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Calculating Photometric Uncertainty

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ABSTRACT

We present a standardized process for the Space Domain Awareness (SDA) community to calculate the photometric uncertainties of the instrumental and standard magnitudes. The SDA data providers currently each have their own approach to calculate the uncertainties of the instrumental and standard magnitudes, or do not report these values at all. The instrumental and standard magnitude uncertainties represent the precision of the data collected, an important measure of the quality of the photometry of Resident Space Objects (RSOs). The standardized process presented was developed from first principles of uncertainty propagation and standard astronomical techniques. We thoroughly reviewed the theoretical background on uncertainty propagation and standard astronomical techniques from the literature and combined these in order to derive expressions for the instrumental and standard magnitude uncertainties. We provide a detailed comparison between the instrumental magnitude uncertainties of the standard astronomical technique (Quadrature) and the Computer Calculation of Uncertainties method. Finally, we derive an expression for the standard magnitude uncertainty that can be used by the SDA community.

1. INTRODUCTION

For Space Domain Awareness (SDA), spatially non-resolved photometric measurements of the brightness of Resident Space Objects (RSOs) are used to make inferences about their status and configuration. These measurements are key for characterizing RSOs that are too far or too faint to be resolved by conventional imaging with optical or radar systems. In order for the inferences to be accurate, it is critical to properly calibrate the photometry and calculate the corresponding uncertainty in the calibrated measurement. The uncertainty of the calibrated measurement characterizes the quality of the data collected. It also allows a user of the data to determine whether changes in the brightness of an RSO are due to intrinsic variations of the RSO or due to measurement noise.

For any set of measurements, errors in those measurements can be prescribed as accuracy and precision. By accuracy, we mean the measure of how close the result of an experiment is to the true value. By precision, we mean the measure of how well the result of an experiment has been determined without reference to its agreement with the true value; it is also a measure of the repeatability of the result in a given experiment. [1] In this paper, we discuss the error in measurements due to precision.

The extraction of the energy as measured in photons or photo-electrons on the detector can be best represented by the instrumental magnitude as expressed in the following equation: $m = -2.5 \log_{10} f$, where m is the instrumental magnitude and f is the flux of the RSO in known physical units. To calibrate the instrumental magnitude to a standard system, two things need to be characterized: 1) Earth's atmosphere and 2) the telescope system. Specifically, we characterize the transmission of light through the atmosphere and the telescope system. When we refer to 'transmission of light' we are referring to the amount of total light transmitted as well as the transmission of light as a function of wavelength. The atmosphere causes some of the light from a source outside of the atmosphere, star or RSO, to not reach the telescope system. Atmospheric characteristics are relevant only to ground-based sensors. The transmission of light through the optical system of the telescope, including mirrors, lenses, spectral filters, etc., comprises the telescope system characteristics.

A passband or response function of a standard system is usually determined by the combination of the reflectivity of the telescope mirror, transmission of the camera optics, filter transmission, and the quantum efficiency of the detector used, all as a function of wavelength. A standard photometric system is defined by a list of stars with standard magnitudes and colors measured at specific passbands; these stars are well distributed around the sky. For any standard photometric system, it is necessary to place the measurements onto a standard physical flux scale by removing the absorption of the Earth's atmosphere and calibrating the transmission of the system at different wavelengths. For a given standard photometric system, observations are calibrated onto that system from measurements of a set of that system's standard stars, rather than laboratory based calibrations lamps of constant temperature. [2]

In order to assign meaning to the instrumental magnitude, these values need to be placed onto a standard photometric system. This process involves compensating for the atmosphere to determine what the brightness of the target would be if there were no atmosphere, as well as transforming the exo-atmospheric brightness measurements to a standard photometric system, such as the Johnson-Cousins photometric system. The resulting transformed magnitudes are then referred to as standard magnitudes. There are a variety of calibration techniques to accomplish this, such as all-sky calibrations, in-frame calibrations, etc. In these approaches, stars that have cataloged brightness measurements on a standard photometric system are observed. The atmosphere is characterized and the brightness measurements of the RSO are determined such that the atmosphere is removed. The observer's system is characterized such that the differences between it and the standard photometric system are determined. Applying those differences to the brightness measurements will transform them to the standard photometric system.

The uncertainty in the instrumental magnitude (brightness measurements) is from multiple sources: 1) the inherent uncertainty due to Poisson noise of collected photons—this is for the source as well as the sky background measurement; 2) sensor noise; and 3) uncertainty due to an imperfect measurement of the sky background. The sources of uncertainty in the standard magnitude expression are from the instrumental magnitude uncertainty and the uncertainties in the calibration terms that characterize the atmosphere and telescope system. The uncertainty from a calibration term may be due to a variety of sources, including but not limited to, instrumental magnitude uncertainty from the star measurements and catalog magnitude uncertainties.

The data providers of the SDA community each have their own approach to calculate the uncertainties of the instrumental and standard magnitudes, or do not report these values at all. One of the goals of this paper is the standardization of the calculation of the photometric uncertainty for the SDA community. This paper describes a methodology for calculating the uncertainties in both the instrumental and standard magnitudes of RSOs that can be utilized by photometry SDA data providers.

We provide an overview of the two theoretical approaches to uncertainty propagation in Section 2. We review the standard astronomical technique from the literature in Section 3. We provide our results and discussions in Section 4, and finally our conclusions in Section 5. Note that throughout this paper we rely primarily on the following references: [3], [4], [1], [5], and [6].

2. UNCERTAINTY PROPAGATION

There are two primary ways to propagate uncertainty. The first is an analytical approach, called Quadrature, which utilizes calculus. For measurement uncertainties that are random, this results in an expression in quadrature under a square-root for the standard deviation (uncertainty) of a quantity. The second is a brute force method called Computer Calculation of Uncertainties [1]. We discuss these two approaches below.

2.1 Quadrature

We provide an overview of uncertainty propagation for two or more variables from [1] (page 41), [4] (pages 48-49), and [5] (page 146), altering the notation slightly. Suppose we have a quantity z that is a function of two or more measured variables, $x, y, ...$, expressed as Equation (1).

$$z = f(x, y, ...) \tag{1}$$

If the uncertainties of the measured variables $x, y, ...$ are random, then the standard deviation of z, σ_z, can be expressed as Equation (2), also known as the 'error propagation equation.'

$$\sigma_z^2 \cong \left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2 + \cdots + 2\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\sigma_{xy}^2 \tag{2}$$

where σ_x is the standard deviation in x, σ_y is the standard deviation in y, and σ_{xy}^2 is the covariance between the variables x and y. Now let us suppose that z is a function of just two variables, x and y, as in Equation (3).

$$z = f(x, y) \tag{3}$$

If the uncertainties in the measurements of x and y are uncorrelated or independent, then the covariance between the variables x and y is zero, as shown in Equation (4).

$$\sigma_{xy}^2 = 0 \tag{4}$$

Finally, we arrive at Equation (5), showing the propagation of uncertainty for two variables whose uncertainties are independent and random.

$$\sigma_z^2 = \left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2 \tag{5}$$

This can be re-expressed as Equation (6).

$$\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2} \tag{6}$$

If there were more than two variables, x and y, and their uncertainties were random and independent, the expression would be altered by adding additional terms under the square-root so that we would have Equation (7).

$$\sigma_z = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 \sigma_y^2 + \cdots} \tag{7}$$

