

# Dempster-Schafer Inspired Association Framework and Tracking Metric for Space Objects.

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## Abstract

Space situational awareness and space domain awareness by necessity must be built on a solid framework of data association, data integrity, and timeliness to support orbit estimation and mission success. Data association is complicated by the often thousands of simultaneous observations and objects being processed at any given time. It is particularly challenging for tracking and custody of multiple objects with similar orbits. The data association problem can result in false and missed detections due to uncertain dynamics, maneuvering objects, closely-spaced objects, and association over long propagation intervals between observations. Orbit estimation is further complicated by the scale of intelligently operating a system with thousands of objects which challenges most processing platforms. This paper proposes a framework to address these issues based on Multi-Bernoulli filter, Dempster-Shafer theory and joint probabilistic data association to better express how well objects are being tracked. In particular, the belief of an object being actively tracked versus actively lost are presented in this paper and directly incorporated into the data association equations. These two measures are shown to provide contextual information on if an object is being actively tracked, while solving some problematic data association situations. The value of this approach is demonstrated by representative use cases based on the live data collection and processing done on the L3Harris SpaceSentry system.

## I. INTRODUCTION

Estimation and tracking of a space object in a standard Bayesian framework assumes the object is explicitly within a defined probability distribution. All space situational awareness (SSA) systems task sensors to acquire observations on those objects, use obtained measurements to update those estimates, and therefore continues to track the objects [1], [2]. Most such systems, at some point in their processing, attempt data association is such a way to assure the observations used for estimation are consistent with the estimate itself; observations don't have license plates so the system itself must determine the association. The most popular such system is joint probabilistic data association (JPDA) which computes the conditional probabilities between  $n$  observations and  $m$  space objects based on Mahalanobis distance [3], [4], [5]. In reality, estimation of orbits can be affected by unmodeled dynamics (such as high area-to-mass ratio (HAMR) objects), constraining statistics (such as Gaussian assumptions) and active maneuvering. From a data standpoint, estimation is muddled by closely-spaced objects, weather, sensor failure, noise in various forms and more. These factors all contribute to inherent object estimates being lost or simply not seen for long periods of time.

This paper acknowledges two states of estimation for a space object: 1) an object is being actively tracked, 2) the object has been lost. Implicitly, there is a third situation: the object is not being actively tracked *but the estimation system doesn't know if its been lost*. The case for one is fairly clear, case two occurs when the system tasks on an object and doesn't find it. Case three can happen due to coverage gaps in a sensor network, weather, sensor failures, or a system operator choosing not to look for a low priority object. Multi-Bernoulli filtering, which has its roots in finite set statistics, handles multi-object tracking by explicitly modeling each object as being able to appear (birth) or disappear (death) at any given time. The first and primary result of this paper is to take multi-Bernoulli framework and use it explicitly to model a systems success at tracking an object.

Dempster-Schafer theory (DST) is a non-Bayesian alternative to estimation which explicitly accounts for the ambiguity described in the three tracking situations discussed [6], [7], [8]. In the simplest form of DST, probability is described as belief split between all possible statistical states and the super sets of those states. In our case, there is belief weight associated with an object being tracked, an object being lost, and the super set: the object is either tracked or lost. This allows the results of an estimation framework to independently assess if the object is being tracked versus if it is lost, and importantly allows the system to say there is no evidence of either. The framework being proposed dynamically estimates belief of tracking and belief of loss. Dempster-Shafer theory and other theoretical frameworks have been used to capture ambiguity in the filtering problem in SSA before and is a current popular research topic [9], [10].

One interesting result of this system is achieved within the new data association equations as the system knows when an object is not being actively tracked. This is best exemplified by a dim and hard to detect object: such an object may pass through a field-of-view of a sensor but produce no observations. A standard JPDA system would erroneously associate this object to noise and incorrect objects. By including belief of tracking in JPDA, objects with no belief of tracking default to no update; noise is rejected as the evidence of false detection is overwhelming. Instead, the mathematics require enough observations to build up belief in tracking before updating the underlying state. This process has greatly reduced the number of cross tags and erroneous associations in the L3Harris SpaceSentry system. Similar work has been done explicitly combining JPDA with Bernoulli filtering to model a completely unknown probability of detection [11].

