Assessing and Minimizing Collisions in Satellite Mega-Constellations

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ABSTRACT

We aim to provide satellite operators and researchers with an efficient and effective means for evaluating collision risk during the design process of mega-constellations as well as insight into how the proper dynamical placement of orbiting systems can greatly reduce collision risk and the need for avoidance maneuvers. Current algorithms and software tools for assessing collision probabilities of satellites are not sufficiently robust for the forthcoming orbital environment with the deployment of many thousands of telecommunications satellites in low-Earth orbit (LEO). We report on satellite constellation configurations that take advantage of the natural dynamics of the Earth-Moon-Sun-satellite system in order to minimize the rate of satellite-satellite close encounters.

First we establish a baseline for evaluating various techniques (Hoots et al. 1984, Gronchi 2005, JeongAhn and Malhotra 2015) for the prediction of satellite-satellite close encounters by carrying out brute-force numerical simulations. We integrate the equations of motion with a sufficiently-small time step, vis-à-vis the relative velocity of the satellites, and then compare their trajectories. Drawing inspiration from Hoots et al. (1984), we implement a sequence of filters, so as to reduce the computational expense of the algorithm. Each subsequent filter consists of integrating the equations of motion with a smaller time step than the previous filter. We test the efficacy of this technique by creating a number of hard-to-detect collision events and propagating the trajectories of the involved objects backwards in time to an initial epoch.

The brute-force approach is then applied to the anticipated orbital environment following the deployment of the OneWeb mega-constellation. Using the satellite-satellite close-encounter baseline, we evaluate the efficacy of the previously cited encounter-prediction and collision-probability algorithms. Of particular interest is a modern formulation of the classical collision-probability technique of Öpik and Wetherill (JeongAhn and Malhotra 2015, 2017). This technique computes the collision probability per unit time of two objects in Keplerian orbits. We modify this technique to address the perturbed-Keplerian dynamics in the LEO environment. Following JeongAhn and Malhotra (2015), we create a distribution of clones to simulate the dynamical evolution of impacting objects having a distribution of initial conditions or of orbital parameters. Next, we compute the total collision probability by taking the sum of collision probabilities of the clones with the target satellites of interest with correction factors to account for the multiplicity of clones per target object in addition to the inflation of the collision radius. The greatest uncertainties in this approach are the generation of the distribution of clones as well as assumptions placed on the motion of the target objects. For various clone distributions and assumptions, we assess the validity of this fast method by means of a comparison with the brute-force close-encounter baseline simulations.

As a further step, in order to respond to increasing space traffic rates in a more dynamical fashion, we investigate the efficacy of so-called Minimum Space Occupancy (MiSO) orbits. MiSO is a generalization of the well-known "frozen orbits" (in the natural dynamics of the Earth-Moon-Sun-satellite system) and are possibly the best solution to truly minimize collisions and/or avoidance maneuvers. Using the aforementioned techniques for collision-probability calculation and close-encounter prediction, we evaluate the ability of MiSO configurations of the proposed OneWeb mega-constellation to reduce the risk of endogenous (intra-constellation) collisions. Preliminary results indicate that the application of the MiSO algorithm can significantly reduce this risk with very minor adjustments to the nominal orbital elements of the constellation satellites.

1. INTRODUCTION

There are currently around 18,200 entries in the space object catalog (SOC) of publicly-available tracked resident space objects (accessed September 2019); however, this number only accounts for those RSOs that are larger than

10 cm in diameter [10, 17]. In reality, there many more objects that are either untracked or are not made publicly available (∼23,000 larger than the LEO-detection threshold and an estimated population of 500,000+ larger than 1 cm). Because these objects are not dispersed uniformly, certain regions of circumterrestrial space are inherently more dangerous than others. In fact, the vast majority of those 18,200 RSOs (around 14,100) reside in low-earth orbit (LEO) and with the inevitable introduction of mega-constellations into near-Earth space, the risk of collision for satellites in LEO will continue to increase (see Fig. 1).

