Improving orbital uncertainty realism through covariance determination in GEO

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ABSTRACT

The reliability of the uncertainty characterization, also known as uncertainty realism, is of the uttermost importance for Space Situational Awareness (SSA) services. Among the different sources of uncertainty related to the orbits of Resident Space Objects (RSOs), the uncertainty of dynamic models is one of the most relevant ones, although it is not always included in orbit determination processes. A classical approach to account for these sources of uncertainty is the consider parameters theory, which consists in including parameters in the underlying dynamical models whose variance aims to represent the uncertainty of the system. However, realistic variances of these consider parameters are not known. The work at hand proposes a method to infer the variance of the consider parameters, based on the distribution of the Mahalanobis distance of the orbital differences between predicted and estimated orbits, which theoretically shall follow a chi-square distribution under Gaussian assumption. This paper presents such methodology together with results in a simulated scenario focusing on geosynchronous (GEO) regimes. The effectiveness and traceability of the uncertainty sources is assessed via covariance realism metrics.

1. INTRODUCTION

Orbital uncertainty plays a key role for the provision of Space Traffic Management (STM) services, including catalogue maintenance, risk assessment or maneuver detection, among others. Therefore its adequate characterization, uncertainty realism, is of paramount importance and focuses on correctly representing the Probability Density Function (PDF) of the orbital state. Under Gaussian assumption, uncertainty realism can be reduced to covariance realism, requiring only the two first moments of the PDF for the proper characterization of the system uncertainty, gathered in the widely known covariance matrix.

Many Orbit Determination (OD) processes in Space Surveillance and Tracking (SST) are based on weighted least-squares algorithms, which rely on available and sufficient measurements to produce an estimate (orbital state and its associated covariance matrix). During this process, the dynamics governing the system are usually assumed deterministic, having the measurements accuracy as the only uncertainty source. Thus the obtained covariance matrix is able to account only for the measurements noise, being known as the noise-only covariance [1]. However, one of the main sources of uncertainty during OD and subsequent propagation arises from the errors in the underlying dynamical models, which is typically disregarded. The impact of this uncertainty is crucial not only for the state estimation but also for the time evolution of its associated uncertainty due to its inherent correlation with the state variables. This leads to an overly-optimistic noise-only covariance matrix time evolution and, eventually, a loss of covariance realism.

Therefore, it is customary for SSA and particularly for SST purposes to characterize and determine such uncertainty and its effects, which is commonly known as Uncertainty Quantification (UQ). Two fundamental problems can be distinguished for UQ: the propagation of uncertainty and the inverse problem (model and parameter uncertainty) [2]. The former concentrates on how to propagate forward an initially given PDF of a state, accurately and efficiently. This is not the focus of the present work. The inverse problem, on the contrary, consists in assessing the differences between the observed behavior of a system and the underlying models and parameters used to represent it. Regarding the uncertainty in the modelling, a common approach is to revisit the deterministic assumption of the equations of motion, recurring to stochastic noise models such as Brownian motion, Ornstein-Uhlenbeck or Gauss-Markov processes [2]. The other target of the inverse problem is the parameter uncertainty, whose objective is to represent the uncertainty in specific terms of the dynamic or measurement equations. This is the core of the work at hand, in this case focusing on uncertain parameters that can be estimated during the OD process as dynamic parameters, not only for the quantification of its uncertainty but also to represent the relationship between the uncertain parameters and the state variance.

There are other techniques conceived to improve covariance realism without focusing on the sources of uncertainty and their modelling. For instance, state representation in mean orbital elements allows the covariance matrix to represent more realistically the uncertainty of Monte Carlo simulations [3]. Other typical representations of the state and covariance in non-linear reference frames that are able to slow down the realism degradation upon propagation are being widely studied, such as the QtW frame in [4]. However, despite the advantage of representing the state of the object in orbital elements for covariance realism, we concern ourselves to Cartesian state representation. Considering the operational goal of this work, it was found unnecessary to use alternate state representations, though the proposed methodology is agnostic to this choice. Other approaches suggest the use of empirical covariance matrices to include all residuals of the estimation process in the covariance computation, regardless of whether the uncertainty has been modelled or not [5] [6] [7]. This proposal claims to account more accurately for noise variations rather than process noise or consider parameter analysis, at the expense of the physical interpretation of the uncertainty.

In an operational environment, simple techniques are required to improve covariance realism since, as previously discussed, the nominal covariance determination methods tend to provide optimistic results. The most applied solutions are:

1) Kalman filter and process noise matrix to introduce the model uncertainty into the system. These UQ methods are gaining confidence over stochastic acceleration methods in the current state of the art since they can account for both dynamic model and parameter uncertainty. However, a physically-based derivation of a process noise is rather challenging [8]. Other authors suggest calibration methods to estimate such process noise [9], but typical solutions lack the physical meaning of the applied correction and are not suitable for batch processing, the common framework of SST.
2) Scaling techniques which inflate the covariance by certain factors. Some authors propose the computation of such scaling based on increasing the initial position uncertainty to match the velocity error [10] whereas other options explore the usage of the Mahalanobis distance of the orbital differences to find the scale factor [11]. However, a common drawback of artificially increasing the covariance is that the physical meaning of the correction is lost, not being able to understand the contributions of each source of uncertainty. These sort of methods are used nowadays in operation centers such as Space Operations Center (CSpOC) [2].

An additional option to be discussed is the consider parameters theory, which is a classical approach for parameter uncertainty analysis in the equations of motion for OD processes [1]. It consists in extending the state space by including parameters in the dynamic models, such as atmospheric force, solar radiation pressure force or measurement models. These parameters are assumed to follow a Gaussian distribution with a null mean and a certain variance. A null mean allows to maintain an unbiased estimation, whereas the uncertainty of the parameter is accounted for in the state covariance matrix. This theory is compatible with both batch and filter applications such as in the Schmidt-Kalman filter [12], [13] and provides the clear advantage of assessing the effect of specific uncertain parameters in the process, maintaining the physical meaning of the covariance correction as opposed to scaling factors techniques. However, one of the main drawbacks of the consider parameter theory is that realistic variances of such parameters are not normally known. This can lead either to overly-sized or underestimated state covariance matrices, failing to model the uncertainty of the system and thus, not achieving covariance realism.

