

Radar-Derived Spin States of Defunct GEO Satellites and Rocket Bodies

Conor J. Benson^a, Charles J. Naudet^b, Daniel J. Scheeres^a, William H. Ryan^c, Eileen V. Ryan^c, Joseph S. Jao^b, Lawrence G. Snedeker^d, and Jeffrey K. Lagrange^d

^a *University of Colorado Boulder, 3775 Discovery Drive, Boulder, CO 80309*

^b *Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, CA 91109*

^c *Magdalena Ridge Observatory, New Mexico Tech, 101 East Road, Socorro, NM 87801*

^d *Goldstone Deep Space Communications Complex, Fort Irwin, CA 92310*

ABSTRACT

Estimating and predicting the spin states of defunct satellites and rocket bodies is important for space situational awareness, active debris removal, and satellite servicing. Observations show that the spin states of defunct geosynchronous (GEO) satellites are diverse and can change significantly over time. Spin state evolution for many defunct GEO satellites is primarily driven by solar radiation torques via the Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) effect. To better understand the YORP-driven evolution of these objects we obtain spin state estimates from a combination of Deep Space Network (DSN) Doppler radar echoes and optical light curves. The resolved nature of Doppler radar echoes allows for clear identification of satellite spin periods and can greatly constrain possible spin pole directions. Observations of the defunct GOES 8-12 GEO weather satellites demonstrate ongoing spin period evolution and pole motion consistent with YORP theory. Observations of two spent upper stage rocket bodies yield spin periods, center of mass offsets, and constraints on spin pole directions.

1. INTRODUCTION

The GEO debris population keeps growing with continued launches and no natural deorbit mechanisms. Understanding the long-term dynamical evolution of these debris is necessary to protect active assets and preserve GEO for future use. Observations show that the spin states of defunct GEO satellites are diverse and evolve significantly over time [1-8]. A better understanding of long-term defunct satellite spin state evolution would aid GEO space situational awareness, material shedding predictions, modeling of attitude-dependent solar radiation pressure, active debris removal (ADR), and satellite servicing. ADR and servicing promise to figure prominently in future efforts to manage the GEO debris population and lower satellite costs. As a result, a number of organizations are developing ADR/servicing missions. These missions will require accurate spin state estimates to grapple and de-spin large target satellites. With evolving spin states and many potential targets, early spin state predictions will be extremely valuable for mission planning and execution. Spin state estimates are also valuable for satellite anomaly resolution and recovery.

Recent observations and dynamical modeling have shown that spin states of some defunct GEO satellites are driven by the Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) effect, spin state evolution due to solar radiation and thermal re-emission torques [5-7,9]. With much still unknown, accurate observations are needed to gain insight and validate general long-term dynamical models. A satellite's rotational angular momentum direction (pole) seems to greatly impact its YORP-driven evolution [7,9]. So both spin period and pole estimates are needed to advance our understanding. Even with complete knowledge of the mechanisms driving long-term evolution, accurate predictions require accurate initial estimates. So prediction hinges on obtaining unambiguous spin state estimates from observations.

Unfortunately, extracting GEO satellite spin periods and poles from non-resolved photometry generally require accurate satellite models, which are usually unavailable. Light curves are very sensitive to optical properties and detailed satellite geometry [5]. So even using high fidelity (e.g. ray-traced) photometric models for satellites with well-constrained geometry and optical properties, inversion can result in numerous local minima. Some defunct GEO satellites are also in non-principal axis rotation (i.e. tumbling), and recent work indicates that the YORP effect

can cause defunct satellites to transition from uniform rotation to tumbling where their dynamical evolution continues [5-7,9]. Tumbling light curve analysis is more challenging than for uniform rotation because there are two fundamental spin periods: one corresponding to precession of the satellite's long axis around the pole ($P_{\bar{\phi}}$) and the second to rotation of the long axis about itself (P_{ψ}) [8]. Dominant tumbling light curve frequencies consist of several or more low-order harmonics of the two fundamental frequencies $f_{\bar{\phi}} = 1/P_{\bar{\phi}}$ and $f_{\psi} = 1/P_{\psi}$ which are a priori unknown [5]. The common analysis approach requires testing numerous candidate tumbling period pairs over the sphere of possible attitude phasing and pole directions [5]. This often results in many similarly well-fitting period pairs and poles within model uncertainty.

