or consistency, becomes a critical factor for protecting space assets.

LeoLabs[1] is a commercial provider of low Earth orbit mapping and Space Situational Awareness services with its own global radar network and data platform. LeoLabs currently tracks objects of size roughly 10 centimeters or greater with its radar network. This network is currently comprised of two phased-array radars - the Poker Flat Incoherent Scatter Radar near Fairbanks, Alaska and the Midland Space Radar in Midland, Texas. LeoLabs' Data Platform[2] offers a variety of products, including radar measurements, orbit determination, and conjunction screening. The platform provides information via a web-based API, a graphical web application, and a command line tool.

A critical component of LeoLab's data products are our orbital state vector covariances. Our orbit determination and conjunction screening, for example, are heavily dependent on the accuracy of our covariances. Thus, it is crucial that we test the realism and reliability of our covariance calculations.

In this work, we assess LeoLabs' covariance realism and self-consistency. We compare the differences in overlapping propagated state vectors from different estimation epochs to the propagated covariances. We use the Mahalanobis distance[5] to assess this. Further, we define metrics to monitor the consistency of states via the Mahalanobis distance distribution. These metrics allow us to recognize inconsistent states when they occur.

This paper starts with a brief introduction about LeoLabs' orbit and covariance propagation systems (Section 2). Then, we describe an overlap analysis and expalin in details with an example in Section 3. In the subsequent section (Section 4), a preliminary study is performed on a single object the covariance realism with a larger data set. Section 5 demonstrates the main results from the covariance realism assessment and presents candiated consistency montoring metrics.

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2. ORBIT AND COVARIANCE PROPAGATION

Orbit and covariance propagation is a nonlinear problem. We address this as follows. First, our orbit fitting and updating is done via an Unscented Kalman Filter (UKF), which is well-suited to describing nonlinear systems[6]. Second, we introduce process noise into the UKF to capture unmodeled physics. Third, we apply scaling factors to the resulting covariances[4]. These scaling factors are derived from the estimated state vectors and covariances of 11 reference satellites.

We use the Orekit open source orbital dynamics library[3] for orbit propagation. The dynamics model includes major perturbations in a LEO regime:

- 42×42 Gravity model (JGM-3),
- Atmospheric drag (NRLMSISE-00),
- Solar radiation pressure,
- Third body forces from the Sun and Moon (JPL DE430).

3. OVERLAP ANALYSIS

3.1 Overview

We use an overlap analysis to validate the realism of off our estimated covariances. Fig. 1 illustrates this method. For a given residential space object (RSO), the method sets a 7-day of propagation window[1] from a estimated epoch of reference state and takes pairs with other states within the window. Then, it propagates each pair of states through an overlapping period, from the epoch of the secondary to the end of the propagation window. If the differences between the two states over that period are not large relative to their covariance matrices, then the two states are consistent. We quantify this by calculating the Mahalanobis distances (Eq. 1) between the two propagated states within the overlapping period.

$$d^2 = \left(\vec{X}(t;t_i) - \vec{X}(t;t_j)\right)^T \cdot \left(\mathbf{P}(t;t_i) + \mathbf{P}(t;t_j)\right)^{-1} \cdot \left(\vec{X}(t;t_i) - \vec{X}(t;t_j)\right), \quad (1)$$

where

d : Mahalanobis distance,

$\vec{X}(t_n)$: fitted state at t_n,

$\vec{X}(t;t_n)$: propagated state at t from t_n,

$\mathbf{P}(t;t_n)$: propagated covariance at t from t_n.

The overlap analysis has the following steps:

1. Propagate a primary state $\vec{X}(t_i)$ to form state vector pairs with all secondary states $\vec{X}(t_j)$ in the propagation window $(t_j - t_i) < 7\,\text{days}$.
2. For a given state vector pair, propagate both state vectors and associated covariance matrices during the overlapping period t_j to $t_i + 7\,\text{days}$.
3. Compute Mahalanobis distance at each propagation step.
4. Select the maximum Mahalanobis distance during each 24-hour period of the time offset $(t - t_i)$.