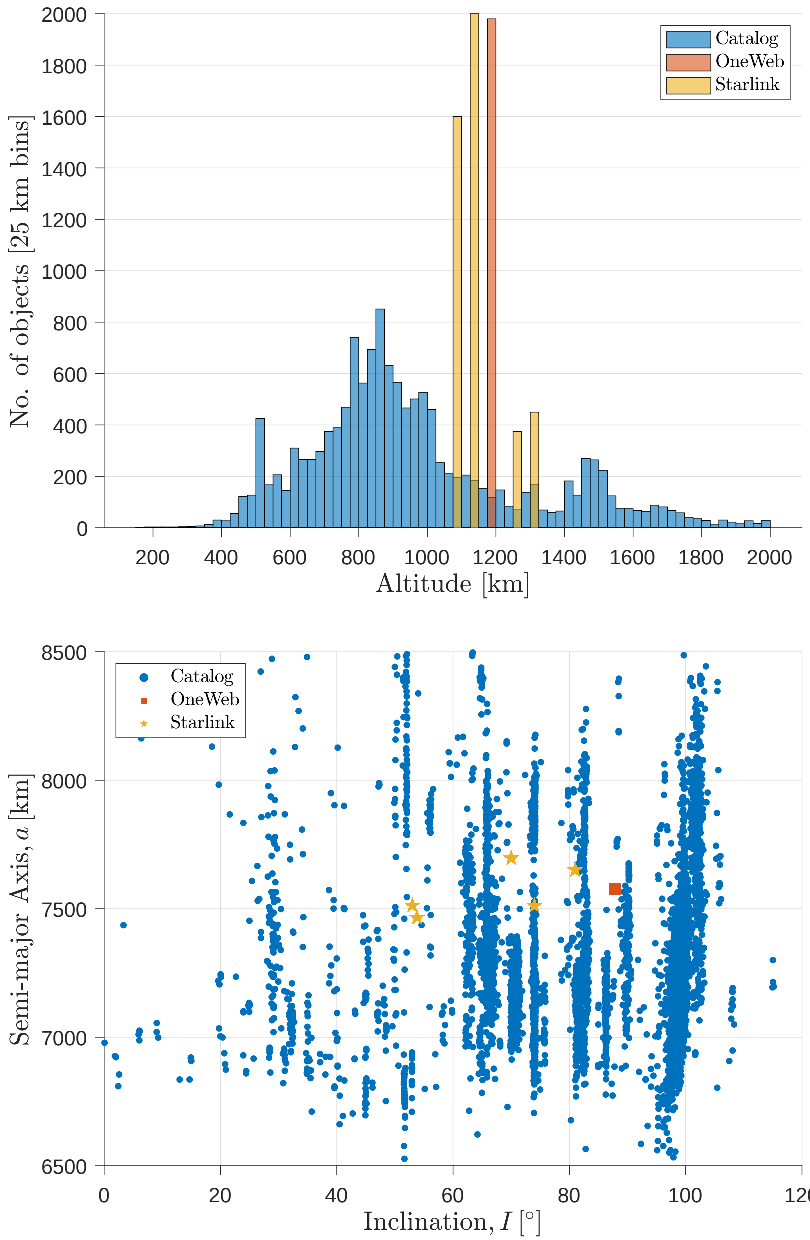


Fig. 1: The addition of the OneWeb LEO and SpaceX Starlink mega-constellations alone will increase the number of objects in LEO by 46 percent.

The threat of collisions between artificial satellites first gained mainstream attention following the landmark 1978 paper by Donald J. Kessler and Burton G. Cour-Palais [15], where the number of artificial satellite collisions per year is predicted. Although Kessler's calculations rely on a significant number of simplifications and assumptions, he accurately predicted that a collision would occur between 1989 and 1997 (recall the 1996 collision between a French military satellite and rocket debris). Moreover, recent events, such as the 2009 Iridium–Cosmos collision between intact satellites serve to further validate his predictions. Although Kessler's methods may tend to inflate the situation, he was able to accurately capture the correlation between the increase in launched objects and in-orbit collisions. Therefore, it is reasonable to assume that the addition of mega-constellations into the already crowded LEO region calls for more rigorous collision-prediction methods, than have been previously employed in order to prevent the realization of Kessler's most drastic conclusion—the conglomeration of artificial orbital debris into a ring system about the Earth [16, 24].

Although these mega-constellations will have the remarkable capability to provide the entire planet with low-latency broadband internet, their addition to the LEO environment could, under certain circumstances, be catastrophic. Currently, the only two mega-constellations that have FCC approval are the OneWeb LEO constellation and the SpaceX Starlink constellations (FCC filing no. SAT-LOI-20160428-00041 and SAT-MOD-20181108-00083, respectively), each of which have constellation satellites currently in orbit. The following work focuses on OneWeb, and utilizes for initial conditions (ICs) the initial orbital elements of the OneWeb LEO constellation made public by OneWeb through the FCC database (SAT-LOI-20160428-00041). It is important to note that the "real" initial placement of the OneWeb LEO satellites will likely be quite different and consequently the goal of this work is not to criticize a "dummy" constellation (although the FCC-reported configuration is indeed dumb). Rather our aim is to demonstrate how several new astrodynamical tools could be used by satellite operators and other researchers to investigate the "safety" (meaning the probability of collision between two satellites belonging to the same group) of mega-constellations, or any arbitrary set of artificial satellites and even reduce the risk of collision with slight modifications to the constellation's ICs. First we develop a brute-force close-approach prediction algorithm utilizing regularized equations of motion to establish "truth" for the number and severity of *endogenous* and *exogenous* satellite-satellite and satellite-debris close approaches experienced by one orbital plane of satellites within the OneWeb LEO constellation. Next, the performance of the Hoots [12] and Malhotra [13, 14] close-approach-probability algorithms are evaluated against this baseline. Finally we compare the collision risk between the same plane of the OneWeb LEO constellation and its corresponding minimum space occupancy (MiSO) variant [8].

Past research efforts have been limited by the computational expense of orbit propagation and as a result have been forced to resort to simplified models and *handwavy* probability calculations. While these methods certainly serve an important purpose and may yield with some level of accuracy; whether or not a collision is *possible*, they are simply ill-suited to deal with day-to-day satellite operations of our modern era where Space Traffic Management (STM) is now imperative. For example, the satellites of mega-constellations such as OneWeb are placed in nearly intersecting orbits by design. Of course, classic analytical collision probability techniques such as Opik (1951) and Whetherhill (1967) will indicate that these objects are at risk [18, 23]; however, when considering two satellites operating within such close proximity, high accuracy orbit propagation is required to determine if there is a significant "real" collision risk that warrants a maneuver as a response.

The nominal implementation of the OneWeb LEO constellation used in this research, hereafter referred to simply as OneWeb LEO, will contain 1980 satellites in highly inclined circular orbits (Fig. 1). These satellites will be distributed evenly throughout 36 planes at an altitude of 1200 km (i.e., 55 satellites per plane), which will be spaced evenly about the earth from 0 to 178.5 degrees in RAAN (right ascension of the ascending node). The modified MiSO variant contains the same number of planes and satellites per plane; however there are slight variation in the altitudes, inclination, and right ascension of the orbital planes relative to one another.

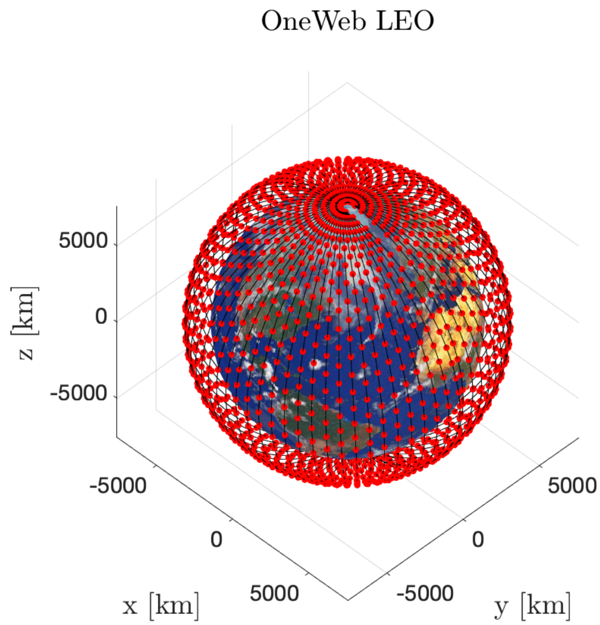


Fig. 2: Nominal OneWeb LEO Constellation according to FCC filing no. SAT-LOI-20160428-00041 and Nominal SpaceX Starlink NSGO Constellation according to FCC filing no. SAT-LOA-20170301-00027.