2.2 Computer Calculation of Uncertainties

The Computer Calculation of Uncertainties method can be found in [1] (pages 47-48) and [6] (page 35). This technique can explicitly calculate the uncertainty of a quantity that is a function of a measured variable, x, such as Equation (8).

$$z = f(x) \tag{8}$$

The value of the function with the uncertainty would then be Equation (9) or Equation (10).

$$z + \Delta z = f(x + \Delta x) \tag{9}$$

$$z - \Delta z = f(x - \Delta x) \tag{10}$$

By taking the difference between Equation (8) and Equation (9), we arrive at Equation (11) and Equation (12).

$$(z + \Delta z) - z = f(x + \Delta x) - f(x) \tag{11}$$

$$\Delta z = f(x + \Delta x) - f(x) \tag{12}$$

Taking the difference between Equation (8) and Equation (10), we arrive at Equation (13) and Equation (14).

$$(z - \Delta z) - z = f(x - \Delta x) - f(x) \tag{13}$$

$$-\Delta z = f(x - \Delta x) - f(x) \tag{14}$$

A general expression that includes Equation (12) for the plus expression and Equation (14) for the minus expression is given in Equation (15).

$$\pm\Delta z = f(x \pm \Delta x) - f(x) \tag{15}$$

This will determine the uncertainty associated with the function, z, given an uncertainty with the measured variable x.

3. STANDARD ASTRONOMICAL TECHNIQUE

We present the standard astronomical technique as found in the literature for calculating the uncertainty associated with a photometric measurement (instrumental magnitude). The standard astronomical technique uses the Quadrature method. We begin by reviewing the Signal-to-Noise Ratio (SNR) for the instrumental magnitude. We finish with a derivation for an expression of the uncertainty of the instrumental magnitude in units of magnitude.

3.1 Signal-to-Noise Ratio (SNR) of the Instrumental Magnitude

We follow the SNR relation from [3] (page 75) to calculate the SNR for the instrumental magnitude; see Equation (16). This expression follows the Quadrature method (see Section 2.1) for the total noise of the instrumental magnitude due to each of the noise terms being independent and random, see [7] (page 176). A detailed derivation of this expression, also known as the 'revised CCD[1] equation,' is given in [7] (pages 176-178).

$$\frac{S}{N} \cong \frac{N_*}{\sqrt{N_* + n_{pix}(1 + \frac{n_{pix}}{n_B})(N_S + N_D + N_R^2 + G^2\sigma_f^2)}} \tag{16}$$

N_* is the signal, i.e., the total number of photons collected from the object of interest. n_{pix} is the number of pixels used in the integration of the signal from the object of interest. n_B is the total number of background pixels used to estimate the mean background (sky) level. N_S is the total number of photons per pixel from the background or sky. N_D is the total number of dark current electrons per pixel. N_R is the total number of electrons per pixel resulting from the read noise. G is the gain of the Charge-Coupled Device (CCD) in electrons per Analog-to-Digital Unit (ADU). σ_f is an estimate of the 1-sigma error introduced within the Analog-to-Digital (A/D) converter and has a value of $\sigma_f = \sqrt{\frac{1}{12}}$ ADU or $\sigma_f \cong 0.289$ ADU [7]. The values used in the SNR expression must be in units of photons, electrons, or photo-electrons[2] as indicated directly above, not in units of counts or ADUs. Photons, electrons, or photo-electrons are what is physically being measured by the detector. Some of these measurements follow Poisson statistics, whereas counts do not.

[1] Charge-Coupled Device (CCD)
[2] These terms are often used interchangeably, see [3] (page 73).

The n_{pix}/n_B and $G^2\sigma_f^2$ terms are sometimes negligible. The n_{pix}/n_B term becomes important if the background level is estimated with fewer pixels than is ideal, or the CCD data are of poor pixel sampling. The $G^2\sigma_f^2$ term becomes important when the CCD gain has a high value (e.g., 100 electrons/ADU). If these terms are important and are not included, then the SNR will be overestimated. By including these terms into the calculation of the SNR, it will ensure that the SNR is being calculated properly every time. If these terms were negligible, then the expression reverts to the traditional SNR relation, often referred to as the 'CCD equation' [3] (page 77).

3.2 Instrumental Magnitude Uncertainty

A general expression for magnitude is given as Equation (17), where m is the magnitude in units of magnitude, I is the source intensity per unit time, i.e., the flux, and C is an appropriate constant determined in such a manner so that the calculated source magnitude is placed on a standard magnitude scale [3] (page 110).

$$m = -2.5\log_{10}(I) + C \tag{17}$$

We define the instrumental magnitude as Equation (18).

$$m_{inst} = -2.5\log_{10}(I) \tag{18}$$

Using Equation (7) and Equation (18), we derive the uncertainty in the instrumental magnitude, $\sigma_{m_{inst}}$. This derivation produces an expression for $\sigma_{m_{inst}}$(in units of magnitude) as a function of the uncertainty in I (source intensity per unit time, i.e., flux).

Using the Quadrature method with Equation (5) for one variable and replacing z with m_{inst} and x with I, we arrive at Equation (19).

$$\sigma_{m_{inst}}^2 = \left(\frac{\partial m_{inst}}{\partial I}\right)^2 \sigma_I^2 \tag{19}$$

We take the partial derivative of Equation (18), which produces Equation (20).

$$\frac{\partial m_{inst}}{\partial I} = \frac{\partial(-2.5\log_{10}(I))}{\partial I} = -2.5\frac{\partial(\log_{10}(I))}{\partial I} \tag{20}$$

We need to take the derivative of a logarithm. We use the derivative of a logarithm from [8] (page 60), as shown in Equation (21).

$$\frac{d}{dx}\log_a u = \frac{\log_a e}{u}\frac{du}{dx}, a \neq 0, 1 \tag{21}$$

Applying Equation (21) to the partial derivative in Equation (20), we arrive at Equation (22).

$$\frac{\partial(\log_{10}(I))}{\partial I} = \frac{\log_{10}(e)}{I}\frac{\partial I}{\partial I} = \frac{\log_{10}(e)}{I} \tag{22}$$

Substituting Equation (22) into Equation (20), we get Equation (23).

$$\frac{\partial m_{inst}}{\partial I} = -2.5\frac{\log_{10}(e)}{I} \tag{23}$$

Substituting Equation (23) into Equation (19), we arrive at Equation (24).

$$\sigma_{m_{inst}}^2 = \left(-2.5\frac{\log_{10}(e)}{I}\right)^2 \sigma_I^2 = \left(2.5\frac{\log_{10}(e)}{I}\right)^2 \sigma_I^2 \tag{24}$$

We can take the square-root of both sides of the expression to eliminate the power of two to get Equation (25).

$$\sigma_{m_{inst}} = 2.5 \log_{10}(e) \frac{\sigma_I}{I} \tag{25}$$

Since $\sigma_I = N$ (noise) and $I = S$ (signal), we can re-express Equation (25) and produce a final relation as Equation (26).

$$\sigma_{m_{inst}} = 2.5 \log_{10}(e) \frac{\sigma_I}{I} = 2.5 \log_{10}(e) \frac{N}{S} \tag{26}$$

The change of base of logarithms from [8] (page 51) is given in Equation (27).