The aim of this work is to better understand defunct satellite spin state evolution by leveraging both optical light curves and ground-based Doppler radar measurements. The spatial resolution afforded by Doppler radar allows for identification of satellite features. The resolved Doppler echoes also provide unambiguous long axis precession period estimates for both uniform rotators and tumblers, greatly constraining possible period pairs for the tumbling case. Leveraging the time-varying antenna to satellite position vector, we can also greatly narrow pole possible directions and in potentially obtain an unambiguous pole solution [8]. This is achieved with just a pair of closely spaced transmitting/receiving antennas. Most importantly, radar-based pole estimation does not require a detailed satellite model, just knowledge of the satellite's radial extent and/or prominent satellite features that can be identified over successive rotations (e.g. a solar panel outer edge).

For this work, we observed both uniformly rotating and tumbling defunct GEO satellites and rocket bodies with Deep Space Network (DSN) antennas at NASA's Goldstone Deep Space Communications Complex in California. Building on our earlier study [8], we first revisit the previously observed GOES 8-12 satellites to compare spin periods/pole directions and gain insight about their ongoing evolution. This will help us better understand how YORP drives defunct satellites spin states. We then present radar observations of two rocket bodies, the first near GEO and the second in a highly elliptic orbit.

2. APPROACH

With Doppler radar providing both period and pole information, we apply several analysis methods to extract spin state information from the observed echoes. For short observation arcs, the long axis precession period $P_{\bar{\phi}}$ is estimated simply by inspection. For sufficiently long arcs, the echoes can be phase-folded over a range of candidate $P_{\bar{\phi}}$ values to determine which minimizes dispersion. To obtain pole information using a closely spaced pair of antennas, both Doppler bandwidth and 2D/3D rotationally phased Doppler (RPD) techniques are used. These techniques assume the satellite is in torque-free uniform rotation over the observing span. Detailed discussions of Doppler bandwidth and RPD pole estimation are provided in Ref. [8]. Here, we briefly summarize these methods.

For Doppler bandwidth, the satellite's known radial extent is used to calculate the maximum possible bandwidth (i.e. the maximum Doppler shift relative to the satellite center of mass) for the observed $P_{\bar{\phi}}$. At each observation epoch, the observed bandwidth then provides two cones around the antenna-satellite line of sight on which the pole lies. Assuming that the pole direction is inertially fixed, viable pole solutions lie at the intersection of cones from all epochs. Doppler bandwidth is relatively robust to mild tumbling and slow spin state evolution. On the other hand, it requires prior knowledge of the satellite's radial extent and yields at least two and often four possible pole solutions.

To further constrain pole solutions, 3D RPD can be used. For a satellite in a torque-free uniform rotation, the rotational velocity of a part of the satellite (e.g. solar panel outer edge) is periodic in $P_{\bar{\phi}}$. Viewing the satellite from three or more well-spaced, non-planar inertial directions at different multiples of $P_{\bar{\phi}}$, one can directly solve for the instantaneous velocity. Computing the velocity at additional rotation phases yields the rotation plane and an unambiguous pole direction. 3D RPD requires simultaneously solving for the sidereal (inertial) $P_{\bar{\phi}}$ which will differ slightly from the observed (synodic) value due to rotation of the line of sight. Importantly, no prior knowledge of the radial extent is needed for 3D RPD. When the observing directions are nearly planar, 2D RPD can be used with knowledge of the radial extent to obtain two possible pole solutions.

3. EXPERIMENT

Two 34 meter DSN antennas were used in a bi-static configuration with one antenna transmitting and the second receiving. The transmitted signal consisted of continuous wave (cw) carrier at X-band. Reflection of the signal off of the rotating target and back to the receiving antenna yielded time-varying Doppler spectra. Each target was observed periodically over the course of several hours to sample different viewing geometries. The targets were observed in this manner on a number of days in early May 2021. For several targets, near-simultaneous optical photometry was collected using the Magdalena Ridge Observatory (MRO) 2.4 m telescope in New Mexico.

4. RESULTS

4.1 GOES 8

Doppler echoes for GOES 8 collected on May 1, 2021 are provided in Figure 1. The outermost sinusoid corresponds to GOES 8's solar sail boom and the smaller amplitude sinusoid 180° out of phase to the solar panel. Phase-folding indicated a precession period $P_{\dot{\phi}} \approx 318$ s. With GOES 8's known radial extent (~18.87 m), the maximum Doppler bandwidth is denoted by the dashed lines at approximately +/-18 Hz. Over the course of the observing arc, we see a notable increase in the observed Doppler bandwidth due to a progressively more side-on view of the pole. At ~3500 s, the antenna transmitter was briefly turned off to prevent unwanted illumination of other spacecraft. Looking closely at the Figure 1 echoes, we can make out additional features that vary in amplitude over successive precession periods, most notably at roughly 200 s, 1800 s, and 3400 s. These features likely correspond to the satellite's magnetometer boom which is attached to the bus. The varying amplitude indicates that the satellite is tumbling.