2. REGULARIZED FORMULATIONS FOR FAST-ORBIT PROPAGATION

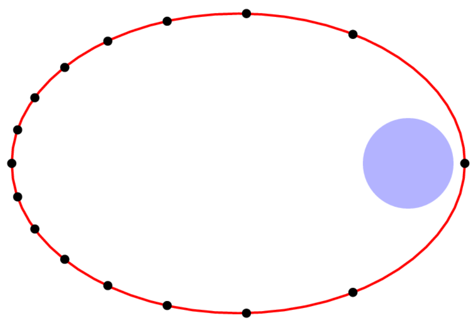
Earth-satellite dynamics is best described by the *perturbed two-body problem*, which, in Cartesian coordinates, can be stated as $\ddot{\boldsymbol{r}} = -(\mu/r^3)\boldsymbol{r} + \boldsymbol{F}$, where $-(\mu/r^3)\boldsymbol{r}$ is the primary (Keplerian) acceleration and $\boldsymbol{F}$ is the vector sum of perturbing accelerations due to the non-sphericity of the Earth's gravitational field, the gravity of external "third" bodies (i.e., lunisolar perturbations), atmospheric drag, etc.

Numerical solutions to the perturbed-Kepler problem are often generated through *Cowell's method*; that is, the integration of the equations of motion in Cartesian coordinates with a numerical solver (the most basic formulation of *special perturbation theory*). This approach, while simple and robust, is computationally inefficient. In particular, the presence of a singularity causes large oscillations in the magnitude of the right-hand side, which are aggravated with increasing eccentricity and unstable error propagation characteristics [9]. These disadvantages can be mitigated or eliminated altogether by employing equations of motion (or *formulations* of the perturbed two-body problem) that have been *regularized*.

In regularized formulations, the independent variable is transformed from the physical time to a fictitious time through *the generalized Sundman transformation*. Using fictitious time as the independent variable gives an immediate advantage: since the fictitious time is an angle-like quantity, meshing the orbit uniformly results in a distribution of points whose density can be adjusted by appropriately choosing numerical parameters. One can select, in particular, a uniform distribution rather than one that is densest at apoapsis, as in Fig. 3. Regularized equations are also stable with respect to the propagation of numerical error, unlike the Cowell method [20], and can be linearized without expressing the perturbations explicitly, thus with no need to truncate expansions in perturbation parameters [22]. Variation of parameters or projective decomposition can also be employed to obtain regularized, nonsingular sets of orbital elements, which are particularly advantageous for weak perturbations [19, 6].

A collection of regularized formulations is contained in the `THALASSA` Earth-satellite orbit propagation tool [4, 3, 2], which is freely available through a GitLab repository.[1] `THALASSA` uses the variable step-size and order (up to 12$^{\text{th}}$) `LSODAR` solver to numerically integrate the equations of motion, which automatically selects the solution algorithm between the implicit Adams-Bashforth-Moulton and backwards differentiation formulas. Even when using such a sophisticated, adaptive solver, regularized formulations have been shown to radically improve computational efficiency

[1] URL: `https://gitlab.com/souvlaki/thalassa`.



(a) Physical time. (b) Fictitious time.

Fig. 3: Uniform spacing of points along an orbit in physical and fictitious time coinciding with the eccentric anomaly. Adapted from [7].

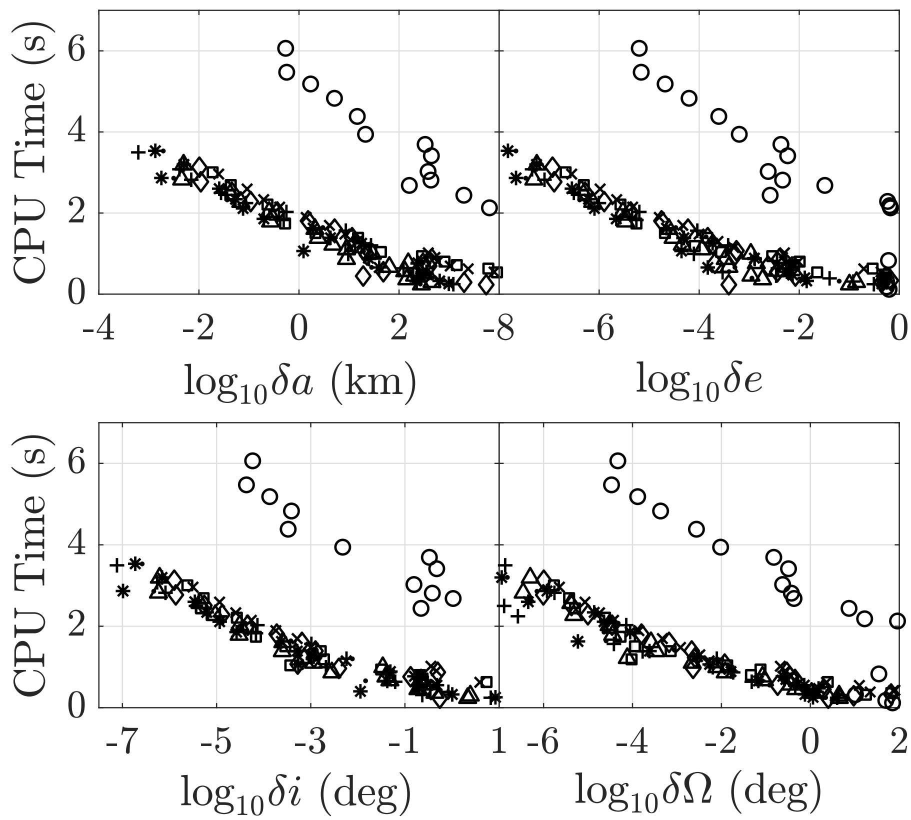


Fig. 4: CPU time for 75-year integrations of a HEO against error in orbital elements. Circles denote Cowell's method, other symbols denote regularized formulations. Adapted from [2], to which we refer for further details.

in the long-term propagation of highly elliptical Earth-satellite orbits (HEOs). Fig. 4 shows that regularized formulations improve accuracy by up to six orders of magnitude with respect to Cowell's in the integration of HEOs.