$$\log_a N = \frac{\log_b N}{\log_b a} \tag{27}$$

From [8] (page 50) we have $\log_e(N) = \ln(N)$. From Equation (27) it can be shown that the logarithm with its base and the argument as the same value is equal to 1, $\log_N(N) = \frac{\log_b N}{\log_b N} = 1$. Using Equation (27) (change of base), letting $b = e$ (exponential), and combining the rest, we can show the following relation in Equation (28).

$$\log_{10}(e) = \frac{\log_e(e)}{\log_e(10)} = \frac{1}{\ln(10)} \tag{28}$$

Equation (28) allows Equation (26) to be re-expressed as Equation (29).

$$\sigma_{m_{inst}} = \frac{2.5}{\ln(10)} \frac{\sigma_I}{I} = \frac{2.5}{\ln(10)} \frac{N}{S} \tag{29}$$

This relationship between the SNR of the instrumental magnitude and the uncertainty of the instrumental magnitude is provided in [3] (page 76) and [9]. The constant, $\frac{2.5}{\ln(10)} \cong 1.0857$, is a correction term between an error in flux and that same error in magnitudes [9]. By combining Equation (16) and Equation (29), we have an expression that provides the uncertainty of the instrumental magnitude in units of magnitude, Equation (30).

$$\sigma_{m_{inst}} = \frac{2.5}{\ln(10)} \frac{\sqrt{N_* + n_{pix}(1 + \frac{n_{pix}}{n_B})(N_S + N_D + N_R^2 + G^2\sigma_f^2)}}{N_*} \tag{30}$$

4. RESULTS AND DISCUSSIONS

We will first derive the instrumental magnitude uncertainty expression using the Computer Calculation of Uncertainties method. Then we provide an example of a calculation for the SNR and the uncertainty of the instrumental magnitude using the two methods of uncertainty propagation, Quadrature and Computer Calculation of Uncertainties. We provide a detailed comparison between the instrumental magnitude uncertainties of the standard astronomical technique (Quadrature) and the Computer Calculation of Uncertainties method. Lastly, we end with a discussion of the standard magnitude uncertainty.

4.1 Instrumental Magnitude Uncertainty using the Computer Calculation of Uncertainties Method

Using the Computer Calculation of Uncertainties method (see Section 2.2), we derive the expected expression for the uncertainty in the instrumental magnitude in units of magnitude starting with the expression for the instrumental magnitude from Equation (18), shown again as Equation (31).

$$m_{inst} = -2.5 \log_{10}(I) \tag{31}$$

Applying this to Equation (15) where z is replaced by m_{inst} and x is replaced by I, we get Equation (32).

$$\pm\Delta m_{inst} = (-2.5\log_{10}(I \pm \Delta I)) - (-2.5\log_{10}(I)) \tag{32}$$

Using an expression from the law of logarithms [8] (page 50), $\log_a \frac{M}{N} = \log_a M - \log_a N$, we arrive at a final expression for the uncertainty in the instrumental magnitude, Equation (33), where σ_I and I are in units of flux (e-/s).

$$\pm\Delta m_{inst} = -2.5\log_{10}\left(\frac{I \pm \Delta I}{I}\right) = -2.5\log_{10}(1 \pm \frac{\Delta I}{I}) \tag{33}$$

Since $\Delta I = N$ (noise) and $I = S$ (signal), $\Delta I/I = N/S = 1/(S/N) = 1/SNR$, we can re-express Equation (33) as Equation (34).

$$\pm\Delta m_{inst} = -2.5\log_{10}(1 \pm \frac{N}{S}) = -2.5\log_{10}(1 \pm \frac{1}{SNR}) \tag{34}$$

This result for the Computer Calculation of Uncertainties method is consistent with [10] (page 78). Note that due to the nature of the logarithm, the plus and minus values of Δm_{inst} will not be the same; thus, Δm_{inst} is not single valued or symmetric. Also, note that Δ is specifically used rather than σ since σ is used to indicate that the uncertainty is the standard deviation, whereas the Δ is not necessarily the standard deviation.

4.2 Example

We demonstrate a calculation of the SNR relation and the uncertainty in the instrumental magnitude using the scenario and values from [3] (page 76). These values are plugged into Equation (16) and Equation (30) to demonstrate the results from the standard astronomical technique (Quadrature method), and then we apply these same values to the derived expression of Equation (34) to demonstrate the results from the Computer Calculation of Uncertainties method. We demonstrate how to express the measured value and its uncertainty in a single expression for both approaches.

The values used for the following example and their units provided in brackets are given below.

- $G = 5$ [e-/ADU]
- $N_{*\_ADU} = 24013$ [ADU]
  - $N_* = N_{*\_ADU} * G$ [e-]
- $N_R = 5$ [e-/pixel]
- $N_D = 1.8$ [e-]
- $N_{S\_ADU} = 620$ [ADU/pixel]
  - $N_S = N_{S\_ADU} * G$ [e-/pixel]
- $n_{pix} = 1$ [pixels]
- $n_b = 200$ [pixels]

Using these values, from the standard astronomical technique, Equation (16) and Equation (30) yield:

- $SNR = 342.053682557 = 342$
- $\sigma_{m_{inst}} = 0.00317416902704\ mag = 0.003\ mag$
  - $m_{inst} \pm \sigma_{m_{inst}} = m_{inst} \pm 0.003$

Applying these same values to Equation (34) from the Computer Calculation of Uncertainties method, we get the following results:

- $\Delta m_{inst(+)} = -0.00316953818073\ mag = -0.003\ mag$
  - $m_{inst} + \Delta m_{inst(+)} = m_{inst} + (-0.003) = m_{inst} - 0.003$
- $\Delta m_{inst(-)} = -0.00317881795977\ mag = -0.003\ mag$
  - $m_{inst} - \Delta m_{inst(-)} = m_{inst} - (-0.003) = m_{inst} + 0.003$

At two significant figures, the final result for $m_{inst} \pm \Delta m_{inst(\pm)}$ is symmetric, and the Quadrature method and the Computer Calculation of Uncertainties method produce the same final result. Our final value of uncertainty is given to one significant figure. This is because the values used to calculate the SNR, and therefore the uncertainty in instrumental magnitude, contained values to one significant figure. See [4] (page 26) for a discussion on significant figures. In a scenario where the values used to calculate the SNR were given to two significant figures, then the final result of the uncertainty would be provided to two significant figures. The plus and minus solutions for Δm_{inst} are distinguished using subscripts of '$(+)$' and '$(-)$' since they are not numerically identical. Both of these results will always give an answer for Δm_{inst} that is negative. However, the final result, $m_{inst} \pm \Delta m_{inst(\pm)}$, will be a plus result and a minus result that are not symmetric. Due to the plus sign associated with the plus result and the minus sign associated with the minus result, the two final results provide an upper and lower bound on the uncertainty as shown, even though the two values of Δm_{inst} themselves are always negative.

We illustrate that Δm_{inst} will always be negative with the following arguments. First, we assume the SNR must be greater than one. This is a valid assumption for a source to be detected. The logarithm of an argument that is greater than one will always be positive, while the logarithm of an argument that is greater than zero but less than one will always be negative. The plus result has the argument of the logarithm as $(1 + \frac{1}{SNR})$, so the argument will always be greater than one, which will result in the logarithm always being positive. The result of the logarithm is then multiplied by -2.5, and thus the final value of $\Delta m_{inst(+)}$ is then negative. The minus result has the argument of the logarithm as $(1 - \frac{1}{SNR})$, so the argument will always be between zero and one, which will result in the logarithm always being negative. The result of the logarithm is then multiplied by -2.5, and then once again by -1 since the term on the left hand side of Equation (34) has a negative sign $(-\Delta m_{inst})$. Thus the final value of $\Delta m_{inst(-)}$ is then negative.