This work presents a novel methodology to determine the variance of the consider parameters included in the analysis. It is based on the orbital differences between estimated and predicted orbits projected into the covariance space (i.e. Mahalanobis distance), which under Gaussian assumptions shall follow a chi-square distribution to achieve covariance realism. Therefore, a minimization process can be designed to obtain the consider parameter variances that provide the best match between the observed Mahalanobis distance distribution and the expected chi-square one. A similar analysis based on the consider parameter theory to improve the covariance realism is performed in [14], a precursor work for this study. There, it is proposed to correct the noise-only covariance with a least squares fitting to a so-called observed covariance, this latter being obtained from Monte Carlo orbital differences aggregation. This approach has a main drawback, which is that to compute such observed covariance, orbital differences at distinct orbital positions are mixed from orbit estimates based on different observation scenarios. This issue is mitigated by the normalization obtained with the Mahalanobis distance concept, which is the cornerstone of this study. In the work at hand, the covariance determination methodology is applied to GEO regimes, continuing the efforts of previous studies that applied the proposed methodology to LEO orbits for drag and range bias uncertainty [15]. The realism of the determined covariance matrices remains as the cornerstone of this study, and thus specific covariance realism metrics such as the covariance containment are analyzed. For a geostationary orbit, two of the main sources of uncertainty that come into play are related to the Solar Radiation Pressure (SRP) and the time bias of the sensors, which have been

The paper is structured as follows: in Section 2, the consider parameter theory is revisited together with the definition of our consider parameters and the proposed covariance determination method. Section 2 also explains the methodology followed for the validation of the discussed methodology. Section 3 shows the results of the simulation campaigns carried out for validation, including a description of the simulation environment, the results and metrics analysis. Finally Section 4 contains the conclusions and future steps of this study.

2. METHODOLOGY

In this section, we describe the consider parameters used for the GEO regime along with the interaction of consider parameter theory with batch least-squares algorithms. Then, the concept of the Mahalanobis distance in the context of the chi-square theory is revisited. Once these theoretical enablers have been properly defined, we propose a procedure to effectively calculate the consider parameter variances that define the uncertainty levels of the dynamic model and compute the consider covariance.

2.1 DYNAMICAL MODELS

As we mentioned previously, two consider parameters of interest are included in our analysis: in the SRP force and the time bias of the sensor. Following the classical definition, the SRP acceleration equation with the first consider parameter reads as follows [1]:

$$\boldsymbol{a}_{SRP} = -P_{SRP} C_R \frac{A}{m} \frac{\boldsymbol{r}}{r^3} AU^2 (1 + c_{SRP}) \tag{1}$$

where P_{SRP} is the solar radiation pressure, C_R is the solar radiation coefficient, A is the cross-sectional area, m is the mass of the object, r is the distance to the sun, AU expresses the magnitude of an astronomical unit and c_{SRP} is the consider parameter associated to the SRP, which is defined to follow a Gaussian distribution of the type:

$$c_{SRP} \sim N(0, \sigma_{SRP}^2) \tag{2}$$

where σ_{SRP}^2 is the variance associated to the consider parameter. The aim of this parameter is to model the uncertainty that can be associated to most of the components of Eq. (2). The mass may vary for maneuverable satellites, the cross-section area is assumed constant along the estimation and propagation, which is not true in general; the solar radiation pressure and the solar radiation coefficient suffer from the Sun's behavior (solar cycles) and satellite surface variability (light reflection and absorption), which are not modelled for most SST applications.

The second consider parameter aims to represent the variability in the time bias present in the sensors when time-stamping the measurements. It is characterized as:

$$t^* = t + c_{TB} \tag{3}$$

where t denotes time and c_{TB} is the consider parameter associated to the time bias, also following a Gaussian distribution:

$$c_{TB} \sim N(0, \sigma_{TB}^2) \tag{4}$$

where σ_{TB}^2 is the variance associated to the consider parameter. The literature suggests that the SRP coefficient exhibits a variability not higher than 5%. In [1], an annual variation of 3.3% is identified. The studies performed on Ajisai and LAGEOS [16] identified variations in the range between 1% and 3%. The characterization of the time bias behavior is more complex. It is strongly dependent on the type of instrument. From [17], [18], we extract typical perturbation levels, ranging from 1 to more than 100 ms.

2.2 CONSIDER PARAMETER THEORY

As discussed previously, the objective of the consider parameters is to include the uncertainty associated to certain parameters into the orbit estimation. The introduction of these elements within the dynamic model leads to several modifications of the classical least-squares OD processes. We will not review the full derivation, which can be found in [1], but rather concern ourselves with the direct results. Without the presence of consider parameters, the noise-only covariance is given by:

$$\boldsymbol{P}^n = (\boldsymbol{H}_X^T \boldsymbol{W} \boldsymbol{H}_X)^{-1} \tag{5}$$

where H_X is the Jacobian of the measurement vector with respect to the reference state and W is the weighting matrix associated to the measurement noise. The introduction of consider parameters leads to a redefinition of the covariance matrix, the consider covariance, which now takes the form:

$$\boldsymbol{P}^c = \boldsymbol{P}^n + (\boldsymbol{P}^n\boldsymbol{H}_x^T\boldsymbol{W})(\boldsymbol{H}_c\boldsymbol{C}\boldsymbol{H}_c^T)(\boldsymbol{P}^n\boldsymbol{H}_x^T\boldsymbol{W})^T \tag{6}$$

where P^n is the original noise-only covariance matrix, H_c is the Jacobian of the observations with respect to the consider parameters and C is a diagonal matrix, defined by:

$$\boldsymbol{C} = \begin{pmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^2 \end{pmatrix} \tag{7}$$

where $\sigma_{1\ldots n}^2$ are the consider parameter variances considered in the model. Eq. (6) can be rewritten in a simplified fashion:

$$\boldsymbol{P}^c = \boldsymbol{P}^n + \boldsymbol{K}^T\boldsymbol{C}\boldsymbol{K} \tag{8}$$

with:

$$\boldsymbol{K} = \boldsymbol{P}^n(\boldsymbol{H}_x^T\boldsymbol{W}\boldsymbol{H}_c); \boldsymbol{K} \in \mathbb{R}^{n_c} \times \mathbb{R}^{n_x} \tag{9}$$

being n_c and n_x the number of consider parameters and the state vector dimension, respectively. K is a matrix which can be calculated directly from the available information in the measurements and dynamic model. Summarizing, the consider covariance is obtained as the noise-only covariance plus a covariance correction, which depends on the consider parameter variances. The goal of the work at hand is to determine the values of C so that the consider covariance realism is improved.