The numerical orbit propagation engine in THALASSA implements the Kustaanheimo-Stiefel [22, ch. 2], Stiefel-Scheifele [22, ch. 5], Dromo [5, 19], and EDromo [6] regularized formulations. These formulations are chosen due to their optimal performance in a wide range of dynamical configurations [2, 1, 20]. The equations of motion are integrated with adaptive numerical solvers in modern Fortran.

3. BRUTE-FORCE APPROACH

The developed brute-force algorithm leverages the `THALASSA` [2] orbit propagator, with the EDromo [6] formulation of the equations of motion. We adapt this high-fidelity astrodynamics tools to be highly parallelizable, enabling rapid and accurate propagation of thousands of RSOs.

A given set of "target" objects and potentially impacting "field" objects are passed through three stages of filters that compare the trajectories in order to determine the occurrence of close approaches within some specified distance. First the set P containing all pairs of potentially colliding objects is defined. Next the vis-viva equation (Eq. 1) is used to calculate the maximum relative velocity, $v_{\max}$, between all objects of the set using the initial osculating orbital elements, where μ is the gravitational parameter of the primary body (Earth), r and v are the satellite's relative position and velocity, respectively, and a is the semi-major axis.

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right). \tag{1}$$

Next, the integration time step is chosen based on the selected approach distance between the target and field objects, D_{ca}, according to Eq. 2, where f_s is a factor of safety (typically a value of 2 is used). This simple step ensures that no close approaches greater than D_{ca} will be neglected.

$$t_{step} = \frac{1}{f_s}\left(\frac{D_{ca}}{v_{max}}\right). \tag{2}$$

The initial states of the target and field objects are then propagated forward over the time span of interest and the Cartesian separation distance (Euclidean norm), r_{sep} between each set of objects in P at each time step is calculated. Objects with trajectories satisfying $r_{sep} < D_{ca}$ are passed on to the next stage. Before beginning propagation of the set of objects not eliminated by the previous filter a new time step of integration is calculated according to Eq. 1 to determine the new v_{max} of the filter stage and a significantly smaller value of D_{ca} is chosen. Performing the computations with this structure greatly increases the efficiency of the algorithm without sacrificing accuracy. The structure outlined in Fig. 3 was implemented using C programming and the MPI message passing interface for parallelization. The resulting program, `FICA` (Fast Integrations For Collision Assessment), was run on the University of Arizona's (UA's) High Performance Computing (HPC) cluster on over 200 CPUs. The structure of the algorithm allows for the efficient parallelization of the propagation and trajectory-comparison portions of the code.

begin subroutine: filter-stage:

1. calculate maximum relative velocity of objects in given set;
2. determine appropriate timestep;
3. propagate object states forward on separate threads;
4. compare sets of object trajectories on separate threads;
5. return objects which require propagation with a smaller timestep;

end

begin algorithm: brute-force:

1. generate set containing all pairs of colliding objects;
2. pass set to **filter-stage** using an intersect distance, d = 20 km;
3. use returned objects to generate a new set of pairs;
4. pass set to **filter-stage** with d = 5 km;
5. use returned objects to generate a new set of pairs;
6. pass set to **filter-stage** with d = 1 km;

end

Fig. 5: Brute-force algorithm pseudo code.

4. HOOTS INSPIRED COLLISION DETECTION

The Hoots method [12] is an "analytic method to determine future close approaches between satellites." This method uses a series of filters to quickly determine which objects could potentially experience a close approach. In this work, the classic Hoots algorithm is expanded upon using the regularized orbit propagator, `THALASSA` (§ 2). First `THALASSA` is used to generate a table of ephemerides for both target and field objects. At each time step of integration, a series of three analytical filters are employed to greatly reduce the number of possible close approaches. The object pairs, which pass through the filters are then further propagated using a much smaller time step in order to eliminate false positives, and determine times of approach as well as approach distances.

The first filter that the target and field objects are passed through is designed to eliminate those field objects whose geometry does not allow the two orbits to ever come within the specified approach distance D_{ca} (e.g., a circular LEO vs circular GEO). This is done by comparing D_{ca} with the maximum of the target and field objects perigee height, q, and the minimum of the target and field object's apogee height, Q. Thus the combination of field and target object must no longer be considered if the following condition is met:

$$q - Q > D_{ca} \tag{3}$$

If the combination of field and target objects is not eliminated by the first filter, then it is passed through a second filter, which uses the well-known concept of minimum orbital intersection distance (MOID) to determine if an approach can occur. The Hoots algorithm utilizes a second-order Newton-Raphson method to calculate the MOID by minimizing the relative separation distance, r_{rel} between the primary and secondary objects. Equation 4 is solved simultaneously to yield the values of true anomaly (θ_p and θ_s for the primary and secondary objects, respectively) where r_{rel} is minimized. In recent years, more advanced methods such as those developed in Gronchi (2005) interpret the equations for the critical points between two Keplerian orbits as a polynomial system in order to calculate *all* of the critical points as well as to avoid the difficulties associated with picking a good starting point for the Newton-Raphson root finding method [11]. Despite this, Gronchi's algorithm has a tendency to struggle with near-circular, nearly-intersecting orbits, which are of course prevalent in the considered problem.

$$\begin{aligned} \frac{\partial {r_{rel}}^2}{\partial \theta_p} &= 0 \\ \frac{\partial {r_{rel}}^2}{\partial \theta_s} &= 0 \end{aligned} \tag{4}$$

The third filter consists of the generation of sets of time windows as the target and field objects pass through the line of intersection of their orbits and are briefly vulnerable. This is done by determining the window of true anomaly values for both target and field objects when the distance from their position to the orbital plane of their counterpart is within D_{ca}. Using Kepler's equation, these angular windows in true anomaly can be converted to eccentric anomaly and finally time. If the time windows of the target and field objects do not ever coincide, then the combination of target and field objects is no longer considered.

The pairs of objects in each time step, which are not eliminated by the filters are then propagated forwards in time and their Cartesian separation distance is calculated at each time step to determine a time of closest approach as well as close-approach distance.