This paper presents two versions of the proposed filter. The first is more classic a Multi-Bernoulli filter based on the JPDA equations which is primarily driven by probability of tracking. The second is a expanded out to include Demster-Shafer theory explicitly in JPDA framework expanding out the applicability. Examples will be shown demonstrating the efficacy of the algorithms on multiple closely spaced objects to highlight some of the best examples.

## II. JOINT PROBABILITY DATA ASSOCIATION WITH BERNOULLI TRACKING PROBABILITY

### A. Mathematics of Joint Probability Data Association

JPDA begins by modeling at a particular time,  $t$ , each measurement  $\mathbf{z}^i(t)$  as a random variables, generated from either background clutter or an object,  $\mathbf{x}^j(t)$  [5]. Then define a particular estimate-to-measurement association,  $d_i^j$ , as the hypothesis that object  $j$  produced measurement  $i$  in the particular measurement frame, using  $i = 0$  to represent a missed detection. The likelihoods of a particular data association is then

$$l(d_0^j) = (1 - p_D)\lambda \quad (1)$$

$$l(d_i^j) = p_D \cdot \mathcal{N}(\mathbf{z}_t^i; \hat{\mathbf{x}}_t^j, \Sigma), i > 0 \quad (2)$$

where  $p_D$  is the probability of detection,  $\lambda$  is the background false detection (clutter) density, and  $\hat{\mathbf{x}}$  is the state projected into the measurement space. The Gaussian distribution,  $\mathcal{N}$ , is assumed in this paper though any PDF evaluation could be used.

We can now define an ‘‘event’’ as a particular set of consistent hypotheses for the all estimates. For an event,  $\theta$ , each estimate  $j$  is given a single association  $d_i^j$ , to unique measurement  $i > 0$  or missed detection  $i = 0$ . Note that each measurement can only be used once, but a missed detection can be assigned to multiple states. A particular hypothesis is then

$$\theta_k = \bigcup_j d_i^j \quad (3)$$

while the likelihood of an event is just the product of the likelihood of each association in  $\theta$ ,

$$l(\theta) \prod_{d \in \theta} (l(d_i^j)). \quad (4)$$

The goal of JPDA is to form a marginal likelihood for each association,  $d_i^j$ , conditional on all other associations. The final marginalized likelihood of a particular association becomes

$$q_t(d_i^j) = \sum_{\theta | d_i^j} l(\theta) \quad (5)$$

or the sums of the likelihoods of all events where  $d_i^j$  occurs. Also note that the likelihood of missed detection is also calculable as

$$q_t(d_0^j) = \sum_{\theta | d_0^j} l(\theta). \quad (6)$$

The final conditional probabilities are then calculated by normalizing the likelihood of a particular association by the total likelihood

$$p_t(d_i^j) = \left( \sum_{\theta | d_i^j} l(\theta) \right) / \left( \sum_{\theta} l(\theta) \right) \quad (7)$$

### B. Probability of Belief and the Bernoulli Filter

An underlying assumption in a Bayesian filter, in particular with regards to probability of detection,  $p_D$ , is that a given object always has a known probability of producing a measurement. In determining probability of detection, all physical reasons an object may not be detected must be accounted for. In historical efforts, a hard coded value of around 0.90 is typically used; for images as an example, this is based on back of the envelope-type calculations of the probability that a detection is obstructed by a star (or other faults in image processing). It is uncorrelated through time/space so it always has that same constant value. On real systems detection is also a function of the brightness of the objects and the sensitivity of the detection algorithm. A detection algorithm’s ability to detect an object can be described as being binary; either the object is bright enough to detect, or is not bright enough to detect (with some edge cases where the objects is just on the boundary). Furthermore, most objects change brightness over the course of the night and therefore may pass the detection threshold in both directions over the course of a night. This section attempts to directly model that behavior with a Bernoulli filter on the belief that an object is being tracked.