2.3 MAHALANOBIS DISTANCE AND CONSIDER COVARIANCE DETERMINATION

The squared Mahalanobis distance is defined as follows [19]:

$$d_M^2 = \left(\boldsymbol{x} - \boldsymbol{x}_{ref}\right)^T\left(\boldsymbol{P} + \boldsymbol{P}_{ref}\right)^{-1}(\boldsymbol{x} - \boldsymbol{x}_{ref}) \tag{10}$$

where $\boldsymbol{x}$ represents the orbital state vector, $\boldsymbol{P}$ its associated covariance and the subscript ref indicates the reference state against which the Mahalanobis distances are calculated, in principle, the true state. If the reference covariance matrix is small enough as compared with the actual state covariance, it can be dismissed.

We can reformulate the Mahalanobis distance definition considering the reformulation of the covariance introduced in Eq. (8):

$$d_M^2 = \left(\boldsymbol{x} - \boldsymbol{x}_{ref}\right)^T\left(\boldsymbol{P} + \boldsymbol{K}^T\boldsymbol{C}\boldsymbol{K} + \boldsymbol{P}_{ref}\right)^{-1}(\boldsymbol{x} - \boldsymbol{x}_{ref}) \tag{11}$$

From the former equation, given an estimated orbits coming from an OD process and an orbit reference, a population of Mahalanobis distances can be computed for a certain variance of the consider parameters, contained inside matrix $\boldsymbol{C}$. It is worth mentioning that the projection of the orbital differences in the covariance space allows to combine samples derived from ODs at different epochs and conditions, which is of paramount importance for an operational scenario.

A realistic covariance is not only required at the OD estimation epoch. On the contrary, the objective of this work is to provide realistic covariance matrices to cover the nominal propagation intervals applied in SST for the orbital regime under analysis, which for GEO regimes are around 15 days. To include such information in our covariance determination method, it is necessary to propagate the consider covariance to any desired epoch. Under Gaussian assumptions, this propagation can be performed by means of the State Transition Matrix (STM).

$$d_M^2 = \left(\boldsymbol{x} - \boldsymbol{x}_{ref}\right)^T\left(\boldsymbol{\Phi}\left(\boldsymbol{P} + \boldsymbol{K}^T\boldsymbol{C}\boldsymbol{K} + \boldsymbol{P}_{ref}\right)\boldsymbol{\Phi}^T\right)^{-1}(\boldsymbol{x} - \boldsymbol{x}_{ref}) \tag{12}$$

where $\boldsymbol{\Phi}$ is the STM. This propagation technique is customary in most operational applications, and was found suitable for the proposed methodology after concluding that linearity was maintained for the proposed periods of analysis using Michael's normality test, whose details may be found in [20] and [21]. More complex and accurate uncertainty propagation methods are out of the scope of this work. It is worth remarking that such propagation must be applied to the full covariance matrix since the effect of the dynamical parameters covariance will affect significantly the position and velocity covariance evolution.

It remains to describe the method to compute the variance of the applied consider parameters that is applied in this work following the previous line of research of [15]. Under linear and Gaussian assumptions, this is, when the differences between the state and the reference are normally distributed and the covariance is representative of such distribution (i.e. realistic), a population of squared Mahalanobis distances should follow a chi-square distribution, whose detailed characteristics may be found in [22]. Eq. (12) allows us to compute such Mahalanobis distance at any desired epoch during the propagation arc by comparing an estimated and posteriorly propagated (predicted) orbit against a reference. Therefore, if a sufficient number of predicted orbits is available, it is possible to look for the variance of the consider parameters represented in matrix $\boldsymbol{C}$ so that the squared Mahalanobis distance population resembles the theoretical chi-square distribution.

In the work presented here, this optimization problem is based on minimizing the differences between two Cumulative Distribution Functions (CDFs), the observed and the theoretical chi-square one, of as many degrees of freedom as elements included in the state vector for the Mahalanobis distance computation. This optimization process can be adapted to any desired number of consider parameters, being able to combine the two included in our analysis.

2.4 VALIDATION SCHEME

The methodology can be proven successful if we can ensure that the outputs of the optimization process are indeed representative of the uncertainty present in the dynamic model. To confirm this, we perform a Monte Carlo iterative scheme in which we simulate a population of orbits. Under simulation conditions, we are able to impose the uncertainty levels in the time bias and SRP. Thus, we condition the outputs of the optimization process to known, controlled values for validation purposes. We can establish a general flow for the data generation:

1. From a GEO reference state associated to an existing Resident Space Object (RSO) and a certain epoch (t_0; estimation epoch), we perform an orbit propagation process to obtain the *reference orbit*. The dynamic model of this propagation is deterministic, without including any perturbation.
2. Iteratively, we introduce SRP perturbations within the dynamic model. The population of perturbations is constructed according to the aforementioned definition of the consider parameter, this is, following a Gaussian distribution of zero mean and a certain variance. We then perform the following steps per iteration:
    a. We propagate the reference state backwards 28 days considering the perturbation in the dynamics. We refer to this propagated orbit as *simulated orbit*.
    b. We generate tracks of the simulated orbits using a model of a ground-based telescope. We introduce the time bias perturbation, defined similarly to its SRP counterpart. Along the generation of these simulated observations, measurement noise is also included.
    c. We perform an OD process based on the simulated tracks. We have inputs from a period spanning between t_0-28 and t_0. The output, including the covariance matrix, is referenced to the estimation epoch t_0. The resulting orbit is known as *estimated orbit*.
    d. The estimated orbit is propagated up to the desired propagation arc, in this case 21 days to have complete coverage of typical prediction periods in GEO.
3. This Monte Carlo process is repeated to have 400 samples, which are expected to suffice for the statistical computations of the proposed methodology, as was found in the simulations performed in previous studies [15]. Between each iteration, the simulation time frame is shifted 1 day forward, so that the observations generation and OD processes vary similarly to an operational environment.

