COOP: A webserver for conjunction management for large satellite constellations using dynamic Voronoi diagrams

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Abstract

Conjunction prediction is critical for economic and safe management of large constellations and geospace. However, accurate and efficient prediction of conjunctions among RSOs remains a mathematical and computational challenge. We propose a unified framework for both conjunction prediction and optimal maneuver planning in an efficient and correct, at least accurate, manner with no missing case. The idea of the proposal can be tested at the COOP (Conjunctive Orbital Objects Predictor and Planner) web server at Hanyang University (http://voronoi.hanyang.ac.kr/coop).

Keywords

Conjunction prediction, Optimal maneuver, Satellite, Constellation, Starlink, Kuiper, Resident Space Object, Spatial reasoning, Moving objects

Introduction

Conjunction management involves the prediction and avoidance of possible conjunctions among orbital objects in a foreseeable future and is one of the most critical issues for space traffic management (STM). Recently, the number of valuable orbital objects is rapidly growing [1]. For example, Starlink plans to deploy 12,000 satellites by the mid-2020, Kuiper has obtained the Federal Communications Commission approval for 3,236 satellites, etc. The narrow space around 550 km altitude will be quickly crowded. Hence, STM is getting more important and so is the accurate and efficient conjunction management.

The more resident space objects (ROSs) there are, the higher collision probability is getting. Despite many prior works, a unified theoretical and computational framework of conjunction management has not been uncovered yet. Accurate and efficient prediction of conjunctions among RSOs remains a mathematical and computational challenge. One of the main challenges has been and will be the combinatorial explosion among the RSOs to be further investigated with astrodynamics considerations during a prediction time window. We want to identify all potential conjunctions for many-to-many cases rather than simple one-to-many cases over the entire prediction time window.

Contributions of this study

I. We propose a unified mathematical and computational theory for conjunction management based on the dynamic Voronoi diagram of RSOs.
II. We developed a testbed web server, COOP (Conjunctive Orbital Objects Predictor and Planner), which performs the following dual functions correctly and efficiently: (i) Predict conjunctions and (ii) plan the best maneuver avoiding conjunctions.

COOP Functions

COOP is a web server of a paradigm-transforming idea based on the dynamic Voronoi diagram (DVD) of moving RSOs. Its initial idea has been presented in the two prior AMOS conferences. In AMOS2017, we reported that DVD can be used to predict a conjunction accurately and efficiently [2], and in AMOS2018, reported that the DVD can be used to find the best maneuver plan by quickly enumerating and evaluating many alternatives during the entire time window [3]. Algorithmic details are elaborated in [4]. COOP implements the algorithms above together with realistic and practical computational considerations. COOP guarantees no missing case of conjunction prediction and computes an optimal maneuver very quickly. Hence, it can be used for efficiently solving diverse spatial reasoning problems among many moving objects including conjunction prediction problem. All challenges are beautifully solved within a single mathematical and computational framework of the DVD of RSOs. Thus, COOP is a necessary and useful tool for the management of mega-constellations.

Brief on Voronoi diagrams. Voronoi diagram is a geometric construct and data structure which can represent the spatial relationship among particles in the most concise manner. Given a Voronoi diagram of particles in the 2D or 3D space, spatial reasoning among the particles can be done most efficiently. See [5] and [6] for details.

Let M_V, M_E, and M_F represent the numbers of vertices, edges, and faces of the Voronoi diagram of N RSOs, respectively. In 3D, $M_V = O(N^2)$, $M_E = O(N^2)$, and $M_F = O(N^2)$ in the worst case. In other words, the Voronoi diagram can have $O(N^2)$ vertices, edges, and faces. The Voronoi diagram of N static RSOs fixed at locations in geospacer corresponding to a particular moment can be constructed in the optimal $\theta(N^2 \log N)$ time. For enhanced robustness, the topology-oriented incremental algorithm which takes $O(N^3)$ time is recommended [7, 8]. On average, however, $M_V = O(N)$, $M_E = O(N)$, $M_F = O(N)$, and the static Voronoi diagram can be constructed in $O(N)$ time for N RSOs. The worst case scenario is hard to imagine for RSOs and from our experience and we do not anticipate to ever encounter the worst case with RSOs. All complexity measures in this paper are in the worst case sense unless otherwise stated.

As a testbed web server, COOP currently provides the answers to the following three types of queries for an entire prediction time window. Solutions are correct and computation is very efficient.