A Bernoulli filter is derived from random finite set (RFS) filters, also referred to as Finite Set Statistics [12]. In particular, a Bernoulli filter is a single object filter with a single important generalization: the object being track randomly switch in and

out of existence [13]. With some simplification of notation, the state is modeled as a Bernoulli RFS which has a probability density

$$p(\mathcal{X}(t)) = \begin{cases} 1 - p_E(t), & \text{if } \mathcal{X} = \emptyset \\ p_E(t) * p(\mathbf{x}(t)), & \text{if } \mathcal{X} = \mathbf{x} \end{cases} \quad (8)$$

where  $p(\mathbf{x}(t))$  is the more standard spacial PDF for the object if it does exist. Similar to all RFS filters, this can be visualized as akin to a pdf but that does not integrate to one; instead the integral over belief is the cardinality of the system (number of objects being tracked). A multi-Bernoulli filter is modeled as the effective sum of several Bernoulli filters each tracking a unique objects. Essentially, this is equivalent to performing estimation of the spacial PDF, just like a normal Bayesian filter, jointly with estimation of the probability of existence,  $p_E(t)$  on each object. Note that the above description is a fairly aggressive simplification of RFS filtering in general and multi-Bernoulli filtering specifically; more can be found at [14], [15].

Classically, Bernoulli filters model “existence” as literal; objects are ceasing to exist and new objects are being born. For the catalog maintenance problem, objects don’t (typically) cease to exist but instead cease to provide measurements, or move out of the sensor’s field. This paper instead assume each Bernoulli has permanence and is explicitly tied to a particular unique space object (much like a standard Bayesian filter). Probability of Existence is instead modeled as “Probability Tracked”, the probability that an object is actively being tracked by the estimation system.

The actual estimation of  $p(\mathcal{X}(t))$  is completely separable between the spatial PDF and the probability of existence. The equations are derived from [13], equations (2.42-2.49), with the added simplification of setting the birth model equal to the propagated spatial PDF. The propagation step is

$$p_{E,k|k-1} = p_b(1 - p_{E,k-1|k-1}) + p_s(p_{E,k-1|k-1}) \quad (9)$$

$$p(\mathbf{x})_{k|k-1} = \int \pi_{k|k-1}(\mathbf{x}|\mathbf{x}')p(\mathbf{x}')_{k-1|k-1}d\mathbf{x}' \quad (10)$$

where (10) is the standard non-linear Bayesian propagation, from which the linear Kalman filter and unscented Kalman filter can be derived. The update step for a standard Bernoulli filter is

$$p_{E,k|k} = \frac{1 - \delta_k}{1 - \delta_k p_{E,k|k-1}} p_{E,k|k-1} \quad (11)$$

$$p(\mathbf{x})_{k|k} = \frac{g_k(\mathbf{z}|\mathbf{x})p(\mathbf{x})_{k|k-1}}{\int g_k(\mathbf{z}|\mathbf{x})p_{k|k-1}(\mathbf{x})d\mathbf{x}} \quad (12)$$

and

$$\delta_k = \left( 1 - \int \frac{g_k(\mathbf{z}|\mathbf{x})}{g_k(\mathbf{z}|\emptyset)} p_{k|k-1}(\mathbf{x})d\mathbf{x} \right) \quad (13)$$

where  $g(\mathbf{z}|\mathbf{x})$  is the likelihood function between the measurement(s) and the state and (12) is the standard nonlinear update equations from which JPDA and Kalman filtering can be derived.

The quantity  $\delta_k$  has an integral which marginalizes the likelihood of the measurements over the estimate prior distribution. This likelihood is normalized by the likelihood of the measurement given the objects does not exist,  $g_k(\mathbf{z}|\emptyset)$ . These likelihoods are equivalent to ones calculated in JPDA:

$$\delta_k^j = \left( 1 - \sum_i \frac{q(d_i^j)}{q(d_0^j)} \right) \quad (14)$$

recalling that  $j$  is the index for a particular object and  $q(d_0^j)$  is the likelihood that an object is not detected. This implies that a standard Bernoulli filter update can be calculated directly with JPDA on a series of Gaussian distributed estimates in a Kalman filter style estimation system.