For further clarification of this scheme, the different kinds of orbits in a relative propagation timeline is shown in Fig. 1 exemplifying 1 Monte Carlo trial. The Mahalanobis distance population can be calculated once all the necessary data for all the orbits is available at the desired analysis epoch. An additional option apart from the aforementioned reference orbit, has been included to use in the orbital differences computation. A precise orbit whose computations can be assumed perfect, for instance derived from precise GNSS data, is not commonly available in most SST operational environments. Thus, we propose for the orbital differences computation an orbit estimation whose determination arc includes the propagation epoch under analysis. In other words, if we are analyzing the first 10 propagation days after the estimation epoch of and OD, we can use as reference another OD performed with measurements in an arc contained inside these 10 days of propagation (see Fig. 1). This option has been defined as the *operational reference orbit*. Of course, the estimation corresponding to this operational reference has certain noise-only covariance due to the measurements uncertainty. Thus, it is necessary to include its noise-only covariance inside the Mahalanobis distance computation of Eq. (12).

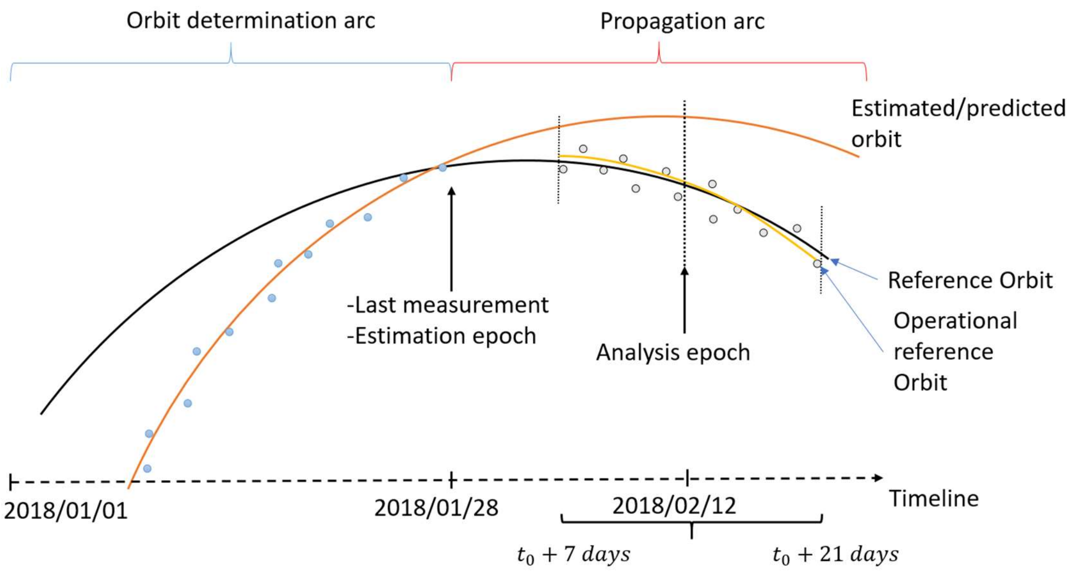


Fig. 1: Monte Carlo scheme

3. RESULTS AND DISCUSSION

3.1 SIMULATION ENVIRONMENT

For the sake of completeness, this section characterizes the conditions under which the validation test sequence has been performed. Table 1 describes the parameters of the simulated telescope used for the generation of orbit tracks (set of measurements). Table 2 contains the information corresponding to the reference RSO.

Table 1: Telescope parameters

Latitude	55.444º
Longitude	-4.638º
Height	800 m
Measurement noise	0.5 mdeg

Table 2: Reference RSO parameters

Semi-major axis	42190.42 km
Eccentricity	$6.826 \cdot 10^{-5}$
Inclination	13.906º
RAAN	339.651º
Argument of pericenter	312.092º
True anomaly	229.845º
Mass	500 kg
Solar radiation area	40 m^2
SRP coefficient	1.3
Reference epoch	2018-01-28 00:00:00.000 UTC

The tracking strategy, using the telescope described in the table, consists on series of three measuring intervals per night. Every night, the telescope takes measurements at intervals of 15 minutes located at the beginning, middle and end of the night, with a time-step of 20 seconds. The exact position of the intervals within the night may vary slightly due to shadow conditions. This approach is taken to resemble telescope availability in an operational scenario. More insight regarding the effect of the tracking condition on the OD process with consider parameters are shown in Section 4.

3.2 SIMULATION RESULTS

The main objective of the validation phase is to verify that the uncertainty introduced in the Monte Carlo simulation is correctly retrieved as consider parameter variances in the optimization process. To verify this, we propose a sequence of validation tests, summarized in Table 3. Only the most relevant test cases are included.