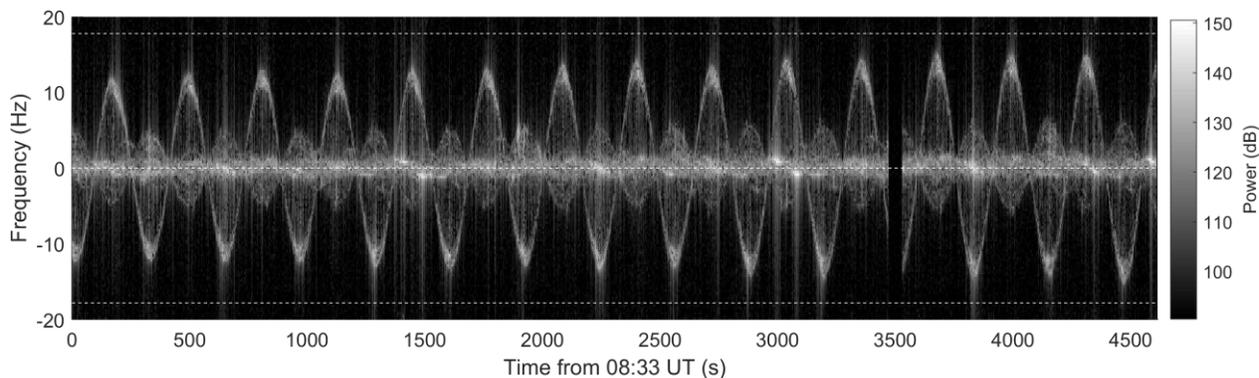


Figure 1: May 1, 2021 GOES 8 Doppler echoes ($P_{\dot{\phi}} \approx 318$ s, 5.3 min)

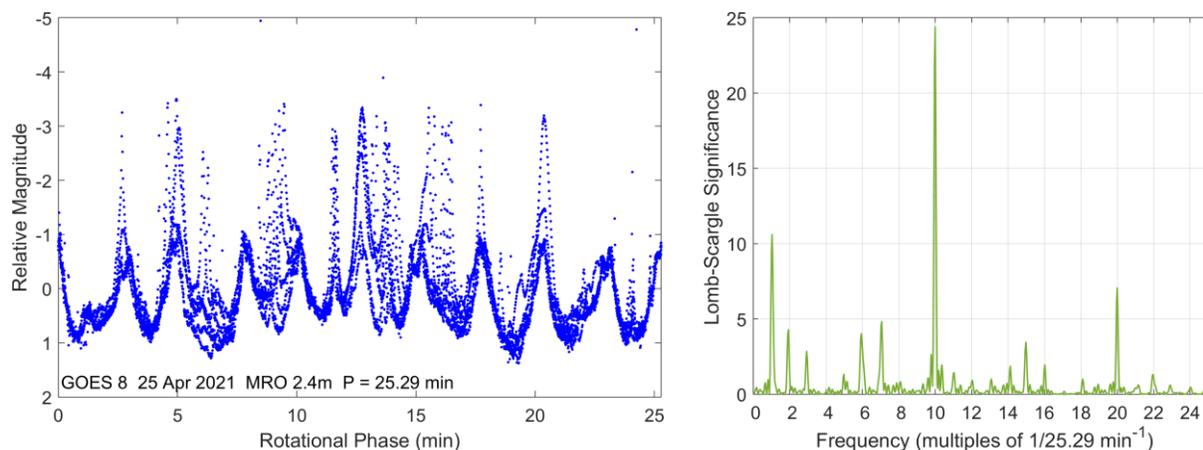


Figure 2: (left) phase-folded April 25, 2021 GOES 8 light curve ($P = 25.29$ min), (right) Lomb-Scargle periodogram

Figure 2 shows GOES 8 light curve data collected on April 25, 2021 with the Magdalena Ridge Observatory (MRO) 2.4 m telescope. Phase-folding the light curve yielded a minimum dispersion period of $P = 25.29$ min. We see that the light curve folds relatively cleanly on this period, with the 170 min observing arc covering ~ 6.5 cycles. Nevertheless, the periodic structure is far too complicated for uniform rotation with 10 primary peaks per cycle. This indicates that the satellite is in a tumbling resonance where long axis rotation period P_ψ is a multiple of $P_{\bar{\phi}}$. The corresponding Lomb-Scargle periodogram is provided in the right plot of Figure 2 and shows that all dominant light curve frequencies are multiples of $f = 1/25.29 \text{ min}^{-1}$, with 1, 10, and $20f$ being most significant.