5. MALHOTRA COLLISION PROBABILITY ALGORITHM

The study of orbital collision probability has its roots within Solar System dynamics and was pioneered by Öpik and Whetherhill in 1951 and 1967, respectively, to study collisions within the asteroid belt [18, 23]. The theory begins with the calculation of the collision probability for two intersecting Keplerian orbits, $P(\tau, \vec{\alpha}_1, \vec{\alpha}_2)$, which is a function of the orbital elements of the target and field objects, $\vec{\alpha}_1$ and $\vec{\alpha}_2$, respectively, and the collision distance, τ. Here, being Keplerian, the orbits are fixed in space and the mean anomalies are assumed to be independent. Over a long period of time the objects will have a well-defined collision probability at their intersection. In JeongAhn and Malhotra (2017), a simplified but equivalent derivation of the collision probability of two objects in Keplerian orbits is developed (Eq. 5), where P_i is the collision probability, τ is the collision distance, $\vec{v}$ is the velocity at the point of closest approach, U is the relative velocity of the objects at the point of closest approach, and T is the orbital period [14].

$$P_i = \frac{\pi \tau U}{2|\vec{v}_1 \times \vec{v}_2| T_1 T_2}. \tag{5}$$

In order to contribute to collision prevention in near-Earth space, the probability with respect to an ensemble of fields is required. In the past, studies utilizing the techniques descended from Öpik and Whetherhill have been in large part limited to the field of Solar System dynamics, where the orbital planes of objects typically change at much slower rates and extremely large sets of objects are considered (the asteroid belt, for example). As such, the semi-major axis (a), eccentricity (e), and inclination (i) of the target and field objects are assumed to be fixed, while the right ascension of the ascending node (Ω), argument of perigee (ω) and τ are assumed to be random stochastic variables. Of course, when considering artificial, Earth-orbiting satellites, such an approach is not suitable as the orbital parameters can change at extremely fast rates and, accordingly, fixing a, e, and i fails to capture the dynamics of the circumterrestrial problem. Following JeongAhn and Malhotra (2015), our solution is to create a distribution of clones by propagating the states of the field and target objects forwards with the full dynamics and randomly sampling the resultant trajectories [13]. In order to keep the problem computationally manageable, *only* the field objects are cloned, however, the target objects *are* forwards propagated and their trajectories *are* randomly sampled. Using Gronchi's algorithm [11], the MOID for each pair of target and field object is calculated. The total collision probability, (P_{total}) of the field and target sets is then computed by summing the individual collision probabilities (5) of all pairs of objects whose MOID is less than or equal to the specified approach distance (Eq. 6), where N_c is the multiplicity of field object clones.

$$P_{\text{total}} = \frac{1}{N_c} \sum P_i(\vec{\alpha}_1, \vec{\alpha}_2). \tag{6}$$

6. RESULTS

In order to determine "truth" for the collision risk of OneWeb LEO, the satellites lying in the orbital plane initially with a right ascension of $\Omega = 0^\circ$ is taken as the set of target objects, while those in the remaining 35 orbital planes with $\Omega = 5.1^\circ$ to $\Omega = 178.5^\circ$ are taken as the set of field objects. A time span of interest of 90 days is considered with close-approach distances of $d_1 = 20$ km, $d_2 = 5$ km, and $d_3 = 1$ km, respectively, for the first, second, and third filters. Running on the UAs HPC cluster with over 200 CPUs for 13 hours registered a *staggering* $54,171$ close approaches of less than 1 km within the nominal OneWeb LEO as well as a minimum approach distance of 6.4 m. As seen in Fig. 8, as many as three thousand close approaches of 1 km or less occur on a *single* day within the nominal constellation. As expected, the relationship between the number of events and approach distance is exponential (Fig. 9), however the addition of a fourth filter stage could increase the frequency of detected approaches with extremely small approach distances. Plotting the frequency of close approaches as a function of the original right ascension of the field object and time illustrates the significantly higher risk between adjacent planes in the absence of station-keeping maneuvers (Fig. 10). In addition, the periodic spikes of the frequency of close approaches experienced by the constellation indicates that the dynamics of the LEO environment play a large role in collision risk between satellites.

The MiSO variant of the OneWeb LEO constellation is then evaluated following the same procedure. In stark contrast to the nominal configuration, after 90 days of operation, the MiSO variant only experiences 9206 close approaches within 1 km (Fig. 8) with a minimum approach distance of 550 m (Fig. 9). Although MiSO significantly outperforms the nominal configuration, it is not without its flaws. As can be seen in Fig. 10, there is an extremely high frequency of close approaches with field objects in the adjacent orbital plane, which does not vary significantly with time. Despite this, the approaches seem to be limited to about 550 meters and would likely not pose a significant threat as compared to the approaches in the nominal constellation. Finally, we should note that the MiSO configuration could be further optimized, resulting in even fewer close approaches, without jeopardizing the geometrical constraints of the mega-constellation.

The Malhotra algorithm is implemented using values for close-approach distance between 50 m and 1 km. In order to investigate the effect of different numbers of clones, runs using both 100 and 1000 field object clones generated over a time span of 90 days are conducted (Fig. 7). Comparing results to the "truth" established by the brute-force algorithm indicates that although the Malhotra approach only slightly under-predicts the probability of close approaches within the nominal OneWeb LEO constellation, because it so greatly overestimates the probability within the MiSO variant, it is likely not reliable enough to be used as a substitute for the brute-force approach. Furthermore, the collision probabilities calculated using both 1000 and 100 field object clones are remarkably consistent, validating the use of the correction factor, N_c, to account for the multiplicity of cloned field objects. Additionally, when considering the nominal configuration, the discrepancy between the close-approach probabilities calculated using the Malhotra and and brute-force algorithms is likely caused by a number of failed cases in the Malhotra approach where Gronchi's algorithm was unable to calculate a MOID for nearly-circular and nearly-overlapping orbits. However, the difference between the close-approach probability calculated for the MiSO configuration appears to be far more systematic as the Malhotra approach is likely not sensitive enough to accurately evaluate the MiSO constellation. It is anticipated that using a more robust method to calculate the MOID would result in a close-approach-probability from the Malhotra algorithm that better aligns with the brute-force "truth", at least for the the case of the nominal configuration. Finally, although the Malhotra approach has flaws, it vastly outperforms the modified Hoot's algorithm, which fails to predict any close approaches within 1 km. The cause of this was likely an overly conservative third filter stage (time window comparison), which, while extremely fast, does not take perturbations into account.