Table 3: Simulation tests summary

Case	Comparison orbit	Analysis epoch	SRP perturbation (σ)		Time bias perturbation (σ)	
			Input	Output	Input	Results
I-A	Ref.	t_0	-	-	-	-
I-B	Ref.	t_0	30%	27.93%	1000 ms	1071.17 ms
II-A	Ref.	t_0+15	30%	28.02%	1000 ms	988.23 ms
II-B	Op. ref.	t_0+15	30%	26.73%	1000 ms	1001.23 ms
III-A	Ref.	t_0+7-21 (interval)	30%	28.26%	1000 ms	1083.46 ms
III-B	Op. ref.	t_0+7-21 (interval)	30%	29.4%	1000 ms	982.75 ms
IV-A	Op. ref.	t_0+7-21 (interval)	5%	4.56%	100 ms	100.22 ms

The tests become gradually more complex. The first three series (I to III) represent the core validation pipeline, in which we subject the dynamic model to perturbations significantly higher than the ones suggested by the literature to verify the robustness of the methodology in highly-perturbed environments where data Gaussianity is stressed to the limits of the methodology assumptions. In the first series, we restrain our analysis to the estimation epoch t_0, starting from null perturbation. We then proceed to include perturbations as uncertainty in the system. In the second series, we extend the analysis to a future epoch, located at 15 days of propagation into the future. The third series is representative of the final complexity of the methodology, combining Mahalanobis distance calculations from epochs located all over an interval of 14 days, with a time-step of one day. Including different epochs of the propagation arc in the same optimization process has the objective of computing a single consider parameter variance that is able to enhance the realism of the covariance in the complete interval of interest, which is the desired operational case. The fourth series is conceptually equal to the third, but show cases with reduced perturbation levels looking for values of uncertainty closer to the ones expected in the literature.

In general, the analyses were performed discarding velocity components (in the optimization phase), due to the accumulation of small errors in the along-track velocity. Apart from the velocity components being several orders of magnitude smaller than the position ones, the along-track velocity expected precision was found to be the smallest covariance term among the velocity components. This leads to ill-conditioning of the covariance and an exponential increase in the Mahalanobis distance for high perturbations. In the end, since a series of different position ephemeris are considered, the dynamics of the system are fully regarded in the computations.

The overall results of the validation test sequence, paying special attention to the last ones due to their increased level of complexity, has shown to provide satisfactory results when computing the consider parameter variances. The deviations between the input perturbations and output consider parameter variances did not exceed 11% for any of the perturbations in any of the cases. Some relevant comments regarding the different tests cases are discussed next.

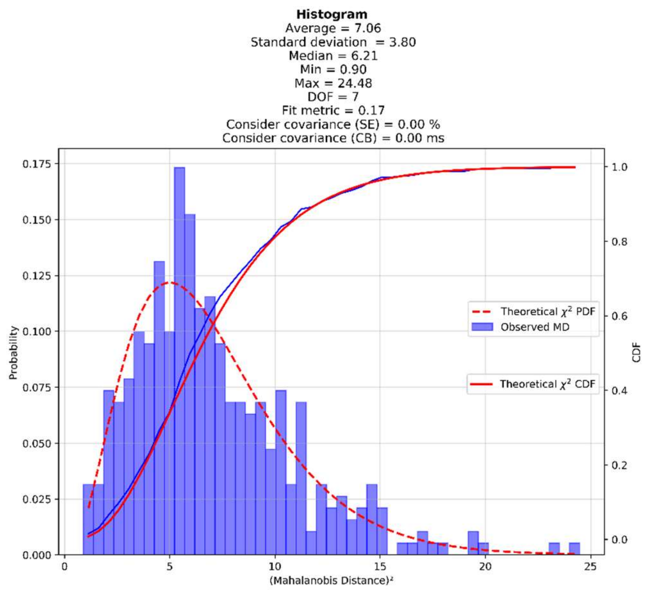


Fig. 2: Case I-A

The starting point of the validation (Case I-A) explores the ideal situation in which no perturbation is present in the dynamic model. Then, if the measurement noise is properly characterized, the noise-only covariance is expected to be representative of the uncertainty of the system, and the squared Mahalanobis distance distribution would directly resemble a chi-square one. Table 3 shows the results of this case, where this fact can be appreciated. The retrieved distribution is very close to the chi-square distribution without applying consider parameter correction. This is a clear indication that in the sole presence of measurement noise, the noise-only covariance is able to represent the uncertainty of the system. This is not the case for any of the other test cases, in which non-zero perturbation levels were enforced. Case I-B shows the results of including both perturbations simultaneously, performing two-variable optimization at the estimation epoch. As seen in Table 3 the methodology is capable of computing consider parameter variances within a margin lower than a 10%.

However, we discussed that from an operational perspective it is desirable to extend this analysis into the prediction region. In the second series (II-A, II-B), we establish the analysis epoch at t_0+15, proving that the quality of the results obtained in the previous series can be maintained when propagating up to this time horizon. Additionally, we introduce the usage of operational reference orbits in the case II-B. Thus, we include a new layer of realism by not relying on an absolutely true representation of the reference orbit, but rather on the result of an OD process with an associated covariance matrix. As was observed in [15], obtaining appropriate results using the operational reference orbit is of high relevance in real case scenarios, allowing the methodology to function using only the observations, not requiring other external information sources.

The third series of cases extend the analysis epoch from a single one to a full interval spanning for 14 days between t_0+7 and t_0+21. Table 3 shows the most relevant subcases, combining both consider parameters and comparing against reference and operational orbits. Mahalanobis distances are calculated at specific points within this period, at time steps of one day. The main leap forward in this series is that a singular optimized consider parameter variance is able to recover the chi-square behavior of the squared Mahalanobis distance population for a wide propagation interval, obtaining consider parameter variances very similar to the introduced perturbations as can be seen in Fig. 3. Again, accurate results are obtained independently on the orbit used to obtain the orbital differences. Besides the fact that adding more comparison epochs increases the analyzed population samples, specific disturbances associated to a certain point in the orbit can be smoothed by including Mahalanobis distances from a wider range of initial epochs.

Fig. 4 corresponds to the same case scenario as Fig. 3, but in this case maintaining the noise-only covariance for the Mahalanobis distance. We find that the values of the distribution are far from resembling the chi-square behavior. It has been included to show the inability of the noise-only covariance to represent the uncertainty of the system when model uncertainty is present. The high values of the distribution indicates that, without the consider parameter variances correction, the covariance is overly-optimistic and the observed orbital differences are much larger than the standard deviation present in the covariance.