[Query I] Find the proximal events of an object-of-interest (OOI): Given an OOI and a threshold distance presented by user, COOP reports all RSOs which approaches closer than the threshold. This is a one-to-many type of query.

[Query II] Find all proximal events in time horizon: Given a threshold value, COOP reports all RSO pairs which approaches closer than the threshold. This is a many-to-many type of query.

[Query III] Find the optimal maneuver plan for an OOI: Given an OOI, an instance t for beginning the maneuver of the OOI, and a set of maneuver alternatives, COOP evaluates the alternatives in order to find the best one in terms of the Time and Distance of Closest Approach (TCA and DCA) of the OOI and the other RSOs.

NOTE: These three queries are the most basic examples that COOP can currently demonstrate. Many other spatial queries among more than two RSOs can be similarly processed.

COOP Algorithms

COOP first downloads the TLE data from Space Catalogue (https://www.space-track.org/) and makes a linear approximation of each orbit. We consider a replica of each RSO moves through the piecewise linear path. Then, COOP constructs the DVD of the RSO replicas and produces an ordered sequence of significant events, abbreviated as EVENTSEQ, for a prescribed prediction time window. The events include those related with the state changes of both Voronoi diagrams (i.e. the flip of Voronoi edges and faces) and RSO replicas (i.e. the change of velocity because a replica transfers to another line segment). Given the EVENTSEQ and the state variables of all RSOs at the beginning of the prediction window, COOP efficiently processes the queries to get correct answers. State variables of an RSO include its location and velocity. We use the replicas because the DVD construction algorithm is not known for the objects following curved paths but is known for linearly moving objects.

The key idea of COOP is the following: The DVD of the RSO replicas which move linearly can be used to reduce the combinatorial solution space to a tiny one which contains the correct solution among the real RSOs on orbits. The computation for finding the reduced solution space is tiny. The construction of the DVD of all RSO replicas for the entire prediction time window requires some significant computation but is acceptable. The specifics of the COOP algorithm follow.

The production of EVENTSEQ consists of three steps: (i) Linear approximation of RSO orbits; (ii) Construction of an initial, static Voronoi diagram of N RSOs at the beginning of the prediction time window; (iii) the computation of EVENTSEQ. Step (i) takes $O(N)$ time for N RSOs. Step (ii) takes O(N) time on average. Step (iii) takes $O(K \log N)$ time, with an $O(N \log N)$ time preprocessing for maintaining a priority queue, where K represents the number of events over the entire prediction time window. The initial Voronoi diagram can be stored in O(N) memory on average and EVENTSEQ can be stored in O(K) memory. It turns out that $K = O(V + \delta + \rho + W)$ where $V,\ \delta,\ \rho$, and W represent RSO velocities, RSO density (i.e., the number of RSOs in a given space), linearization resolution (i.e. the number of line segments to approximate an orbit), and the length of prediction time window, respectively.

The idea of the Query I algorithm is as follows. Given an OOI, COOP locates the Voronoi cell of OOI from the initial, static Voronoi diagram. Then, COOP collects all Voronoi faces of the Voronoi cell of OOI by scanning EVENTSEQ over the entire prediction time window. As a Voronoi face defines the distance between two nearby RSO replicas, the spatial reasoning for OOI with its neighbors can be correctly performed by simply investigating the set of collected Voronoi faces. As a Voronoi face can be born for or can be removed from a Voronoi cell by an event in EVENTSEQ, Query I for an OOI can take $O(M_F + K)$ time. Surprisingly enough, Query II can also be performed with the same time complexity of Query I. Two reasons for this speed. First, there exist total $M_F + K$ Voronoi faces during the entire prediction time window. Second, once the lifespan of a Voronoi face (i.e. the adjacency of an RSO replica pair) is given, the TCA and DCA of the pair of RSO replicas can be easily found from the lifespan. The TCA and DCA for real RSOs are closely related with those of RSO replicas and can be computed correctly and efficiently. We skip details.

The idea of Query III algorithm is as follows. Suppose that COOP receives an input of M_A maneuver alternatives of an OOI and the moment t of the beginning of the maneuver. The key idea is to find the best alternative which maximizes the minimum miss distance (i.e. DCA) between each of the alternatives and the RSOs. The core algorithmic issue resides in reasoning the closest neighbor RSO of the maneuver alternative during the entire prediction time window.