4.3 A Comparison of the Standard Astronomical Technique and the Computer Calculation of Uncertainties Method

We perform a comparison of the instrumental magnitude uncertainties produced by the standard astronomical technique (Quadrature) (Equation (30)) and those produced by the Computer Calculation of Uncertainties method (Equation (34)). Fig. 1 shows the instrumental magnitude uncertainty as a function of SNR for the two uncertainty propagation methods: the standard astronomical technique (Quadrature) (Equation (30)), $\sigma_{m_{inst}}$, and the absolute value of the Computer Calculation of Uncertainties method (Equation (34)), $|\Delta m_{inst(+)}|$ and $|\Delta m_{inst(-)}|$. The red circles are the instrumental magnitude uncertainties for the standard astronomical technique, and the green squares are the absolute values of the plus results and the blue diamonds are the absolute values of the minus results for the Computer Calculation of Uncertainties method. The x-axis has values of SNR from 2 to 20, in increments of one. Recall that the detection threshold is around $SNR \sim 3$. The two methods, yielding three values of instrumental magnitude uncertainty, differ significantly for small values of SNR but converge rapidly as the SNR increases. Note that the absolute value of the minus result of the Computer Calculation of Uncertainties method has a larger value than the plus result.

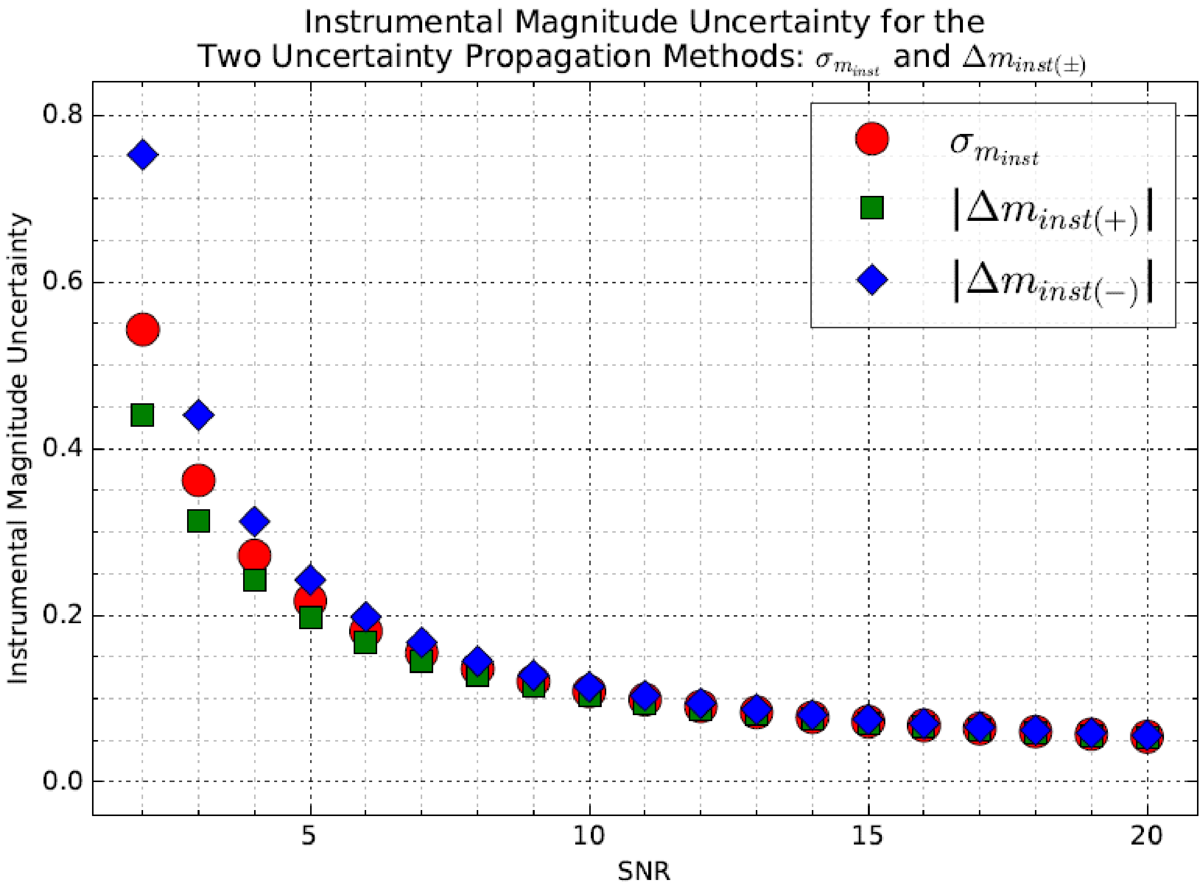

Fig. 1. Instrumental Magnitude Uncertainty for the Two Uncertainty Propagation Methods: Standard Astronomical Technique ($\boldsymbol{\sigma_{m_{inst}}}$) and Computer Calculation of Uncertainties Method ($|\Delta \boldsymbol{m_{inst(+)}}|$ and $|\Delta \boldsymbol{m_{inst(-)}}|$)

The standard astronomical technique expression for the instrumental magnitude uncertainty is derived using the Quadrature method (Equation (2)), where the Quadrature method is an approximation. The Computer Calculation of Uncertainties method is a direct calculation of the uncertainty for the instrumental magnitude (Equation (34)). The distinction between the two uncertainty propagation methods accounts for the differences observed in Fig. 1 between the two methods.

Fig. 2 shows the differences in instrumental magnitude uncertainty between the two uncertainty propagation methods: the standard astronomical technique (Quadrature) and the Computer Calculation of Uncertainties method. The green squares with a red border are the absolute value of the instrumental magnitude uncertainty difference between the standard astronomical technique and the absolute value of the Computer Calculation of Uncertainties plus result. The blue diamonds with a red border are the absolute value of the instrumental magnitude uncertainty difference between the standard astronomical technique and the absolute value of the Computer Calculation of Uncertainties minus result. The differences are large for small values of SNR, but rapidly approach very small values as the SNR increases. The differences decrease to less than 0.01 mag for $SNR \geq 8$. At $SNR = 20$, the differences are as small as 0.0014 mag. The difference for the Computer Calculation of Uncertainties minus result has a larger value than the plus result; this asymmetric behavior decreases as the SNR increases.