Figure 3 shows GOES 8 light curve data collected on May 1, 2021. While this is a shorter arc, phase-folding still shows clear repetition on $P = 26.53$ min. This is almost exactly five times the radar-derived May 1 $P_{\bar{\phi}}$ of 5.3 min, strongly suggesting GOES 8 was in a $P_\psi/P_{\bar{\phi}} \sim 5:1$ tumbling resonance during late April and early May 2021. This resonance has been observed previously for GOES 8 in Apr. 2018 and Feb. 2020 [5,8]. On Apr. 25, 2021, the phase-fold period of 25.29 min indicates $P_{\bar{\phi}} \approx 303 \text{ s}$ (5.06 min).

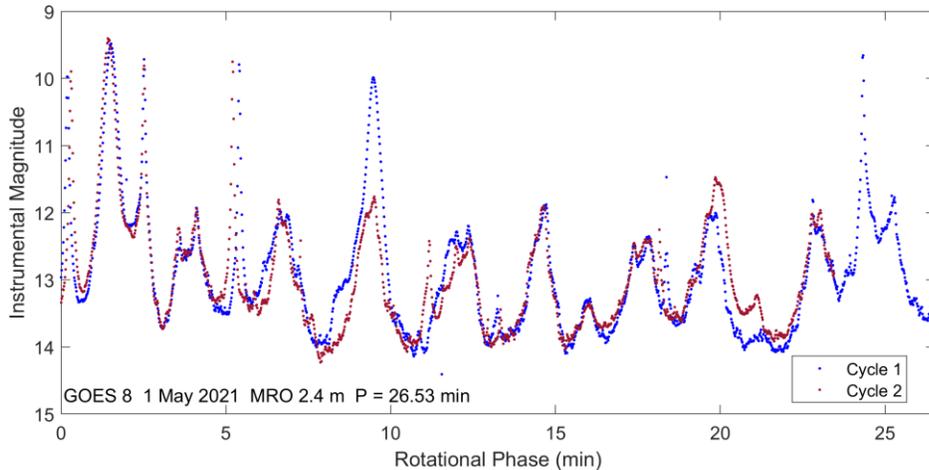


Figure 3: Phase-folded May 1, 2021 GOES 8 light curve ($P = 26.53$ min)

The candidate pole directions for GOES 8 on May 1 and 9, 2021 are provided in Figure 4. Here, the colored curves denote the Doppler bandwidth solutions. Again, viable Doppler bandwidth solutions lie at the intersection of curves from all epochs. Using 2D RPD, these were narrowed to two possible solutions on each day denoted by black x's. On both days, the candidate solutions are in very similar locations with a slight increase in longitude from May 1 to 9.

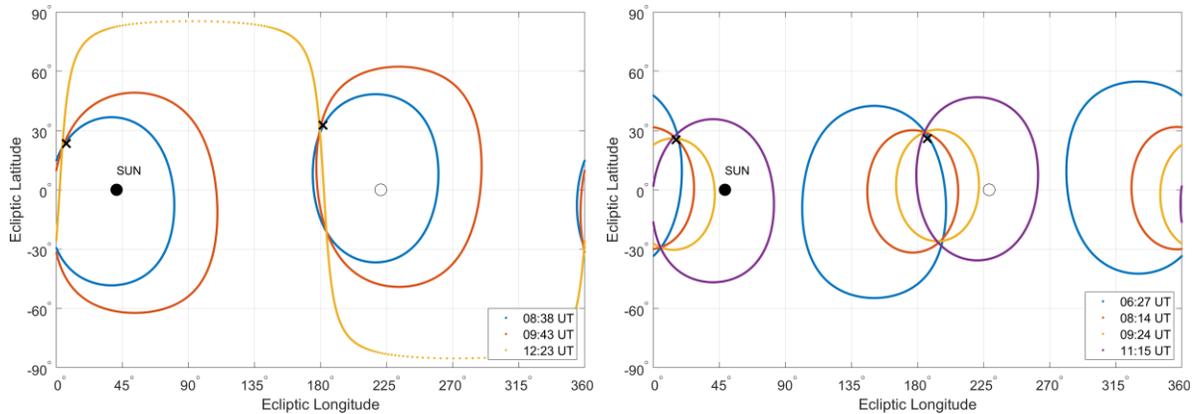


Figure 4: (left) May 1, 2021, (right) May 9, 2021 GOES 8 candidate poles in the J2000 ecliptic frame