Using a 150 second propagation interval, we produce at most 576 Mahalanobis distances per 24 hour period. Only the maximum of these is kept during the final step of the analysis. The maximum is chosen because it is the most sensitive statistic to outliers. Thus we are finding the worst-case inconsistency between the state vector pair.

[1]The window length is chosen to match our conjunction services, which provide notifications up to 7 days prior to the time of closest approach of a possible collision.

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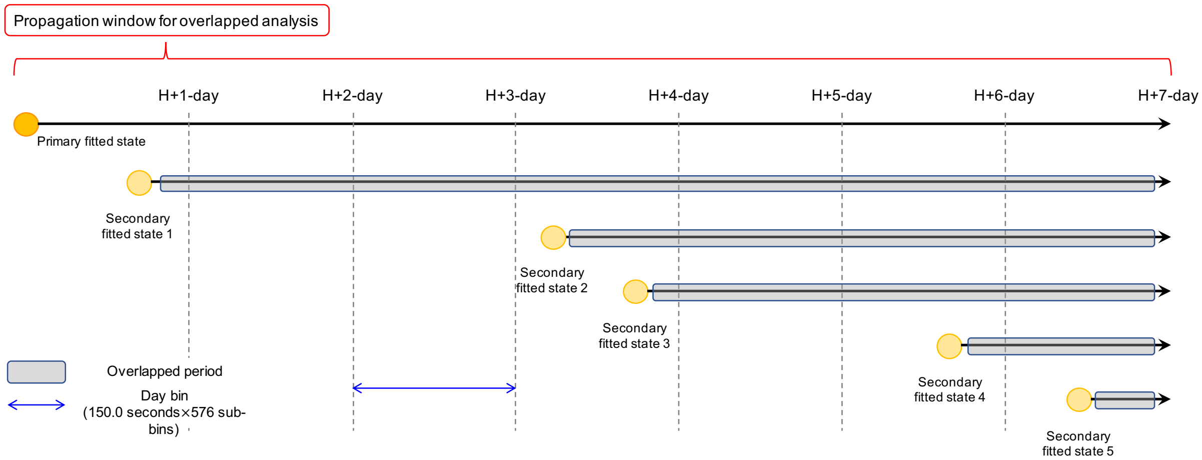

Fig. 1: Overlap analysis for a single object with all pairs of state vectors within a 7-day of propagation window.

3.2 Application

We provide an illustrative example of the overlap analysis works and what information comes through it. A single pair of state vectors from the satellite COSMOS 2263 (NORAD ID 22802). Fig. 2 shows the propagation window, overlapping period, and the selection of maximum Mahalanobis distance from each 24 hour period..