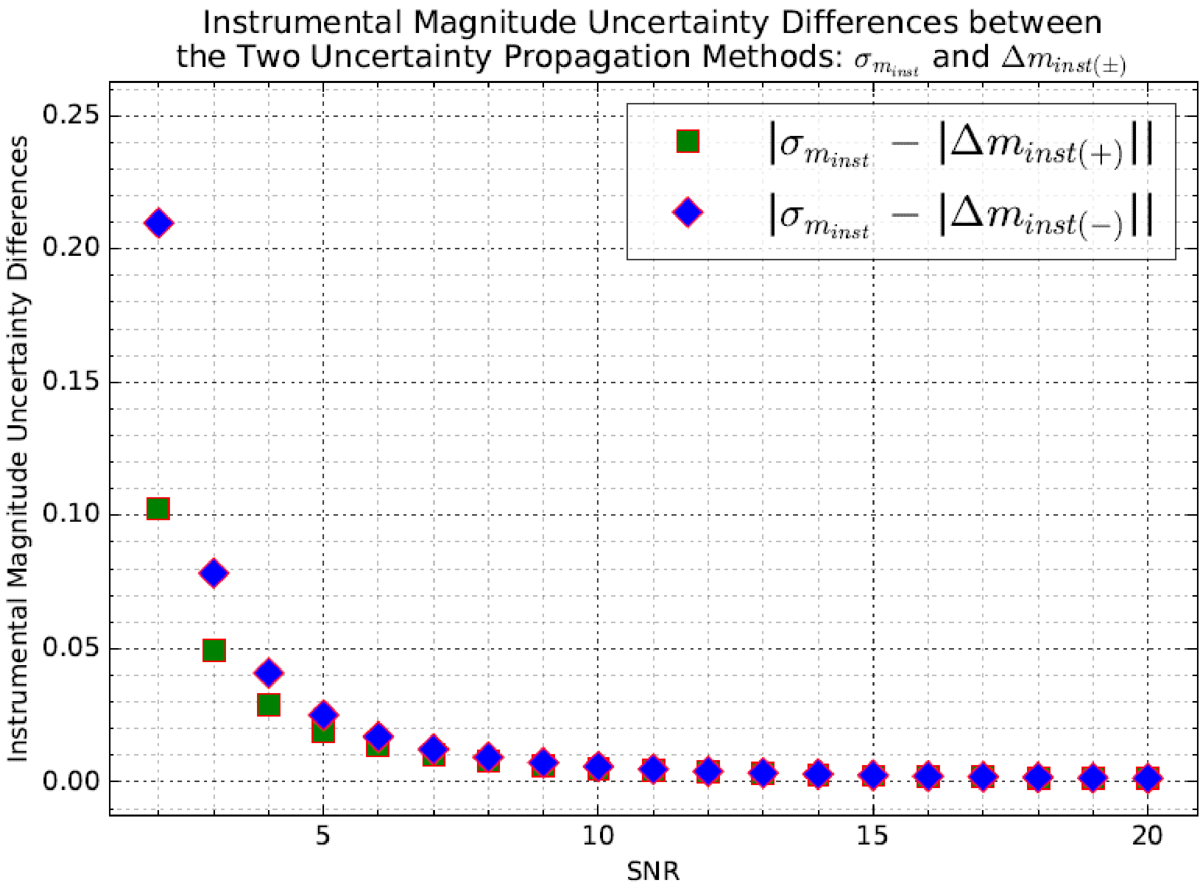

Fig. 2. Instrumental Magnitude Uncertainty Differences between the Two Uncertainty Propagation Methods: Standard Astronomical Technique and Computer Calculation of Uncertainties

To assess how asymmetric the Computer Calculation of Uncertainties result is, we have created Fig. 3 that shows the instrumental magnitude uncertainty difference between the plus and minus results. The gray hexagons are the absolute value of the difference between the absolute value of the plus and minus result. The y-axis value represents the asymmetry of the Computer Calculation of Uncertainties method as a function of SNR. The asymmetric characteristic of the plus and minus result is large for small values of SNR and rapidly decreases as the SNR increases. The asymmetry decreases to less than 0.01 mag for $SNR \geq 11$. At $SNR = 20$, the asymmetry is as small as 0.0027 mag.

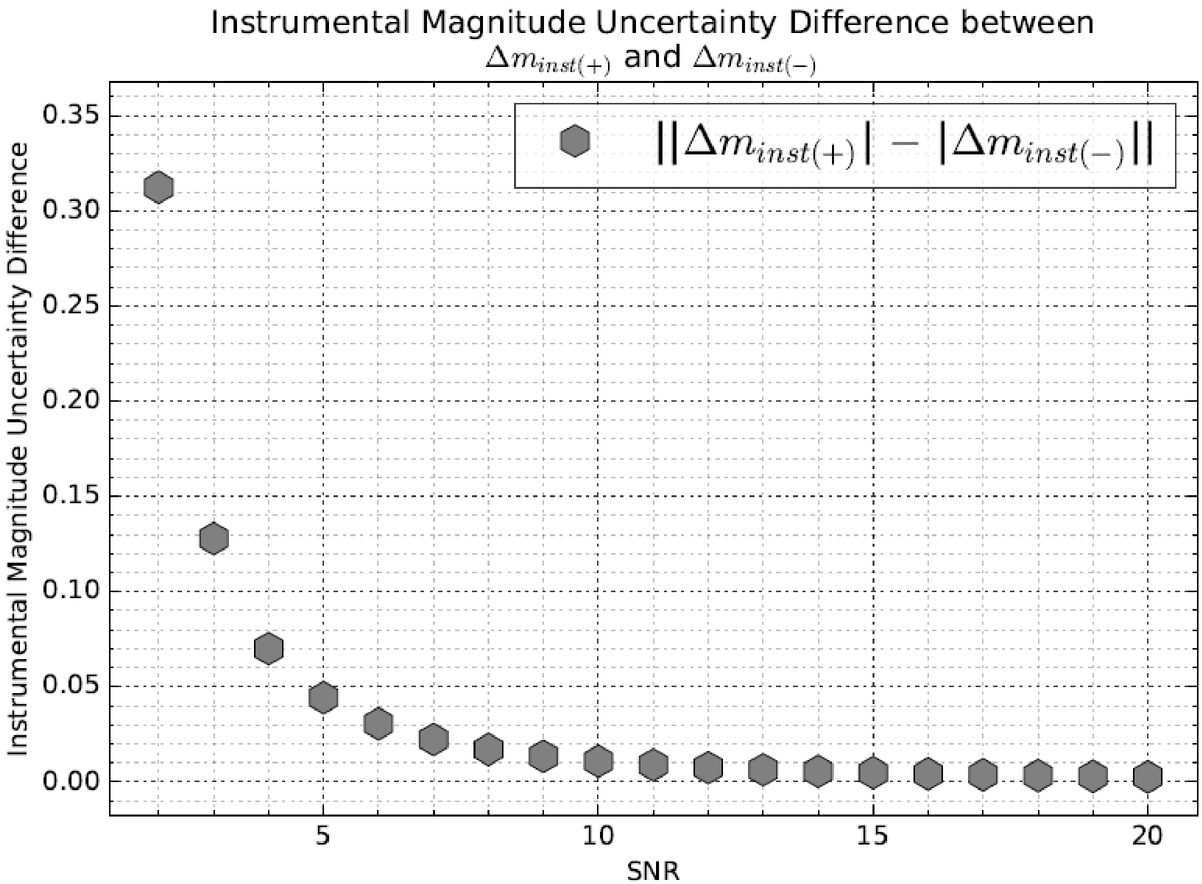

Fig. 3. Instrumental Magnitude Uncertainty Difference between $\Delta \boldsymbol{m}_{\boldsymbol{inst}(+)}$ and $\Delta \boldsymbol{m}_{\boldsymbol{inst}(-)}$

The instrumental magnitude uncertainty for the two uncertainty propagation methods, the standard astronomical technique (Quadrature) and the Computer Calculation of Uncertainties method, do not produce the same results for small values of SNR, but quickly converge for larger values of SNR. At $SNR \geq 8$, the differences between the two uncertainty propagation methods is less than 0.01 mag. At $SNR = 20$, the differences are as small as 0.0014 mag. The Computer Calculation of Uncertainties method has a plus and minus result that has large asymmetry for small values of SNR and an asymmetry that rapidly decreases as the SNR increases. The asymmetry decreases to less than 0.01 mag for $SNR \geq 11$. At $SNR = 20$, the asymmetry is as small as 0.0027 mag.