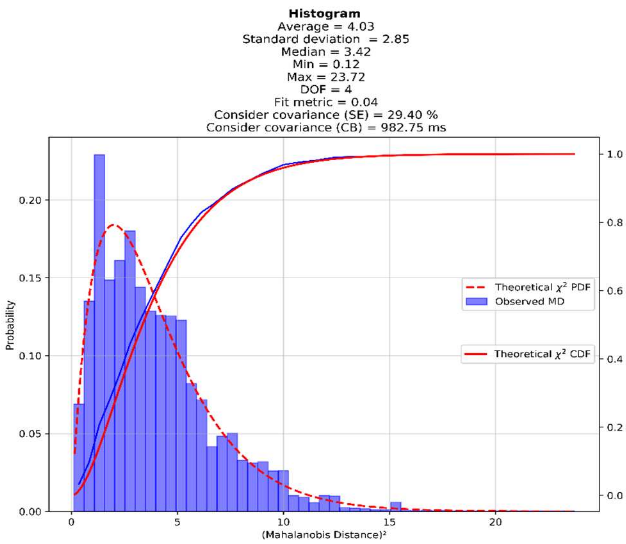


Fig. 3: Case III-B

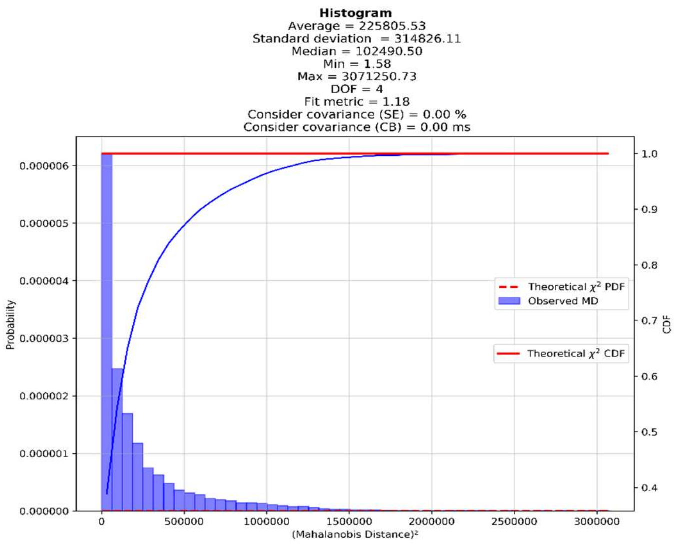


Fig. 4: Case III-B without consider parameter correction

The fourth series of tests does not introduce new elements into the methodology. It serves to prove that the model is sensitive to lower perturbation levels which are aligned with those suggested by the literature. Fig. 5 contains the fitting results of the last test case, IV-A, which is representative of the full complexity of the methodology proposed in this work. Again, the differences between the input perturbation and the results of the consider parameter variances is lower than a 10%, showing that the methodology is able to recover the theoretical chi-square behavior.

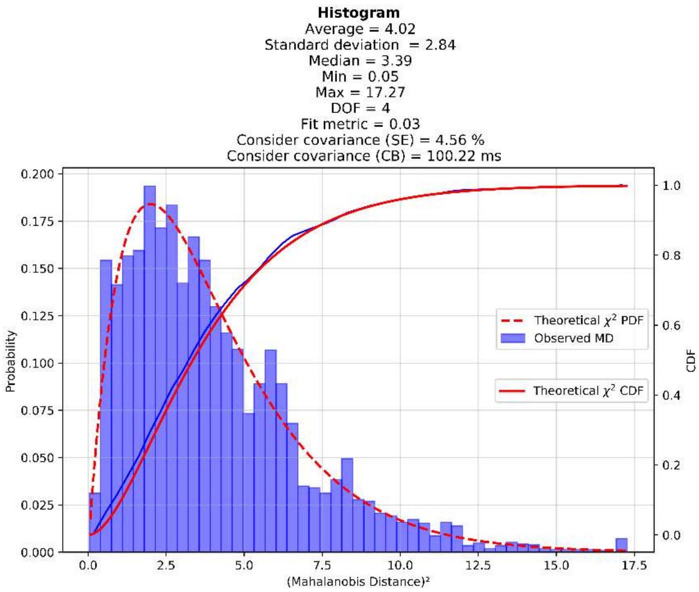


Fig. 5: Case IV-A

3.3 CONTAINMENT ANALYSIS

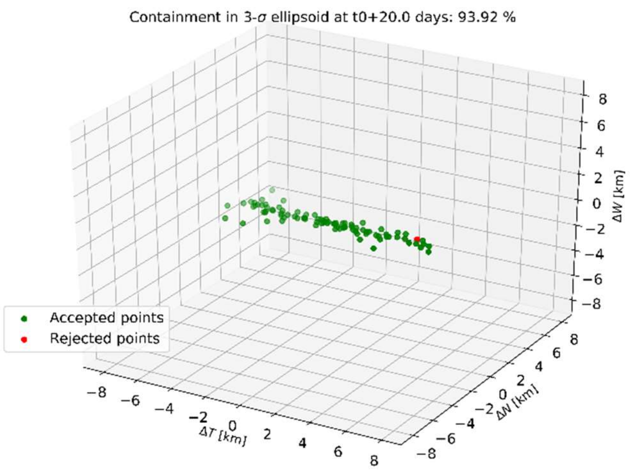
To obtain a physical sense and a proper visual representation of the effectiveness of proposed covariance determination method, covariance containment tests such as the one proposed in [23] are provided in this section. To evaluate if the covariance is representative of the orbital differences (i.e. realistic), the Mahalanobis distance can be used as a metric to see the amount of points that lay inside a k-σ ellipsoid and compare it against the theoretical expected fraction for a Gaussian distribution of the same number of Degrees Of Freedom (DOF). In all tests presented previously, only position and the SRP coefficient of the state vector were included in the Mahalanobis distance computation. Gaussianity tests of the orbital differences were performed (Michael's normality test), to ensure the fulfillment of Gaussian assumptions in the proposed methodology, though those results have been omitted for the sake of the length of this paper. Table 4 presents the evolution of such containment matrix at different epochs, comparing both noise-only and consider covariance matrices against the 4 DOF theoretical containment results.