The next big innovation in this paper is remodeling probability of detection to reflect the new model of object existence. Assuming that an object is bright enough to be detected, there is still a chance that the object is behind a star, etc, in a given frame,  $p_D^-$ . However, getting a detection is conditional on the object actually being successfully tracked by a sensor with sufficient sensitivity. Then the actual probability of detection is then the probability that the sensor can detect it and it is not blocked by a star,

$$p_D(t) = p_E(t) * p_D^- \quad (15)$$

which in practice is used in (2). Because  $p_D$  is used in JPDA, the prior  $p_{E,k|k-1}$  is used to compute likelihoods which are then used to update probability of existence and the spatial PDF.

TABLE I  
TRACKING BERNOULLI VARIABLE IN DEMPSTER-SHAFER FRAMEWORK

	mass	belief	plausibility
$\mathcal{T}$ (tracked)	0.1	0.1	0.7
$\mathcal{L}$ (Lost)	0.3	0.3	0.9
$\mathcal{T} \cup \mathcal{L}$ (Unknown)	0.6	1.0	1.0

### C. Interpreting the Bernoulli Update

Equation (15) is an important part of the results presented in this paper and warrants further discussion. Probability of existence is a term from multi-Bernoulli filtering and is being used here for consistency. In the next section, it will change to “belief of tracking” which is a better over all name. This probability is needed because SSA systems are flawed. Objects may not be trackable due to weather, visibility, flaws in the detection algorithms, seasonal changes in solar phase angle, or more. They also may not be detected because the SSA system no longer has a valid orbit for the object. Regardless, this untrackability is different than the normal probability of detection used for these systems; its is explicitly correlated with time and is better modeled as a Markov switch between tracked and not tracked. Probability of existence models that phenomenology and gives JPDA the ability to adapt to such a situation without changing its core mathematics. In practice, when  $p_E(t)$  is low on object  $j$  it forces all the likelihood into  $q_t(d_0^j)$  (The missed detection hypothesis). Good detections increase  $p_E(t)$  but are unable to claim associations; only when enough detections have associated to raise  $p_E(t)$  is the spacial distribution updated. False detections from noise or incorrect nearby objects do not persist long enough to raise  $p_E(t)$  and therefore protect the system from bad updates.

The  $p_{E,k}$  equations have several input parameters which may be new to the reader. The probability of birth,  $p_b$  is interpreted as objects randomly appearing in the state space modeled as a Poisson in time and (often) uniform spatially. This new model still assumes that birth is Poisson but instead is a known object becoming trackable again so spatial distribution is preserved from the prior estimate,  $p(\mathbf{x})_{k|k-1}$ . Probability of survival,  $p_s$ , is much more simply the probability that an object that is giving measurements will continue to in any given frame. Both  $p_b$  and  $p_s$  are then probabilities between 0 and 1. In practice,  $p_b$  acts as a floor to  $p_{E,k}$  and  $p_s$  acts as a ceiling to  $p_{E,k}$ , assuring that it never over converges in either direction. The  $p_{E,k}$  updates is based around a ratio of likelihoods in (14). If the ratio is even,  $\delta_k = 0$  and  $p_{E,k|k} = p_{E,k|k-1}$ . As the ratio becomes greater than one, the probability of existence goes up, while a ratio lower than one causes probability of existence to fall.

## III. DEMPSTER-SHAFER BERNOULLI FILTER

### A. Dempster-Schafer Theory as it Applies to the Tracking Problem

As described above,  $p_E(t)$  explicitly models whether an estimation system is successfully tracking an object or not. However, being unable to track an object could be caused by two possible scenarios. The first is that weather, visibility, and other normal operational issues have caused a particular sensor to fail to detect the object. Operators may still know where the object is but need a different sensor. The second is that, for whatever reason, the object has been lost. This could be due to a variety of reasons including the object actively maneuvering and is one of the most important and challenging events facing the Space Domain Awareness community. There is an obvious gulf between those two scenarios and  $p_E(t)$  does not provide a way to distinguish between them.