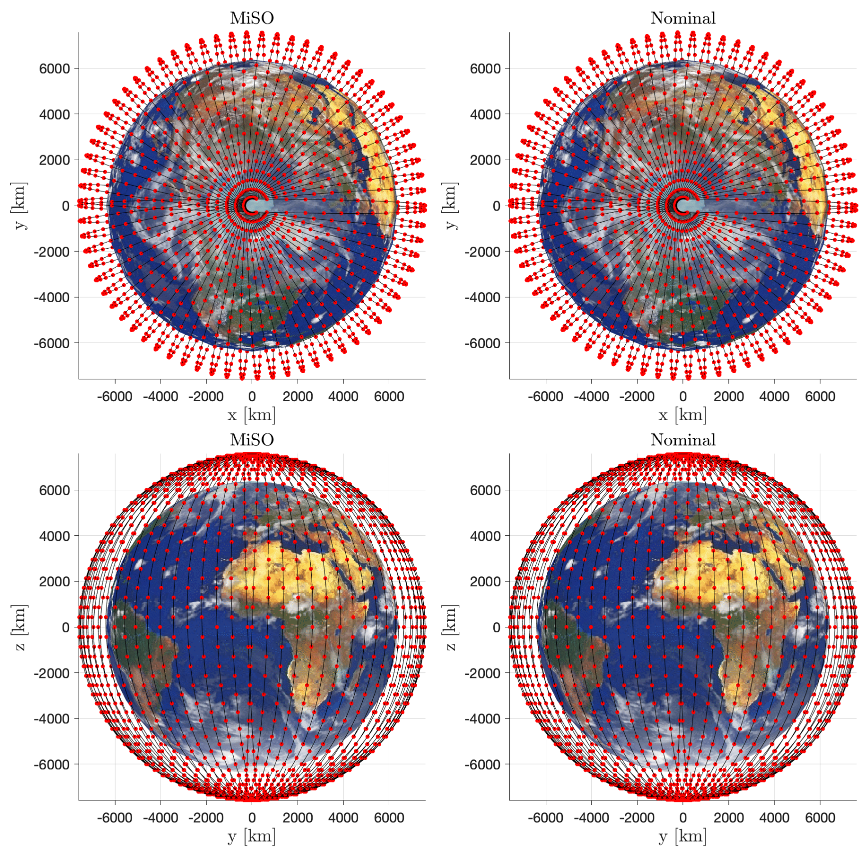


Fig. 6: The MiSO and nominal variants of the OneWeb LEO constellation are visually identical, however miniscule changes to the initial orbital elements result in a much safer design.

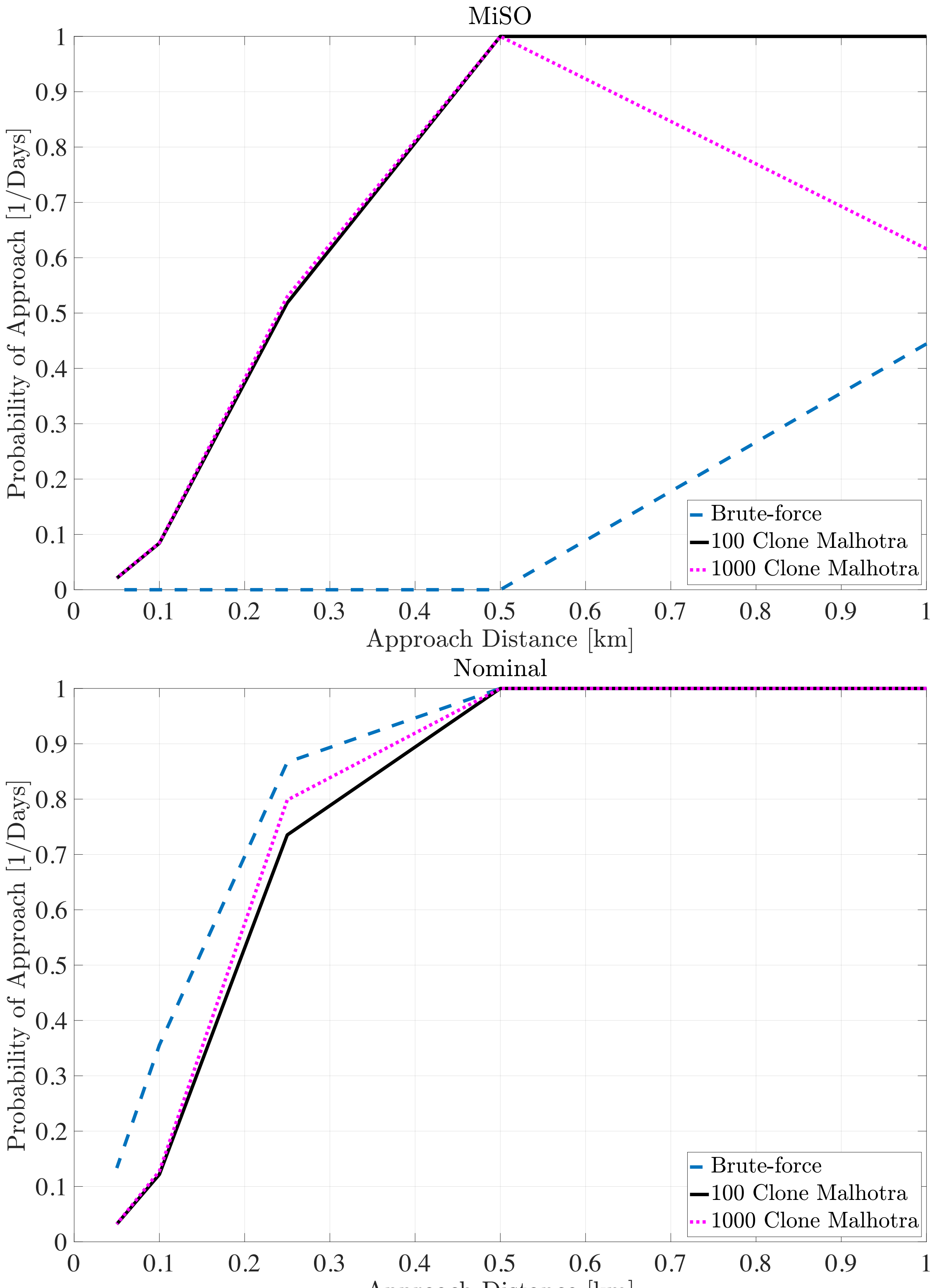


Fig. 7: Although the Malhotra approach accurately represents the probability of close approaches in the nominal constellation, it greatly over-estimates probability of close approaches within the MiSO variant.