Since observations for SDA may have modest SNRs, we recommend using the Computer Calculation of Uncertainties method to calculate the instrumental magnitude uncertainty (Equation (34)) for the most accurate reporting of uncertainties. This method produces two values that are asymmetric, with a plus and minus result. Although this is more cumbersome to deal with than the standard astronomical technique (Equation (30)), which produces a single value that is symmetric, this ensures that the instrumental magnitude uncertainty value is correct for all values of SNR. In situations where an approximation of the instrumental magnitude uncertainty in the form of a single valued result would be acceptable, the standard astronomical technique (Equation (30)) may be used to achieve this. It is also appropriate to use the standard astronomical technique for large values of SNR. The two methods produce virtually identical results in this case.

4.4 Standard Magnitude Uncertainty

The terms that exist in the standard magnitude uncertainty expression in addition to the instrumental magnitude uncertainty, $\sigma_{m_{inst}}$, depend on the standard magnitude expression used and the calibration process involved in calculating the standard magnitude. Generally speaking, if the standard magnitude is a function of N variables whose errors are random, then the theoretical framework for the propagation of the uncertainty is as discussed in Section 2.1. If we re-express Equation (2) by replacing z with $m_{standard}$ and the variables $x, y, \ldots$ with $variable_1, variable_2, \ldots$, then the uncertainty in the standard magnitude can be expressed as Equation (35).

$$\sigma_{m_{standard}} = \sqrt{\left(\frac{\partial m_{standard}}{\partial variable_1}\right)^2 \sigma^2_{variable_1} + \left(\frac{\partial m_{standard}}{\partial variable_2}\right)^2 \sigma^2_{variable_2} + \cdots + 2\left(\frac{\partial m_{standard}}{\partial variable_1}\right)\left(\frac{\partial m_{standard}}{\partial variable_2}\right)\sigma^2_{variable_1 variable_2}} \tag{35}$$

Equation (35) may also be expressed as Equation (36) using summation notation. Note that the summations for the covariance terms do not have a factor of two when using summation notation.

$$\sigma_{m_{standard}} = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial m_{standard}}{\partial variable_i}\right)^2 \sigma^2_{variable_i} + \sum_{i,j:i\neq j}^{N}\left(\frac{\partial m_{standard}}{\partial variable_i}\right)\left(\frac{\partial m_{standard}}{\partial variable_j}\right)\sigma^2_{variable_i variable_j}} \tag{36}$$

For the particular situation where the errors between all of the variables are independent, the covariance between each variable would be zero, and we can express the uncertainty in the standard magnitude as shown in Equation (37).

$$\sigma_{m_{standard}} = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial m_{standard}}{\partial variable_i}\right)^2 \sigma^2_{variable_i}} \tag{37}$$

4.4.1 Applying Theory to the Standard Magnitude Expression

The final expression for the standard magnitude uncertainty depends on the form of the standard magnitude expression used and the calibration technique used. Some data providers choose to not use the entire standard magnitude expression, with the determination or assumption that some terms are negligible or zero. An example of this would be the second-order extinction coefficient for the Johnson-Cousins R band. The calibration technique, all-sky calibrations, in-frame calibrations, or some other approach that may involve components of both, will determine for some of the variables whether the measurement uncertainties are independent or not, and therefore determine whether the covariance between pairs of variables is zero or not.

We first review the standard magnitude expression and briefly discuss each of its terms. Then we derive the standard magnitude uncertainty using the entire standard magnitude expression, leaving the covariance terms in generic summations. We simplify the summations of the covariance terms with a few assumptions about what variable uncertainties are independent, regardless of the calibration technique used. Thereafter, we use the all-sky calibration technique and describe how it would affect the covariance terms.

Δm_{inst} is recommended in Section 4.3 because it is exact, whereas $\sigma_{m_{inst}}$ has error in the uncertainty value for low values of SNR compared to Δm_{inst}. However, the best approach to add the uncertainties of the instrumental magnitude with the other sources of uncertainty to calculate the standard magnitude uncertainty is to add them using the Quadrature method, which uses σ. Recall that Δm_{inst} converges to $\sigma_{m_{inst}}$ for sufficient values of SNR. As we derive the standard magnitude uncertainty in this section using the Quadrature method with $\sigma_{m_{inst}}$, if the SNR is low, then $\sigma_{m_{inst}}$ is a rough approximation and will be in error due to the difference between $\sigma_{m_{inst}}$ and Δm_{inst}. Thus, for low SNR conditions, the final value of the standard magnitude uncertainty will be less accurate.

4.4.1.1 Photometry Theory

We begin by presenting the expression for calculating the magnitude on a standard system ($m_{standard}$), as shown in Equation (38):

$$m_{standard} = m_{inst} + m_{first-order\_ext} + m_{second-order\_ext} + m_{zp} + m_{ct} \tag{38}$$

The instrumental magnitude (m_{inst}) is the brightness of the object as measured from the sensor. The first-order extinction ($m_{first-order\_ext}$) and second-order extinction ($m_{second-order\_ext}$) characterize the atmosphere, while the zero-point magnitude (m_{zp}) and the color term (m_{ct}) characterize the sensor. The first-order extinction is a measurement of the light lost due to the atmosphere, while the second-order extinction accounts for the differing extinction of objects as a function of instrumental color index. The zero-point magnitude is a shift needed to put the photometry onto a standard system. The color term characterizes the difference in spectral response between the system used to observe and that of the standard system; it is a function of the target's standard color index.

The following provides a detailed description of each term in the above expression:

- $m_{standard}$ is the magnitude on the standard system.
- m_{inst} is the instrumental magnitude.
  - $m_{inst} = -2.5 * \log_{10}(flux_{target})$
- $m_{first-order\_ext}$ is the first-order extinction.
  - $m_{first-order\_ext} = -k' * X$
    - k' is the first-order extinction coefficient.
    - X is the airmass.
- $m_{second-order\_ext}$ is the second-order extinction.
  - $m_{second-order\_ext} = -k'' * X * \mathrm{CI}_{inst}$
    - k'' is the second-order extinction coefficient.
    - X is the airmass.
    - CI_{inst} is the instrumental color index, the observed color index.
      - The instrumental color index is expressed as $\mathrm{CI}_{inst} = m_{inst_{\lambda_1}} - m_{inst_{\lambda_2}}$.
- m_{zp} is the zero-point magnitude.
- m_{ct} is the color term.
  - $m_{ct} = T * CI_{standard}$
  - T is the color coefficient.
  - $CI_{standard}$ is the standard color index, the color index on the standard system.
    - The standard color index is expressed as $CI_{standard} = m_{standard_{\lambda_1}} - m_{standard_{\lambda_2}}$.