Dempster-Shafer Theory is a system for handling epistemic uncertainty by splitting the typical probability measure of a system into two separate measures, belief and plausibility [7]. This work will only model a single Bernoulli variable with DS Theory which allows this introduction to be very brief. First, for each object define the set of possibilities as tracked,  $\mathcal{T}$ , lost,  $\mathcal{L}$ , or either tracked or lost,  $\mathcal{T} \cup \mathcal{L}$ . For each set, including  $\mathcal{T} \cup \mathcal{L}$ , we assign a probabilistic mass,  $m(\mathcal{T})$  and so on. Note that the masses as a whole must sum to one. The belief of a particular set is defined as the sum of masses of all subsets,

$$b(\mathcal{A}) = \sum_{\mathcal{B} \subseteq \mathcal{A}} m(\mathcal{B}) \quad (16)$$

while the plausibility is instead 1 minus the sum of masses of all sets that don't contain it

$$pl(\mathcal{A}) = \sum_{\mathcal{A} \not\subseteq \mathcal{B}} m(\mathcal{B}). \quad (17)$$

This is much easier to see in tabular format; imagine we estimate the probabilistic mass that an object is being tracked to be 0.1 and the mass that an object has been lost to be 0.3. The resulting beliefs and plausibilities are shown in Table I.

A few things to note about the above framework. First, mathematically it is entirely defined by  $m(\mathcal{T})$  and  $m(\mathcal{L})$ ; everything else is a dependent variable. Second, in terms of results what we care about is only the belief we are tracking,  $b(\mathcal{T})$ , and the belief we have lost the object,  $b(\mathcal{L})$ . The entire framework going forward is designed to estimate these two probabilistic masses as largely independent values. Finally,  $b(\mathcal{T} \cup \mathcal{L})$  represents the idea that we do not have sufficient information. The estimation system is either correctly tracking the object or has lost the object, but our knowledge is more limited and requires this third option.

## B. Bernoulli updates in a Dempster-Shafer Framework

In order to develop the proposed filter, the state existence must be modeled as a demster schaffer variable. Consider the spacial belief representation

$$b(\mathcal{X}(t)) = \begin{cases} b(\mathcal{L}; t), & \text{if } \mathcal{X} = \emptyset \\ b(\mathcal{T}; t) * p(\mathbf{x}(t)), & \text{if } \mathcal{X} = \mathbf{x} \end{cases} \quad (18)$$

where  $p(\mathbf{x}(t))$  is still the standard spacial PDF for the object if the system is tracking it. Note that the beliefs do not need to sum to one, and the remaining belief is the epistemic uncertainty.

The actual estimation of  $p(\mathcal{X}(t))$  is still assumed to be separable from the beliefs. Note that when  $b(\mathcal{T}) + b(\mathcal{L}) = 1$ , the DST formulation collapses to the standard probability of existence formulation, where  $b(\mathcal{L})$  is equivalent to  $1 - p_E$ . The propagation step is still governed by birth of new tracks and survival of existing tracks, but the belief now flows between  $b(\mathcal{T})$  and  $b(\mathcal{L})$ ,

$$b_{k|k-1}(\mathcal{T}) = p_b * b_{k-1|k-1}(\mathcal{L}) + p_s b_{k-1|k-1}(\mathcal{T}) \quad (19)$$

$$b_{k|k-1}(\mathcal{L}) = (1 - p_b) * b_{k-1|k-1}(\mathcal{L}) + (1 - p_s) b_{k-1|k-1}(\mathcal{T}) \quad (20)$$

$$(21)$$

where  $p_b$  and  $p_s$  still represent birth and survival.

The update step requires considering the likelihoods for each situation. The structure of the update equation is still valid for both beliefs, by the same argument as for propagation:  $b(\mathcal{T})$  is equivalent to  $p_E$  while  $b(\mathcal{L})$  is equivalent to  $1 - p_E$ . That gives

$$b_{k|k}(\mathcal{T}) = \frac{1 - \delta_k(\mathcal{T})}{1 - \delta_k(\mathcal{T}) b_{k|k-1}(\mathcal{T})} b_{k|k-1}(\mathcal{T}) \quad (22)$$

$$b_{k|k}(\mathcal{L}) = \frac{1 - \delta_k(\mathcal{L})}{1 - \delta_k(\mathcal{L}) b_{k|k-1}(\mathcal{L})} b_{k|k-1}(\mathcal{L}) \quad (23)$$

$$(24)$$

where there are two versions of the likelihood ratio,  $\delta_k$ . The likelihood ratio for  $\mathcal{T}$  doesn't change, remaining the evidence in support of tracking the object over the evidence against tracking the object. The likelihood ratio for  $\mathcal{L}$  is the opposite: the evidence against  $\mathcal{T}$  is explicitly evidence for  $\mathcal{L}$ . This gives

$$\delta_k^j(\mathcal{T}) = \left( 1 - \sum_i \frac{q(d_i^j)}{q(d_0^j)} \right) \quad (25)$$

$$\delta_k^j(\mathcal{L}) = \left( 1 - \sum_i \frac{q(d_0^j)}{q(d_i^j)} \right) \quad (26)$$