Table 4: Containment tests

Epoch	Without consider parameter correction [%]				With consider parameter correction [%]			
	1σ	2σ	3σ	4σ	1σ	2σ	3σ	4σ
t0+4	0	0	0.25	0.51	8.33	57.07	92.68	99.75
t0+6	0	0	0.25	0.51	8.84	56.82	92.42	99.49
t0+8	0	0	0.25	0.25	8.84	56.31	93.18	99.75
t0+10	0	0	0.25	0.25	8.33	56.82	92.93	99.75
t0+12	0	0	0.25	0.25	7.58	57.32	93.69	99.75
t0+14	0	0	0.25	0.25	8.61	57.47	93.92	100
t0+16	0	0	0.25	0.25	8.61	58.73	93.92	100
t0+18	0	0	0.25	0.25	8.61	58.99	93.92	100
t0+20	0	0	0	0.25	9.11	58.99	93.92	99.75
Average	0	0	0.22	0.31	8.54	57.61	92.96	99.80
Theoretical	9.0	59.40	93.90	99.70	9.0	59.40	93.90	99.70

The results found in the table were obtained using the optimum consider parameter variances found in the results of case III-B. It is easily appreciated the covariance containment improvement that is achieved whith the consider covariance computed with the presented methodology. The results at all epochs are closely adjusted to the theoretically expected ones, particularly at 3σ and 4σ. There are several implication of these results. Firstly, the average containment along the complete propagation interval resembles closely its theoretical expectation. Recalling the operational goal of the methodology, this shows that a unique consider parameter variance is able to improve substantially the covariance realism in the interval of interest for GEO scenarios. Secondly, the degree of similarity of the containment in all sigma ellipsoids indicates that the consider covariance is not over-sized, showing that the proposed methodology is able to tackle the model uncertainty properly and maintain the traceability of its sources.

On the contrary, it is directly observed the lack of covariance realism in the absence of consider parameter correction when model errors are present. Noise-only covariance matrices are too optimistic and fail to represent the PDF of the state. Another way to visualize this phenomenon can be found in Fig. 6, where we compare the amount of orbital differences included in the 3σ ellipsoid (in green) in both cases: with consider parameter (left) and without consider parameter (right). The orbital differences are shown in the relative frame TNW (Along-track, normal and cross-track directions)



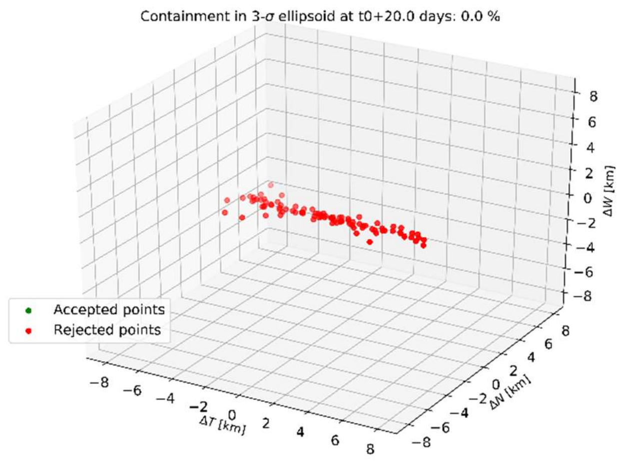


Fig. 6: Containment at t_0+20 using the consider covariance (left) and the noise-only covariance (right)

Fig. 6 shows that the orbital differences are larger in the along-track direction. The consider covariance is elongated in such direction, representing the actual distribution. The ellipsoidal shape is not easily discerned in Fig. 6 due to the significantly larger standard deviation of the along-track component. It is clearly observed that the consider parameter methodology correction with our determined variance elongates the ellipsoid in the Along-track direction, which is the one showing a higher dispersion.

Fig. 7, Fig. 8 and Fig. 9 below are included to show the effect on the position covariance that enable the consider parameter correction, focusing on a single orbit and separating the contribution of each of the consider parameters included in the present analysis. The figures compare the standard deviation of the Noise-only covariance and the consider covariance in the three main directions of the local TNW reference frame. Again, the optimum consider parameter variances obtained in case III-B (29.4% and 982.75 ms for SRP and clock-bias consider parameter standard deviations, respectively) have been used for consistency. The shown epochs range from estimation (t_0) to 21 days of propagation for a single Monte Carlo iteration case.