Applying these definitions to Equation (38), we get Equation (39):

$$m_{standard} = m_{inst} + (-k' * X) + (-k'' * X * CI_{inst}) + m_{zp} + (T * CI_{standard}). \tag{39}$$

Equation (39) may also be expressed as Equation (40):

$$m_{standard} = m_{inst} - k' * X - k'' * X * CI_{inst} + m_{zp} + T * CI_{standard}. \tag{40}$$

The measurement of the standard magnitude may be over a specific spectral region with an equivalent wavelength [10] (page 356) of λ_2. Then we can re-express Equation (40) as Equation (41).

$$m_{standard_{\lambda_2}} = m_{inst_{\lambda_2}} - k'_{\lambda_2} * X - k''_{\lambda_2} * X * CI_{inst} + m_{zp_{\lambda_2}} + T_{\lambda_2} * CI_{standard}. \tag{41}$$

Many of the terms now have a subscript of λ_2 indicating that they pertain to this specific spectral region, e.g., the Johnson-Cousins V band. Using the expression for instrumental color index, Equation (41) can be re-expressed as Equation (42).

$$m_{standard_{\lambda_2}} = m_{inst_{\lambda_2}} - k'_{\lambda_2} * X - k''_{\lambda_2} * X \left(m_{inst_{\lambda_1}} - m_{inst_{\lambda_2}}\right) + m_{zp_{\lambda_2}} + T_{\lambda_2} * CI_{standard}. \tag{42}$$

The standard color index itself can be calculated using Equation (42), for both λ_1 and λ_2, taking the difference, and solving for the standard color index ($CI_{standard}$). In the next section, we will show how to derive the standard magnitude uncertainty ($\sigma_{m_{standard_{\lambda_2}}}$). A similar analysis needs to be performed beforehand to calculate a value for the uncertainty in the standard color index ($\sigma_{CI_{standard}}$). See [10] for additional details on the theory of photometry.

4.4.1.2 Derivation of the Standard Magnitude Uncertainty

The standard magnitude expression of Equation (42) has eight variables. We can re-express Equation (36) by writing out the first summation for $N = 8$ to arrive at Equation (43).

$$\sigma_{m_{standard_{\lambda_2}}} = \sqrt{\begin{matrix} \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_1}\right)^2 \sigma^2_{variable_1} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_2}\right)^2 \sigma^2_{variable_2} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_3}\right)^2 \sigma^2_{variable_3} + \\ \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_4}\right)^2 \sigma^2_{variable_4} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_5}\right)^2 \sigma^2_{variable_5} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_6}\right)^2 \sigma^2_{variable_6} + \\ \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_7}\right)^2 \sigma^2_{variable_7} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_8}\right)^2 \sigma^2_{variable_8} + \\ \sum_{i,j:i\neq j}^{8} \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_i}\right)\left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_j}\right) \sigma^2_{variable_i variable_j} \end{matrix}} \tag{43}$$

Next, we define the variables in our standard magnitude expression. These are given in Equation (44), Equation (45), Equation (46), Equation (47), Equation (48), Equation (49), Equation (50), and Equation (51).

$$variable_1 = m_{inst_{\lambda_2}} \tag{44}$$

$$variable_2 = k'_{\lambda_2} \tag{45}$$

$$variable_3 = X \tag{46}$$

$$variable_4 = k''_{\lambda_2} \tag{47}$$

$$variable_5 = m_{inst_{\lambda_1}} \tag{48}$$

$$variable_6 = m_{zp_{\lambda_2}} \tag{49}$$

$$variable_7 = T_{\lambda_2} \tag{50}$$

$$variable_8 = CI_{standard} \tag{51}$$

Inserting the variables into Equation (43), we arrive at Equation (52).

$$\sigma_{m_{standard_{\lambda_2}}} = \sqrt{\begin{matrix} \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial m_{inst_{\lambda_2}}}\right)^2 \sigma^2_{m_{inst_{\lambda_2}}} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial k'_{\lambda_2}}\right)^2 \sigma^2_{k'_{\lambda_2}} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial X}\right)^2 \sigma^2_X + \\ \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial k''_{\lambda_2}}\right)^2 \sigma^2_{k''_{\lambda_2}} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial m_{inst_{\lambda_1}}}\right)^2 \sigma^2_{m_{inst_{\lambda_1}}} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial m_{zp_{\lambda_2}}}\right)^2 \sigma^2_{m_{zp_{\lambda_2}}} + \\ \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial T_{\lambda_2}}\right)^2 \sigma^2_{T_{\lambda_2}} + \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial CI_{standard}}\right)^2 \sigma^2_{CI_{standard}} + \\ \sum_{i,j:i\neq j}^{8} \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_i}\right)\left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_j}\right) \sigma^2_{variable_i variable_j} \end{matrix}} \tag{52}$$

The partial derivatives are given in Equations (53) - Equation (60).

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_1} = \frac{\partial m_{standard_{\lambda_2}}}{\partial m_{inst_{\lambda_2}}} = 1 + k''_{\lambda_2} * X \tag{53}$$

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_2} = \frac{\partial m_{standard_{\lambda_2}}}{\partial k'_{\lambda_2}} = -X \tag{54}$$

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_3} = \frac{\partial m_{standard_{\lambda_2}}}{\partial X} = -k'_{\lambda_2} - k''_{\lambda_2}\left(m_{inst_{\lambda_1}} - m_{inst_{\lambda_2}}\right) = -k'_{\lambda_2} - k''_{\lambda_2} * \mathrm{CI}_{inst} \tag{55}$$

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_4} = \frac{\partial m_{standard_{\lambda_2}}}{\partial k''_{\lambda_2}} = -X\left(m_{inst_{\lambda_1}} - m_{inst_{\lambda_2}}\right) = -X * \mathrm{CI}_{inst} \tag{56}$$

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_5} = \frac{\partial m_{standard_{\lambda_2}}}{\partial m_{inst_{\lambda_1}}} = -k''_{\lambda_2} * X \tag{57}$$

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_6} = \frac{\partial m_{standard_{\lambda_2}}}{\partial m_{zp_{\lambda_2}}} = 1 \tag{58}$$

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_7} = \frac{\partial m_{standard_{\lambda_2}}}{\partial T_{\lambda_2}} = CI_{standard} \tag{59}$$

$$\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_8} = \frac{\partial m_{standard_{\lambda_2}}}{\partial CI_{standard}} = T_{\lambda_2} \tag{60}$$

Applying these partial derivatives, we arrive at Equation (61).

$$\sigma_{m_{standard_{\lambda_2}}} = \sqrt{\begin{array}{c}\left(1 + k''_{\lambda_2} * X\right)^2 \sigma^2_{m_{inst_{\lambda_2}}} + (-X)^2 \sigma^2_{k'_{\lambda_2}} + \left(-k'_{\lambda_2} - k''_{\lambda_2} * \mathrm{CI}_{inst}\right)^2 \sigma^2_X + \\ (-X * \mathrm{CI}_{inst})^2 \sigma^2_{k''_{\lambda_2}} + \left(-k''_{\lambda_2} * X\right)^2 \sigma^2_{m_{inst_{\lambda_1}}} + (1)^2 \sigma^2_{m_{zp_{\lambda_2}}} + \\ (CI_{standard})^2 \sigma^2_{T_{\lambda_2}} + \left(T_{\lambda_2}\right)^2 \sigma^2_{CI_{standard}} + \\ \sum_{i,j:i\neq j}^{8} \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_i}\right)\left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_j}\right) \sigma^2_{variable_i variable_j}\end{array}} \tag{61}$$