Note that when  $b(\mathcal{T}) + b(\mathcal{L}) = 1$ , Equations (20) and (23) fall away and the update structure reverts to the original Bernoulli filter structure. The update equation as it stands only serves to inject certainty into the system and eventually forces the system to revert to a probabilistic Bernoulli filter. This is a good result as it means that with evidence the epistemic uncertainty is driven down.

## IV. ANALYSIS AND RESULTS

### A. Results on the SpaceSentry System

The SpaceSentry system has been using the Bernoulli version of this filter for approximately a year. This simulation results below are for the DST version of the filter which is still being tested and developed but is not live on the system.

The scenario shown is a single night of updates for a closely-spaced cluster of space objects (ANIK Cluster). These objects are tracked and estimated every night and the shown example is one nominal night in the middle of a long series of nights. All three objects are in the field-of-view of a SpaceSentry wide angle staring sensor. In the given example, object 1 is seen by our staring array sensor over the entire night, object 2 is seen for a period in the middle of the night, and object 3 is only seen at the beginning of the night. The first thing to note is that belief of tracking in Figure 3 matches the times when objects are receiving updates as seen in Figure 1. This illustrates the first reason we implemented this algorithm: belief of tracking and belief of loss are explicit mathematical measures of if we are actively seeing an object. This is primarily valuable for large scale catalog maintenance where belief of tracking over long periods of time provides a quick glance measure of catalog stability. The second important benefit is illustrated by the close approach between, highlighted in Figure 2. The two objects bottom out at a separation of only 4 pixels but when this separation occurs only one object is receiving updates. In other words, the estimates of the objects converge in the measurement space (image) but only one set of observations is visible. The normal JPDA implementation not only splits the association during the close approach, but the unobserved object consistently splits association at every time step even long after the close approach. In the new version of JPDA, the unobserved object