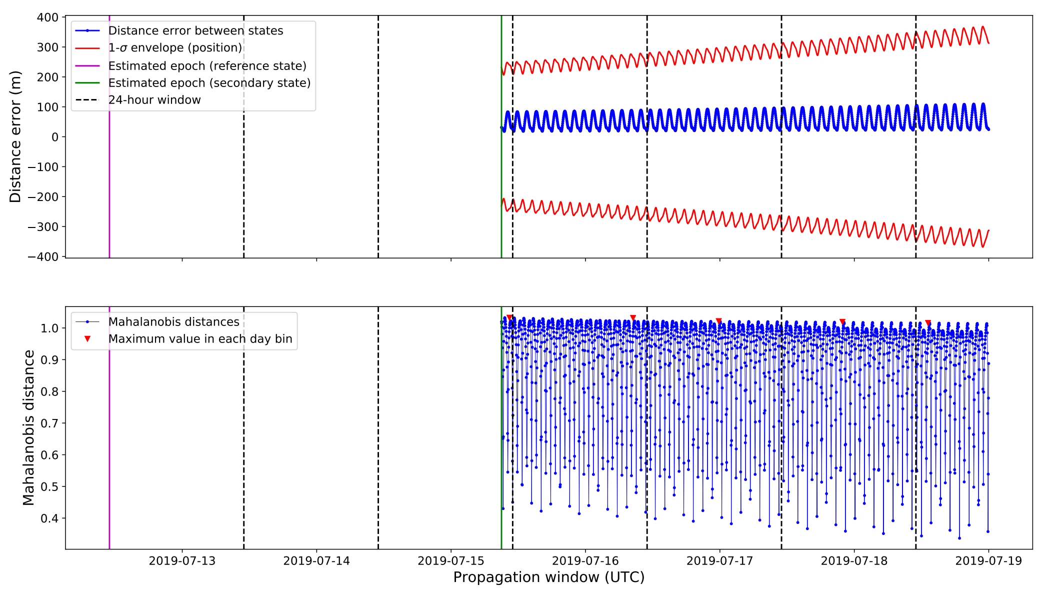

Fig. 2: The overlap test period for a pair of states of COSMOS 2263. The epoch of the secondary state (green) was about 3 days after the reference epoch (magenta). The overlapping period was about 4 days.

The top panel shows the position distance between two states and the expected 1-σ error predicted by the combined covariance matrices over the overlapped period (steps 1 and 2). The bottom panel shows the 3D position Mahalanobis distances calculated from the pair of states (step 3). The calculated Mahalanobis distances are separated into 24 hour periods, and the maximum values within each 24 hour period are selected (step 4). These maxima are shown as red points in the bottom panel. In this example, five such maxima are selected. In general, the number of values in the output depends on the separation of the epochs of the two states.

For the covariance realism assessment, the overlap analysis collects representative Mahalanobis distances from all objects and all pairs of states. This is done for both the 3D position and the 6D state vector Mahalanobis distances.

4. MAHALANOBIS DISTRIBUTION FOR A SINGLE OBJECT

We now consider the distribution of Mahalanobis distances for a single RSO. We do this by collecting data from all paired states within the propagation window. The same object, COSMOS 2263, is used for this study. For this case, we get 2,196 Mahalanobis distances by considering all paired states from 40 state vectors. The propagation window of this example is from 07/12/2019 to 07/19/2019.

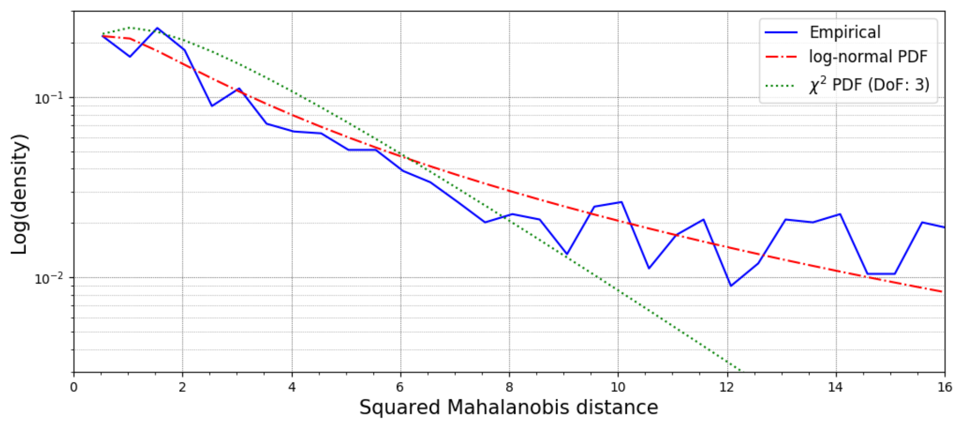

Fig. 3: Logged density of the overlap test results (blue) from all pairs of states (COSMOS 2263): theoretical PDFs for log-normal (red) and χ^2 (green) distributions are overlapped on the result

Fig. 3 shows the probability distribution function (PDF) of 3D positions Mahalanobis distances and two theoretical PDFs of log-normal and χ^2 distributions. Note that 13.5% of Mahalanobis distances had values greater than 4 and are not shown in this figure.

Theoretically, it is typically expected that the distribution of Mahalanobis distances will follow the χ^2 distribution. However, the figure shows the log-normal distribution gives a better fit. The χ^2 distribution assumes that the state vector covariance is Gaussian and that the Mahalanobis distances values are uncorrelated with each other. However, the nonlinear state and covariance propagation does not guarantee Gaussian covariance, and the overlap analysis does not guarantee uncorrelated Mahalanobis distances. The log-normal distribution, on the other hand, is preferred by the central limit theorem in the case where a random number is the multiplicative product of many random variables. Therefore, the appearance of the log-normal distribution in this analysis is not surprising given the complexity of our orbit propagation system and Mahalanobis distance calculation procedure.