Equation (61) simplifies to Equation (62).

$$\sigma_{m_{standard_{\lambda_2}}} = \sqrt{\begin{array}{c}\left(1 + k''_{\lambda_2} * X\right)^2 \sigma^2_{m_{inst_{\lambda_2}}} + X^2 * \sigma^2_{k'_{\lambda_2}} + \left(-k'_{\lambda_2} - k''_{\lambda_2} * \mathrm{CI}_{inst}\right)^2 \sigma^2_X + \\ \left(X^2 * \mathrm{CI}_{inst}{}^2\right) \sigma^2_{k''_{\lambda_2}} + \left({k''_{\lambda_2}}^2 * X^2\right) \sigma^2_{m_{inst_{\lambda_1}}} + \sigma^2_{m_{zp_{\lambda_2}}} + \\ {CI_{standard}}^2 * \sigma^2_{T_{\lambda_2}} + {T_{\lambda_2}}^2 * \sigma^2_{CI_{standard}} + \\ \sum_{i,j:i\neq j}^{8} \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_i}\right)\left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_j}\right) \sigma^2_{variable_i variable_j}\end{array}} \tag{62}$$

Equation (62) is a generic result for the uncertainty of the standard magnitude expression of Equation (42).

A simplification can be made if the airmass (X) is assumed to be a constant. If the positional information for the object of interest is used to calculate airmass, then the uncertainty of the airmass (σ_X) should be negligible. In this scenario, the uncertainty would be zero ($\sigma_X = 0$), and the airmass (X) would no longer be a variable. Thus, $N = 7$ and we would get Equation (63).

$$\sigma_{m_{standard_{\lambda_2}}} = \sqrt{\begin{gathered} \left(1 + k''_{\lambda_2} * X\right)^2 \sigma^2_{m_{inst_{\lambda_2}}} + X^2 * \sigma^2_{k'_{\lambda_2}} + \\ \left(X^2 * \mathrm{CI}_{inst}{}^2\right) \sigma^2_{k''_{\lambda_2}} + \left(k''_{\lambda_2}{}^2 * X^2\right) \sigma^2_{m_{inst_{\lambda_1}}} + \sigma^2_{m_{zp_{\lambda_2}}} + \\ CI_{standard}{}^2 * \sigma^2_{T_{\lambda_2}} + T_{\lambda_2}{}^2 * \sigma^2_{CI_{standard}} + \\ \sum_{i,j:i \neq j}^{7} \left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_i}\right)\left(\frac{\partial m_{standard_{\lambda_2}}}{\partial variable_j}\right) \sigma^2_{variable_i variable_j} \end{gathered}} \tag{63}$$

4.4.1.3 Covariance Terms

The covariance terms need to be calculated explicitly for each pair of variables. If the uncertainty of the pair of variables is independent, then the covariance between the variables is zero. In a general manner, using Equation (62), the variables whose uncertainties are independent may be determined.

The uncertainty of the airmass (X) should be independent of all other variable uncertainties, therefore, all of the covariance terms involving airmass (X) should be zero, $\sigma^2_{X variable_j} = 0$. The instrumental magnitudes ($m_{inst_{\lambda_2}}$ and $m_{inst_{\lambda_1}}$) are brightness measurements of the RSO, and their uncertainties should be independent of the uncertainty of the other variables that are not a measurement of the RSO's brightness. This implies that the covariance between the instrumental magnitudes ($m_{inst_{\lambda_2}}$ and $m_{inst_{\lambda_1}}$) and all of the other variables is zero, except for $CI_{standard}$ ($\sigma^2_{m_{inst_{\lambda_2}} CI_{standard}} \neq 0$ and $\sigma^2_{m_{inst_{\lambda_1}} CI_{standard}} \neq 0$) since $CI_{standard}$ is the difference of the brightness of the RSO on the standard system in the two different spectral regions, λ_2 and λ_1. These assumptions should be verified by an explicit calculation of the covariance between these pairs of variables.

Beyond these simplifications, there is a dependency upon the calibration technique used to put the instrumental magnitude onto a standard photometric system, e.g., all-sky calibrations or in-frame calibrations. Depending on the calibration approach used, it may be possible to make assumptions that eliminate the covariance for some of the other pairs of variables. Let us examine an approach that uses all-sky calibrations.

For all-sky calibrations, standard stars (constant brightness) of a photometric system are followed throughout the night as they transit the sky and their airmasses vary. The calibration variables (k'_{λ_2}, k''_{λ_2}, $m_{zp_{\lambda_2}}$, and T_{λ_2}) are solved from the measurements of these stars. We cannot make any assumptions that the uncertainties of the calibration variables are independent since they will be dependent on the measurement uncertainties of the same stars; therefore, the covariance terms need to be solved for each pair of calibration variables. All-sky calibrations do not allow additional assumptions that eliminate any of the covariance terms.

5. CONCLUSIONS

We have presented a theoretical framework to calculate the uncertainties for the instrumental magnitude and the standard magnitude. We have provided the theoretical background on the techniques for uncertainty propagation, the Quadrature method and the Computer Calculation of Uncertainties method. We also reviewed the literature of the astronomical community to determine the standard astronomical technique for calculating the instrumental magnitude uncertainty. The standard astronomical technique, determined using the Quadrature method, includes an expression for calculating the SNR and an expression for the uncertainty of the instrumental magnitude as a function of SNR, the latter of which we derive.

We provided an example of a calculation of the SNR and each expression of the uncertainty of the instrumental magnitude using a specific scenario and values. This example demonstrates how to calculate these values as well as how to represent the measurement and its uncertainty in a single expression for both the standard astronomical technique and the Computer Calculation of Uncertainties method.

We performed a detailed comparison between the standard astronomical technique (Quadrature) and the Computer Calculation of Uncertainties method. We found that the instrumental magnitude uncertainty for the two uncertainty propagation methods do not produce the same results for small values of SNR, but quickly converge for larger values of SNR. The Computer Calculation of Uncertainties method has a plus and minus result that has large asymmetry for small values of SNR; this asymmetry rapidly decreases as the SNR increases. We recommend using the Computer Calculation of Uncertainties method to calculate the instrumental magnitude uncertainty (Equation (34)) for accurate results. The standard astronomical technique (Equation (30)) is appropriate for situations where an approximation of the instrumental magnitude uncertainty in the form of a single valued result would be acceptable, or for large values of SNR where the two methods produce virtually identical results.

We provide a generic expression to calculate the standard magnitude uncertainty in Equation (36). Using the entire standard magnitude expression, Equation (42), we derive the standard magnitude uncertainty with the covariance terms in generic summations, Equation (62). We simplify the summations of the covariance terms with a few assumptions about what variable uncertainties are independent, regardless of the calibration technique used. We conclude the standard magnitude uncertainty discussion with a scenario for all-sky calibrations and how it would affect the covariance terms. While a data provider may not use the full standard magnitude expression used in our standard magnitude uncertainty derivation, the standardized process shown provides an example that a data provider may follow.

We conclude with our recommendations that the Computer Calculation of Uncertainties method (Equation (34)) be used for the instrumental magnitude uncertainty and the expression of Equation (62) be used for the standard magnitude uncertainty.

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