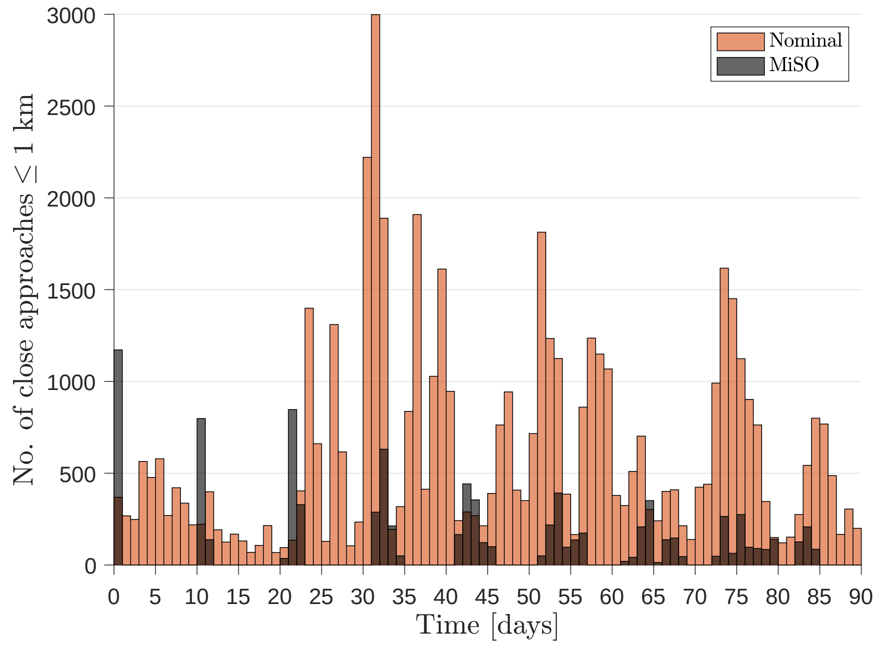


Fig. 8: After 90 days of operation, the nominal OneWeb LEO constellation will experience close approaches within 1 km at much higher frequencies than the MiSO variant.

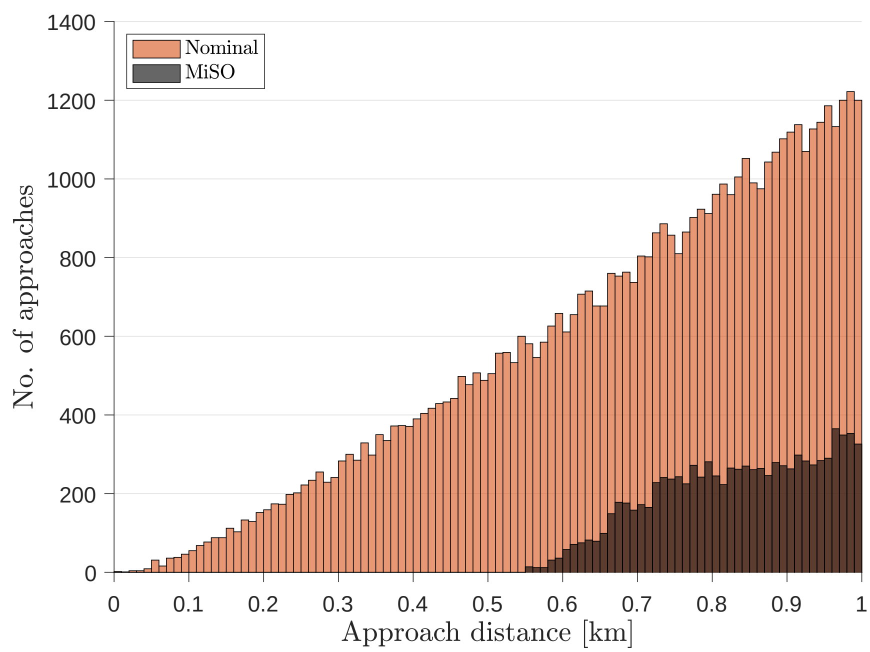


Fig. 9: The nominal OneWeb LEO constellation experiences 54,171 close approaches after 90 days of operation with a minimum approach distance of 6 m, while the MiSO variant experiences only 9,206 close approaches with a minimum approach distance of 550 m.

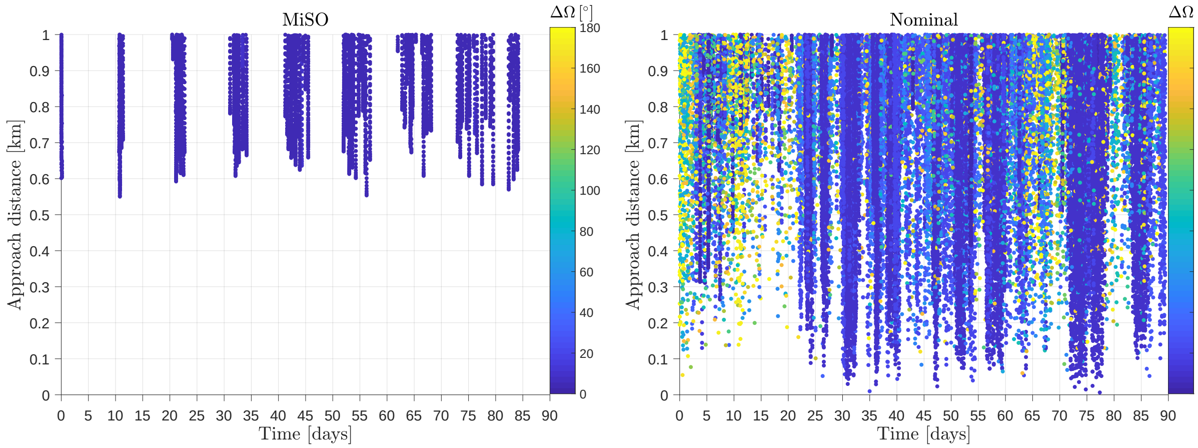


Fig. 10: Time evolution of close-approach distance in OneWeb LEO based on initial nodal placement.

7. CONCLUSION

The space object catalog currently contains around 20,000 objects, and when the planned "space fence" radar network becomes operational this number is expected to exceed one hundred thousand. It has often been assumed that even the current SOC is too large to permit the application of such a brute-force approach to close-approach prediction and collision-probability estimation. Indeed, a systematic study also accounting for the uncertainty in the state-estimation of each object would represent a formidable task with significant computational requirements; yet, the problem becomes tractable if modern developments in astrodynamics (i.e., regularization, nonsingular orbital element formulations, perturbed collision-probability algorithms, etc.) are properly leveraged with sophisticated computing resources.