4.2 GOES 9

Doppler echoes for GOES 9 from May 9, 2021 are provided in Figure 5. Analysis indicated $P_{\bar{\phi}} \approx 715$ s (11.9 min). The Doppler bandwidth being roughly half the maximum indicates a somewhat pole on view. Doppler echoes collected on May 2, 2021 indicated a slightly longer precession period of $P_{\bar{\phi}} \approx 724$ s (12.1 min). This suggests that the satellite spin rate increased during this span.

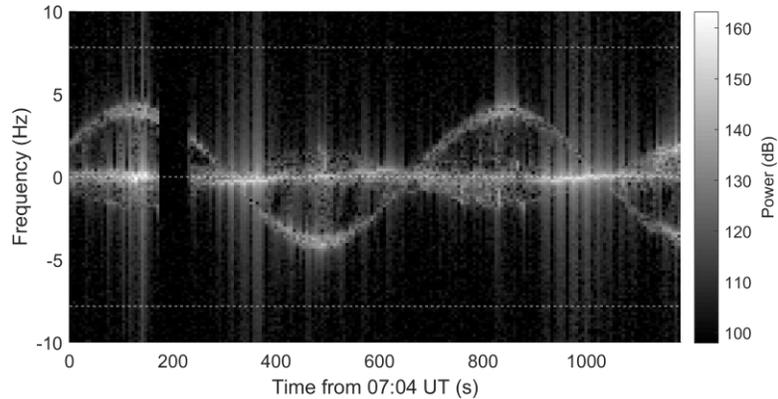


Figure 5: May 9, 2021 GOES 9 Doppler echoes ($P_{\bar{\phi}} \approx 715$ s, 11.9 min)

The left plot of Figure 6 shows GOES 9 MRO photometry from May 1, 2021. There is no clear periodicity indicating that the satellite is tumbling. With $P_{\bar{\phi}}$ well-constrained by radar, a fourth order ($m=4$) two-dimensional Fourier series [10] was fit to the light curve over a range of candidate P_{ψ} values. The resulting best-fit solution ($P_{\bar{\phi}} = 12.10$ min, $P_{\psi} = 49.25$ min) is provided in the left plot of Figure 6. The right plot shows the Lomb-Scargle periodogram of the light curve with the dominant frequencies labeled with their corresponding harmonics. Unlike GOES 8, GOES 9's May 1 tumbling state is non-resonant given the incommensurability of the best-fit period pair and many of the periodogram peaks being non-multiples. As for the May 1, 2021 GOES 8 light curve in Figure 2, $2f_{\bar{\phi}}$ is the most significant light curve frequency.