Assume that each alternative, say A, consists of connected line segments in the same resolution with those for the RSO replicas. COOP starts by finding the Voronoi cell of OOI from the static Voronoi diagram at t. Then, COOP propagates the DVD by both scanning EVENTSEQ and processing each line segment of A with a little bit of careful synchronization effort. As the computation requirement for scanning each event in EVENTSEQ and processing each line segment of A takes $O(1)$ time, this propagation takes $O(M_F + M_V + K)$ time. This is because it is necessary (i) to propagate the Voronoi diagram by processing K_1 events up to t in $O(M_F + K_1)$ time, (ii) to evaluate the Voronoi diagram at t by calculating the coordinates of M_V Voronoi vertices in $O(M_V)$ time, and (iii) to propagate the Voronoi diagram and the OOI replica through the line segments to the end of the prediction time window by processing $K - K_1$ events in $O(K - K_1)$ time. Therefore, the evaluation of M_A maneuver alternatives can be done in $O(M_A(M_F + M_V + K))$ time. If maneuver alternatives are represented in a finer resolution, computational effort is increased slightly, but not significantly at all. Be aware that the M_A maneuver alternatives are scalable.

Researchers can freely download both the initial Voronoi diagram and EVENTSEQ from the COOP web server in order to solve spatial reasoning problems among RSOs using his or her own application programs. The COOP web server will soon post a library with useful API functions so that users can easily embed in their application programs. By tracking the events in EVENTSEQ, users can replay the propagation of RSOs accurately and efficiently. Many critical applications are expected to be found. Therefore, COOP will be a key engine of constellations such as Starlink, Kuiper, etc.

Scalability

The COOP algorithms have several scalability features. First of all, the time-window scalability, TW-scalability, is to divide the prediction time window into a set of mutually exclusive time fragments where each can be processed independent of each other. Hence, this property scales with the number of CPU cores. TW-scalability applies to both the generation of EVENTSEQ and the processing of queries. The flip-influence scalability, FI-scalability, refers to the number of re-calculations to update the priority queue of events in order to reflect a new flip of either Voronoi edge or face in the neighborhood. FI-scalability applies to the generation of EVENTSEQ. The maneuver planning scalability, MP-scalability applies to the evaluation of maneuver alternatives and is scalable to the minimum of the number of alternatives and the number of CPU cores. There are other scalable features in the COOP algorithms.

Experimental Statistics

The performance of the COOP server is measured through an experiment of the following setting. The test data set consists of 1,000 Low Earth Orbit RSOs in Space Catalogue (From the first to the 1,000-th one) and was downloaded on 07:50, Aug 28, 2020, UTC. Computation platform is as follows: Core i7-6700 3.4GHz/16GB RAM (Quad-core CPU used). The prediction time window was set as 24 hours from the download. To take advantage of the time-window scalability and the quad-core platform, we divided the prediction time window of 24 hours into a 1,440 (=60*24) time segments (each with the length of 60 second) and assigned each segment to one of the four cores.

The computation of EVENTSEQ took about 11,700 seconds (3 hours and 15 minutes).

We tested Query II and found that the global TCA/DCA occurs between COSMOS 830 (8894) and COSMOS 1393 (13380) with distance 0.94 km at 1:17:0, Aug. 28, 2020, UTC. It took 1,242 seconds (20 minutes and 42 seconds). While we solve Query II, we also found other nine TCAs and the associated distances. Table 1 summarizes them. From the solution of Query II, we extracted the solution of Query I using NOAA-1 (4793) as the OOI: Table 2 summarizes the six TCAs and their distances.

No.	Primary (ID)	Secondary (ID)	Distance (km)	TCA
1	COSMOS 830 (8894)	COSMOS 1393 (13380)	0.94	1:17:0, Aug. 28, 2020, UTC
2	COSMOS 422 (5238)	COSMOS 1027 (10991)	1.13	10:43:27, Aug. 28, 2020, UTC
3	COSMOS 828 (8892)	COSMOS 1637 (15619)	1.32	12:11:46, Aug. 28, 2020, UTC
4	COSMOS 529 (6264)	COSMOS 1856 (18117)	1.39	20:43:46, Aug. 28, 2020, UTC
5	COSMOS 619 (6987)	COSMOS 1193 (11876)	1.4	1:14:56, Aug. 28, 2020, UTC
6	COSMOS 342 (4389)	COSMOS 1999 (19790)	1.45	15:27:12, Aug. 28, 2020, UTC
7	COSMOS 422 (5238)	COSMOS 1428 (13757)	1.69	4:38:27, Aug. 28, 2020, UTC
8	COSMOS 1500 (14372)	COSMOS 1825 (17566)	1.71	14:33:49, Aug. 28, 2020, UTC
9	OSCAR 5 (4321)	COSMOS 1131 (11539)	1.92	21:12:40, Aug. 28, 2020, UTC
10	COSMOS 791 (8607)	COSMOS 1037 (11046)	2	4:7:55, Aug. 28, 2020, UTC