5. CATALOG ANALYSIS

We apply the overlap analysis to all objects in the LeoLabs catalog during the 2 week period of 07/07/2019 to 07/21/2019. This includes 83,787 state vectors and 1,878,827 Mahalanobis distances for 8,100 RSOs. In this analysis, we use 3D position Mahalanobis distances and 6D state vectors Mahalanobis distances without considering cross correlation terms.

In order to test the covariance realism, we perform a containment analysis. We estimate an empirical PDF of our Mahalanobis distances, and we fit a log-normal distribution to that PDF. We use the empirical PDF to calculate what percentage of Mahalanobis distances fall below certain Mahalanobis distance thresholds. Further, we use the fitted log-normal parameters as covariance consistency monitoring metrics.

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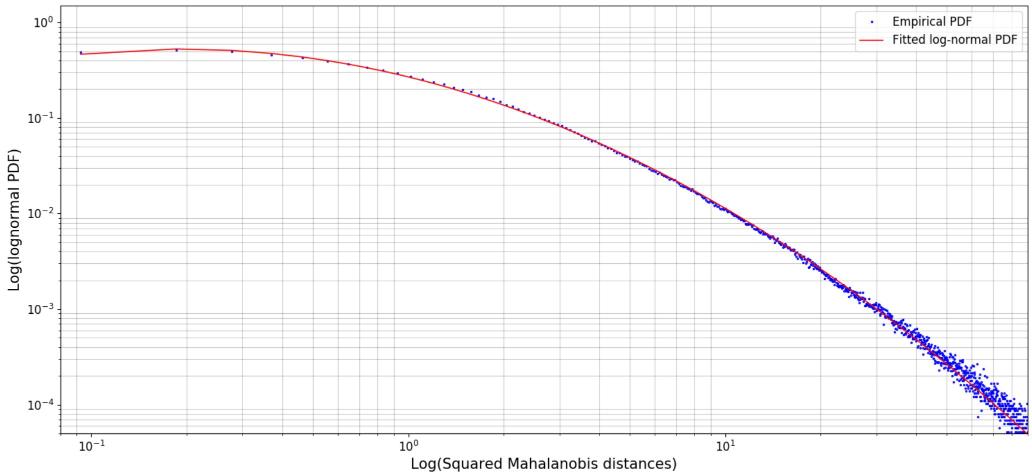

Fig. 4: Log scaled PDFs with all Mahalanobis distances during the proagation window: empirical distribution vs. log-normal distribution

Our empirical PDF and the fitted log-normal PDF for 3D position Mahalanobis distances are shown in Fig. 4. The log-normal PDF is described by this equation:

$$f(d^2) = \frac{1}{\sqrt{2\pi}\sigma d^2} \exp\left[-\frac{\left(\ln d^2 - \ln m\right)^2}{2\sigma^2}\cdot\right], \tag{2}$$

where σ and m are related to the width and mean of the distribution, respectively.

The log-normal PDF was fit to our empirical distribution using only those Mahalanobis distances less than 10. 99.8% of the Mahalanobis distances fall within that range. Table 1 shows the fitted parameters for the 3D position Mahalanobis distances and for the 6D state vector Mahalanobis distances. These are denoted $N = 3$ and $N = 6$ degrees of freedom, respectively.

Table 1: Fitted parameters for the log-normal PDF (N: degrees of freedom)

N	σ	m
3	1.4294	1.4687
6	1.4954	2.4828

Table 2 shows the containment percentiles for various Mahalanobis distance values. The table shows this for the 3D position and 6D state vector Mahalanobis distances (ie, $N = 3$ and $N = 6$ degrees of freedom). Percentiles for the χ^2, log-normal, and empirical distributions are shown. Note the close agreement between the log-normal and empirical values.