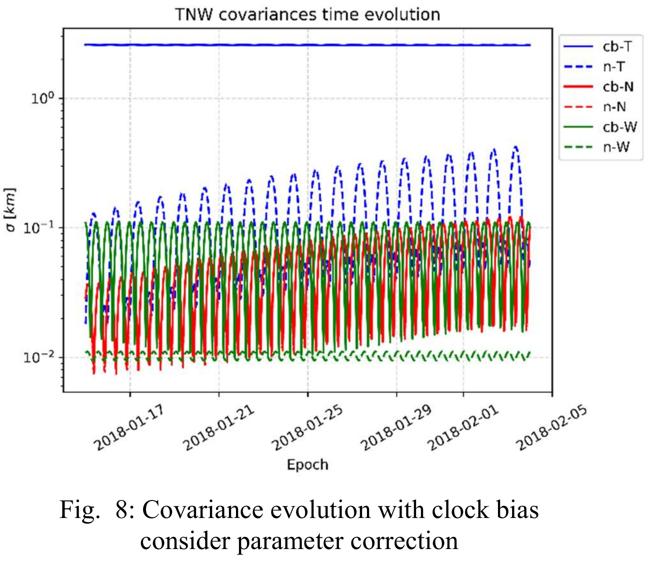


Fig. 8: Covariance evolution with clock bias consider parameter correction

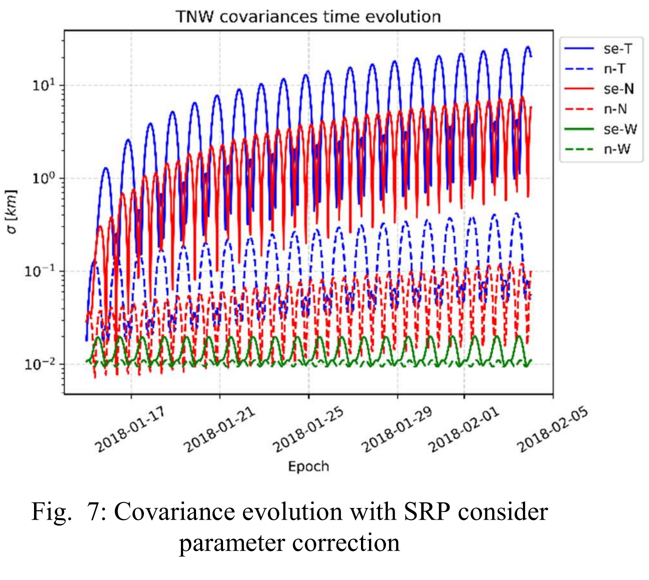


Fig. 7: Covariance evolution with SRP consider parameter correction

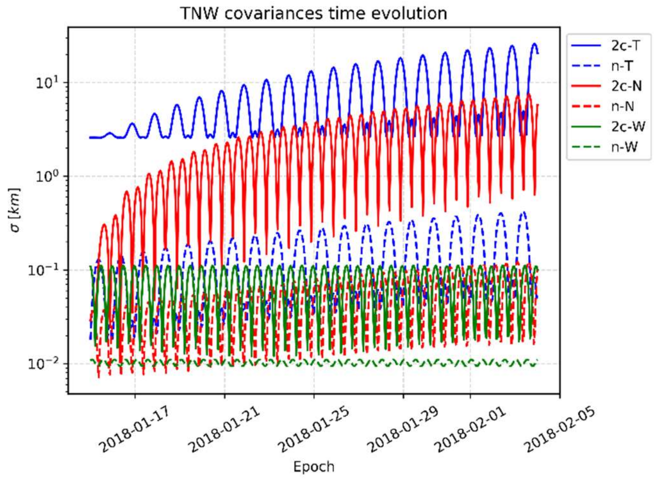


Fig. 9: Covariance evolution with both consider parameters correction

Where prefix "n" refers to the Noise-only covariance, "cb" to clock-bias individual correction, "se" to SRP individual correction and "2c" to double consider parameter correction. SRP model correction causes a fast covariance growth in along-track and normal directions as compared to the noise-only, mildly affecting the out-of-plane component. However, at t_0 its effects are barely visible in the state covariance. This is caused by the consider parameter correction, which at t_0 for the SRP is mostly applied to the SRP coefficient variance, and is transmitted to the state via the linear propagation of the STM (which includes the complete augmented state with the SRP coefficient), due to the correlation between the SRP coefficient term and the state vector. This causes the observed growth in the along-track and normal directions.

Clock bias correction, on the contrary, provokes a remarkable growth of the along-track covariance at t_0 without any further time evolution. This is expected due to the nature of the clock bias perturbation, which is only affecting the measurements model, being observed mostly at the output of the estimation. It is concentrated in the along-track component, since any error in the observation time is directly translated into an uncertainty in the satellite position in the direction of the motion. For this reason the along-track covariance appears constant in the CB-only case, since the orbital oscillations from the noise-only covariance are several orders of magnitude smaller. When both corrections are combined (Fig. 9), the clock bias provides not only a higher initial value of the along-track covariance, but also a lower uncertainty limit for this direction along the orbit oscillations. Again, the growth of both along-track and normal directions with time is dominated by the SRP correction.

4. CONCLUSIONS AND FUTURE WORK

The results shown in this work indicate that the presented covariance determination methodology is capable of accurately capturing the model error present within the dynamics of an RSO on a simulated SST scenario. It has been successfully applied to the GEO regime, considering SRP and time bias perturbations. We have tested the robustness of the model by enforcing large perturbation levels, while also ensuring that sensitivity to lower values is maintained. The deviation between the perturbation input and consider parameter output tends to escalate to the input magnitude and has remain bounded to an 11% throughout the development of this work. Additionally, successful results have been obtained when estimated orbits are used as reference to compute the Mahalanobis distance, indicating the operational suitability of the methodology for operational scenarios. Relevant metrics for covariance realism assessment such as the covariance containment tests have shown that the proposed methodology is able to determine a realistic covariance, applicable to the complete propagation region of interest in GEO regimes and without oversizing.

The presented methodology allows to maintain the traceability of the different sources of uncertainty, being capable of determining the uncertainty accordingly in different consider parameters. Regarding the impact of the corrections induced by the retrieved consider parameter variances on the covariance matrices, it was observed that the time bias uncertainty accumulates in the along-track component at estimation epoch, but does not introduce a significant degree of growth rate in the long term. On the contrary the SRP uncertainty correction, though small at t_0, increases the growth rate in the along-track and normal direction with time.

The next natural step is to use real data from a GEO RSO to further study the capabilities of the methodology. However, the efforts carried out towards realistic simulated scenarios, and the real data results of previous lines of research [15] suggest that the applicability of the proposed methodology for GEO regimes in a real scenario is possible. Further research is under development regarding the optimization process. Robust statistics widely used for the comparison of CDFs such as Cramer-von-Mises or Kolmogorov-Smirnov are being tested as cost functions to retrieve the optimum consider parameter variance, allowing also to determine bounds for variances that are statistically consistent to a chi-square behavior. Additional analysis should be performed to increase the amount of consider parameters for different regimes, with the objective of quantifying as much as possible all sources of uncertainty

It is worth reminding that the consider parameter methodology assumes a constant error model for its derivation, even though having a certain variance. More complex noise models such as purely Gaussian, Ornstein-Uhlenbeck, Gauss-Markov or other time and space correlation processes are also a current line of research, focusing on how to adapt or generalize the consider parameter methodology to more complex representation of the orbit uncertainty sources. Another relevant line of work is connected to the characterization of the methodology accuracy, exploring the minimum amount of orbital data required to achieve a successful model error estimation. The less data is required, a smaller time-region can be faithfully analyzed, which can allow to capture seasonal variability in the model uncertainties, as is expected due to solar weather, for instance. Finally, parametrization and benchmarking to select appropriate consider parameter corrections as a function of the orbital regimes and space conditions to improve catalogue covariance realism is a relevant future goal to improve SST products quality.

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