Table 1. Solution of Query 2: Top 10 TCAs and their distances

No.	Secondary (ID)	Distance (km)	TCA
1	COSMOS 1433 (13765)	8.68	10:16:11, Aug. 28, 2020, UTC
2	COSMOS 2010 (19904)	9.59	15:32:47, Aug. 28, 2020, UTC
3	COSMOS 2010 (19904)	2.69	16:30:13, Aug. 28, 2020, UTC
4	COSMOS 979 (10586)	8.05	18:38:49, Aug. 28, 2020, UTC
5	COSMOS 878 (9595)	8.6	18:29:57, Aug. 28, 2020, UTC
6	COSMOS 979 (10586)	9.35	19:36:8, Aug. 28, 2020, UTC

Table 2. Solution of Query 1: The primary is NOAA-1 (4793). The TCAs with distances less than 10km.

Conclusion

We report a paradigm-transforming method for conjunction management, i.e. both predicting and planning conjunctions, of resident space objects by using on the dynamic Voronoi diagram of moving objects. There will be many more important queries involving more than two RSOs which are difficult to answer unless the proposed COOP method is used. The COOP method guarantees (i) correct solutions, (ii) no missing case, and (ii) efficient computation. We recommend readers to visit and test the COOP (Conjunctive Orbital Objects Predictor and Planner) web server (http://voronoi.hanyang.ac.kr/coop/).

We anticipate the COOP web server and the COOP API library will be the core tool for Space Traffic Management. Code optimization will significantly improve both solution quality and computation speed. Mega-constellations such as Starlink and Kuiper must take advantage of the capability demonstrated by COOP for safe and economic operation of their satellites.

References

[1] How many space debris objects are currently in orbit?, European Space Agency, 2013. (http://www.esa.int/Our_Activities/Space_Engineering_Technology/Clean_Space/How_many_space_debris_objects_are_currently_in_orbit)

[2] Jehyun Cha, Drew McNeely, Joonghyun Ryu, Misoon Mah, Moriba Jah, Deok-Soo Kim, DVD-COOP: Innovative Conjunction Prediction using Voronoi-filter based on the Dynamic Voronoi Diagram of 3D Spheres, *Advanced Maui Optical and Space Surveillance Technologies Conference(AMOS), September 19-22, Wailea Marriot, Maui, Hawaii*, USA, 2017.

[3] Jehyun Cha, Joonghyun Ryu, Mokwon Lee, Chanyoung Song, Youngsong Cho, Paul Schumacher, Misoon Mah, Deok-Soo Kim, DVD-COOP for Optimal Design of Maneuver Path for Conjunctive Objects, *Advanced Maui Optical and Space Surveillance Technologies Conference(AMOS), September 12-14, Wailea Marriot, Maui, Hawaii, USA*, 2018.

[4] Chanyoung Song. Jehyun Cha, Mokwon Lee, Deok-Soo Kim, Dynamic Voronoi Diagram for Moving Disks, *IEEE Transactions on Visualization and Computer Graphics*, DOI: 10.1109/TVCG.2019.2959321, 2019.

[5] A. Okabe, B. Boots, K. Sugihara, and S. N. Chiu. 1999. Spatial Tessellations: Concepts and Applications of Voronoi Diagrams (2nd ed.). John Wiley & Sons, Chichester.

[6] F. Aurenhammer, R. Klein, D.-T. Lee, Voronoi Diagrams and Delaunay Triangulations, World Scientific, 2013.

[7] Kokichi Sugihara. 1992. A simple method for avoiding numerical errors and degeneracy in Voronoi diagram construction. IEICE Transactions, Fundamentals E75-A (1992), 468–477.

[8] Mokwon Lee, Kokichi Sugihara, and Deok-Soo Kim, Topology-oriented Incremental Algorithm for the Robust Construction of the Voronoi Diagrams of Disks, ACM Transactions on Mathematical Software, 2016.