We now wish to investigate the covariance realism as a function of state pair time separation. We separate the state vector pairs into groups based upon their time separation, and we perform the containment analysis on each group separately. This allows us to calculate the containment for states that are separated by less than 1 day, separated by between 1 and 2 days, and so on. Fig. 5 shows the resulting containment analyses for a Mahalanobis distance threshold of 4. Containment vs. time separation of paired states is quite flat, indicating that our covariances retain their consistency over a 7 day propagation timescale.

Throughout this research, we get two applicable metrics, containment percentage and fitted parameters of the log-

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Table 2: Containment Analysis for overall propagation window
(N: degrees of freedom, d: Mahalanobis distance)

	$N=3$			$N=6$		
d	χ^2	Log-Normal	Empirical	χ^2	Log-Normal	Empirical
1.0	19.9	39.4	40.0	1.4	27.2	27.6
2.0	73.9	75.8	76.5	32.3	62.5	64.4
3.0	97.1	89.8	89.9	82.6	80.5	82.9
4.0	99.9	95.3	95.2	98.6	89.4	91.3
5.0	100.0	97.6	97.4	100.0	93.9	95.3
6.0	100.0	98.7	98.5	100.0	96.3	97.3
10.0	100.0	99.8	99.7	100.0	99.3	99.6

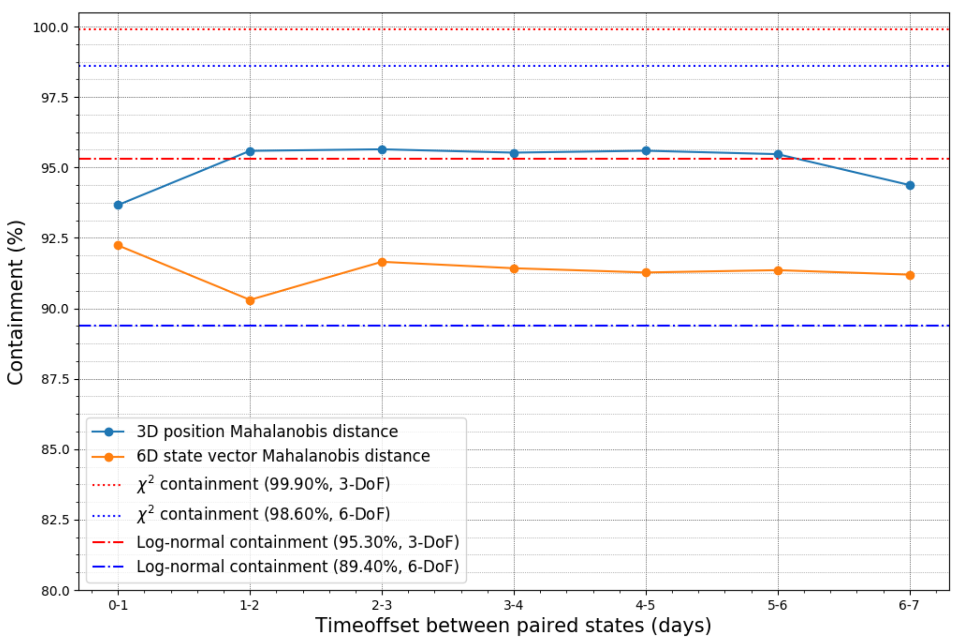

Fig. 5: Containment analysis from 3D and 6D state vector Mahalanobis distances for each 24-hour period group for a Mahalanobis distance theshold 4.

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normal PDF, that can be used for consistency monitoring. Both of these metrics inform a general tendency of LeoLabs' data distribution for a given data collection window. Therefore, periodic updates of those metrics through a sliding window iteration enables us to monitor the consistency of LeoLabs' data.

6. CONCLUSION

We present the covariance realism of LeoLabs data and define the consistency monitoring metrics to improve reliability. We perform a containment analysis to quantify the consistency of our state covariances. This test shows that 95.2% of LeoLabs' propagated states are self-consistent within Mahalanobis distances of 4 (97.4% if the criteria is 5). We note that the empircal Mahalanobis distance PDF is well-described by a log-normal distribution. The consistency monitoring metrics are the containment percentage and fitted parameters of the log-normal PDF. The periodic updates of the metrics allow us to monitor the consistency of LeoLabs' data. In future works, we will investigate the time variations of the consistency metrics.

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