As near-Earth space becomes increasingly congested and Space Traffic Management becomes a necessity, the current techniques based on crude approximations and assumptions must be properly vetted against a truth model. Yet, hitherto, to our knowledge at least, there has been no such truth established for assessing close approaches and collision probabilities. Indeed, even the debris-evolution models employed by space agencies, which all have the same underlying simplifications, are only compared against each other. The only instance of a satellite-satellite collision has been the famed Iridium-Cosmos; given the states and maneuver histories of all LEO satellites pre collision, however, could the existing algorithms have pinpointed the doom of Iridium.

Although the results can be expanded upon, they are conclusive. The brute-force approach offers detailed insight into the collision risk of operational satellites, and can be a very useful tool for constellation designers as well as the STM community. Furthermore, the modified Hoot's algorithm has been shown to be far too conservative for the new orbital environment, while more modern developments such as the Malhotra approach, although able to better capture the problem's dynamics are not reliable enough to be used as a stand-alone method for evaluating collision risk.

Our developed methods can be used in all of the above scenarios, and have also been adapted for a Department of Commerce (DOC) commissioned study on new approaches for STM [21], particularly for launch-window risk assessment.

ACKNOWLEDGEMENTS

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8. REFERENCES

[1] D. Amato, G. Baú, and C. Bombardelli. Accurate orbit propagation in the presence of planetary close encounters. *Monthly Notices of the Royal Astronomical Society*, 470:2079–2099, 2017.

[2] D. Amato, C. Bombardelli, G. Baú, V. Morand, and A. J. Rosengren. Non-averaged regularized formulations as an alternative to semi-analytical orbit propagation methods. *Celestial Mechanics and Dynamical Astronomy*, 131:21 (38 pp.), 2019.

[3] D. Amato, A. J. Rosengren, and G. Baú. What happened to Luna-3? A numerical exploration of cislunar dynamics. In *John L. Junkins Dynamical Systems Symposium*, Texas A&M University, College Station, TX, 2018.

[4] D. Amato, A. J. Rosengren, and C. Bombardelli. THALASSA: a fast orbit propagator for near-Earth and cislunar space. In *Proceedings of the AIAA/AAS Space Flight Mechanics Meeting, AIAA SciTech Forum*, Kissimmee, Florida, Paper AIAA 2018-1970, 2018.

[5] G. Baú, C. Bombardelli, and J. Peláez. A new set of integrals of motion to propagate the perturbed two-body problem. *Celestial Mechanics and Dynamical Astronomy*, 116:53–78, 2013.

[6] G. Baú, C. Bombardelli, J. Peláez, and E. Lorenzini. Non-singular orbital elements for special perturbations in the two-body problem. *Monthly Notices of the Royal Astronomical Society*, 454:2890–2908, 2015.

[7] M. Berry and L. Healy. The generalized Sundman transformation for propagation of high-eccentricity elliptical orbits. In *Proceedings of the AAS/AIAA Space Flight Mechanics Meeting*, San Antonio, TX, Paper AAS 02-109, 2002.

[8] C. Bombardelli, G. Falco, and D. Amato. Analysis of space occupancy in low-earth orbit. Paris, France, 2018.

[9] V. R. Bond and M. C. Allman. *Modern Astrodynamics: Fundamentals and Perturbation Methods*. Princeton University Press, Princeton, 1996.

[10] Committee for the Assessment of the U.S. Air Force's Astrodynamic Standards; Aeronautics and Space Engineering Board; Division on Engineering and Physical Sciences; National Research Council. Continuing Kepler's quest: Assessing Air Force Space Command's astrodynamics standards (2012), 2012. The National Academies Press, Washington, DC.

[11] G. Gronchi. An algebraic method to compute the critical points of the distance function between two Keplerian orbits. *Celestial Mechanics and Dynamical Astronomy*, 93:295–329, 2005.

[12] F. R. Hoots, L. L. Crawford, and R. L. Roehrich. An analytical method to determine future close approaches between satellites. *Celestial Mechanics and Dynamical Astronomy*, 33:143–158, 1984.

[13] Y. JeongAhn and R. Malhotra. The current impact flux on Mars and its seasonal variation. *Icarus*, 262:140–153, 2015.

[14] Y. JeongAhn and R. Malhotra. Simplified derivation of the collision probability of two objects in independent Keplerian orbits. *The Astronomical Journal*, 153:235 (11 pp.), 2017.

[15] D. J. Kessler and B. G. Cour-Palais. Collision frequency of artificial satellites: The creation of a debris belt. *Journal of Geophysical Research*, 83:2637–2646, 1978.

[16] J. C. Liou and N. L. Johnson. Risks in space from orbiting debris. *Science*, 311:340–341, 2006.

[17] Limiting future collision risk to spacecraft: An assessment of NASA's meteoroid and orbital debris programs, 2011. The National Academies Press, Washington, DC.

[18] E. J. Öpik. Collision probabilities with the planets and the distribution of interplanetary matter. *Proceedings of the Royal Irish Academy*, 54:165–199, 1951.

[19] J. Peláez, J. M. Hedo, and P. Rodríguez de Andrés. A special perturbation method in orbital dynamics. *Celestial Mechanics and Dynamical Astronomy*, 97:131–150, 2007.

[20] J. Roa. *Regularization in Orbital Mechanics: Theory and Practice*. De Gruyter, Berlin, 2017.

[21] B. D. Seery, K. Hilser, K. Toner, R. Carden, S. Kordella, A. J. Rosengren, and C. Frueh. Testbed to evaluate new approaches for STM. In *Proceedings of the Advanced Maui Optical and Space Surveillance Technologies Conference*, Maui, Hawaii, 2019.

[22] E. L. Stiefel and G. Scheifele. *Linear and Regular Celestial Mechanics*. Springer-Verlag, Berlin, 1971.

[23] G. Wetherill. Collisions in the asteroid belt. *Journal of Geophysical Research*, 72:2429–2444, 1967.

[24] A. Witze. The quest to conquer Earth's space junk problem. *Nature*, 24:25–26, 2018.