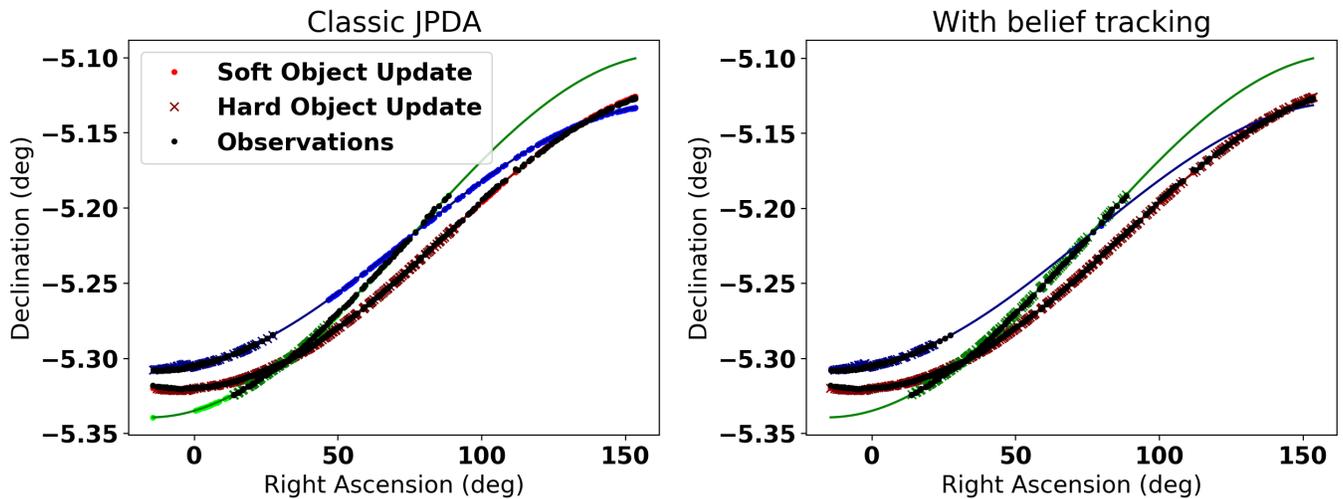


Fig. 1. Tracks of all three objects projected into right ascension and declination. Hard updates refer to association weight  $\zeta$  0.999, while soft updates refer to association weight  $\zeta$  0.001. Note that classic JPDA produces large numbers of soft updates which are eliminated by including belief of tracking (due to the adjusted  $p_D$ ).

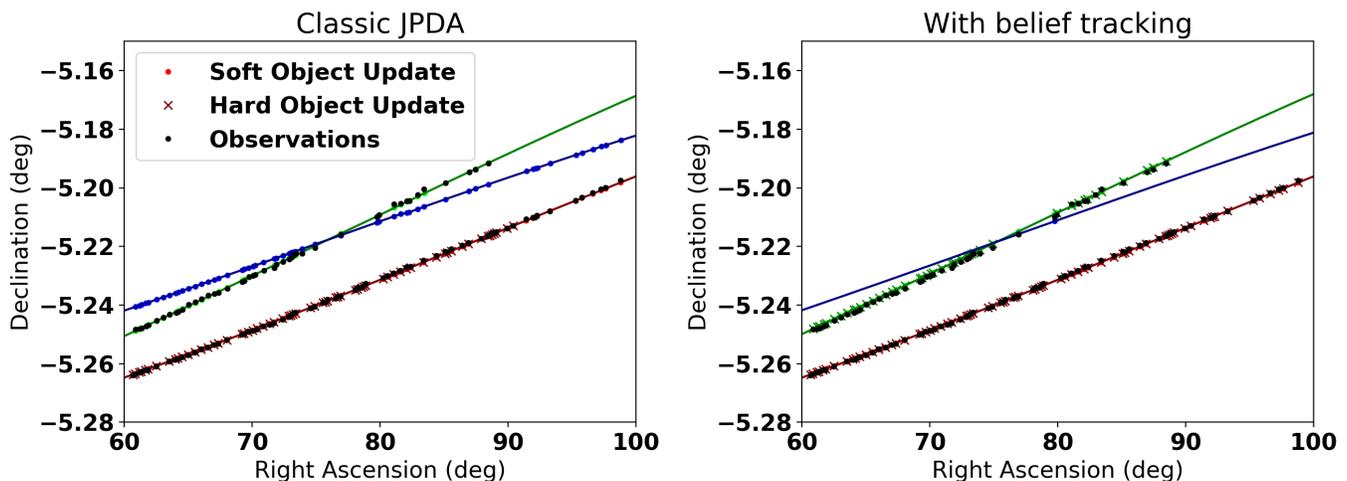


Fig. 2. More detailed view of a close pass between objects. Hard updates refer to association weight  $\zeta$  0.999, while soft updates refer to association weight  $\zeta$  0.001. Note that new JPDA still produces soft updates, but only when the objects are very close together.

has a belief of tracking of zero so its probability of detection is also zero and JPDA fully (correctly) associated with only the actively tracked objects. Finally, note in Figure 2 that even with the dynamics probability of detection, the new JPDA still produces split associations at the close approach, but only during the close approach. This means that JPDA can still allow for uncertainty in association but only when the association is truly uncertain. This is better than arbitrarily forcing associations which can lead to incorrect associations; instead the dynamics and data must drive decisions between closely spaced objects. Belief of tracking and belief of loss mirror each other in this scenario because the objects are being actively tracked and information is readily available.

## V. CONCLUSIONS AND FUTURE WORK

This paper provides the framework to integrate the most important results from Bernoulli filtering into a standard JPDA driven bank of Kalman filters. The intention is to distill the mathematics and technical requirements of these extremely capable filters to make them more widely usable on system with limited computational and algorithmic resources. These results were also born of trying to better solve the closely space object problem by better modeling the underlying statistical assumptions of optical tracking. The final results represent a more robust and advanced tracking algorithm that does not require wholesale restructuring of resources.

The results must be put into the context of other available methods for solving the underlying problems. The issues with closely spaced objects can be solved with other existing methods such as updating on tracks or explicit rules disabling updates

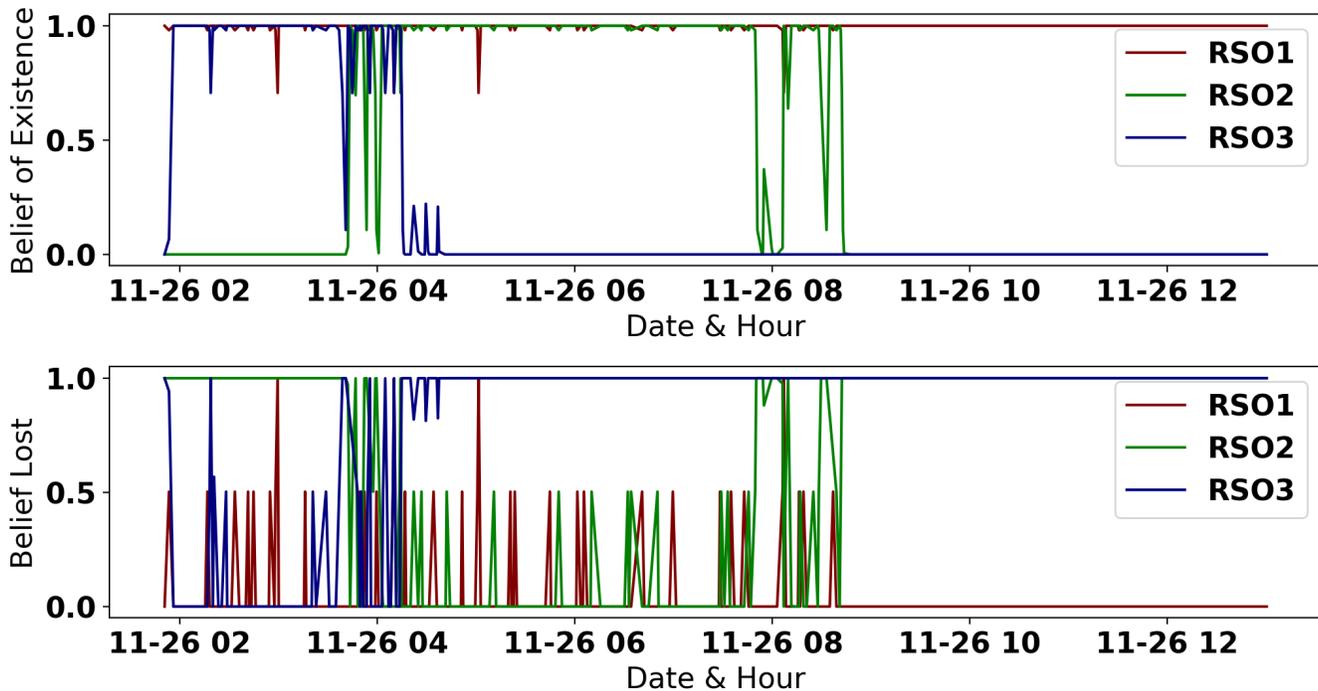


Fig. 3. Belief of tracking and belief of loss over the same night of updates as Figure 1. Note how the the objects naturally move between zero and one as updates begin and end.

when objects are close. Custody of objects can be determined by simply analyzing raw numbers of updates. The advantage of this approach is a single upgrade to how the system is modeled which naturally provides better results on ambiguous situations. It applies to our entire global SSA system and the thousands of objects it tries to track. We have seen it provide anecdotal results on specific situations we have found difficult to automatically process and have seen an overall improvement in the performance of our system.

There is plenty of future work, in particular for the Dempster-Shafer filter. First, birth and survival models should be upgraded to better represent long term evolutions of the tracking problem. In particular they should be derived as functions of time and should explicitly model increasing epistemic uncertainty. The authors have experimented with such techniques but do not have a sufficiently refined algorithm to present. Second, the DST algorithms do not sufficiently capture ambiguity in measurements. Situations where one observation fits with two different space objects is inherently ambiguous and improvements should exist to capture that within the DST framework.

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