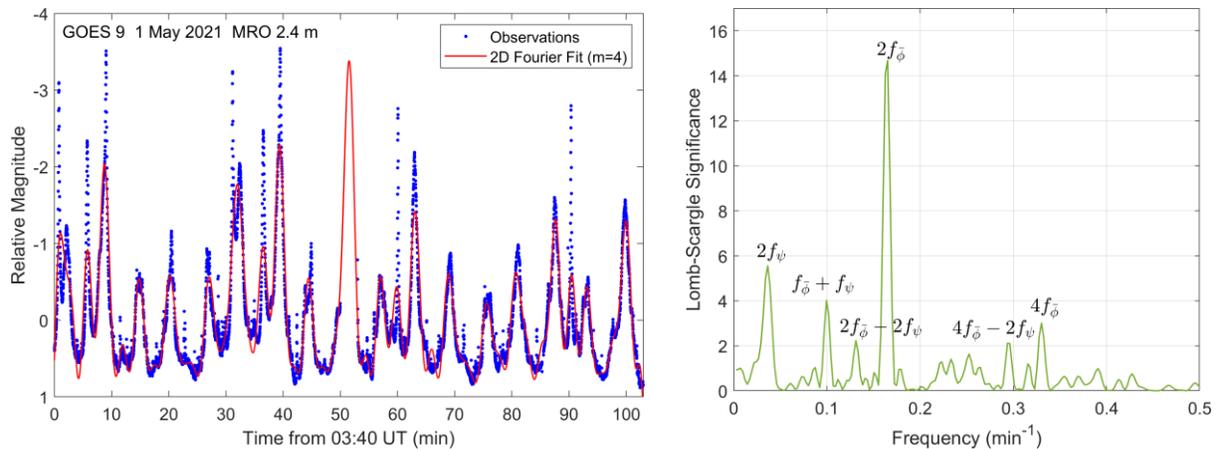


Figure 6: (left) May 1, 2021 GOES 9 light curve with best-fit 2-D Fourier series ($P_{\bar{\phi}} = 12.10$ min, $P_{\psi} = 49.25$ min), (right) Lomb-Scargle periodogram

Candidate pole solutions for GOES 9 on May 9, 2021 are provided in Figure 7. The Doppler bandwidth and 2D RPD solutions (denoted by x's) are quite consistent with each other. The ~5.5 hr observation arc was sufficiently

long to attempt 3D RPD analysis. The resulting tentative 3D RPD solution is given by the black diamond, suggesting the pole is within $\sim 45^\circ$ of the anti-sun direction.

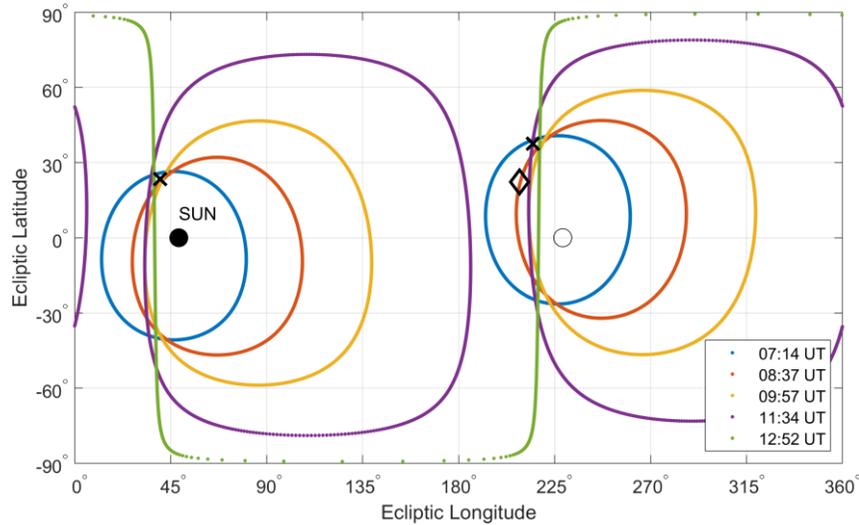


Figure 7: May 9, 2021 GOES 9 candidate poles in the J2000 ecliptic frame

4.3 GOES 11

Doppler echoes for GOES 11 obtained on May 1, 2021 are provided in Figure 8. They indicate a precession period of $P_{\bar{\phi}} \approx 799$ s (13.35 min).

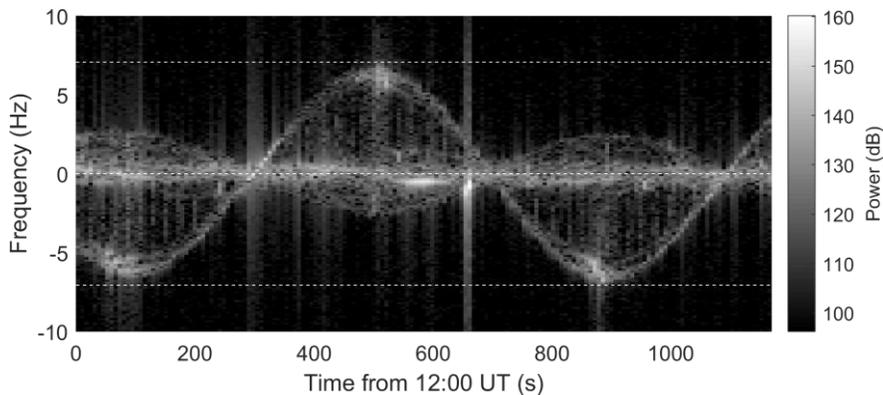


Figure 8: May 1, 2021 GOES 11 Doppler echoes ($P_{\bar{\phi}} \approx 799$ s, 13.35 min)

Figure 9 shows GOES 11 MRO 2.4 m photometry from May 1, 2021. There is no clear periodicity in the light curve which indicates the satellite is tumbling. This is further confirmed by the Lomb-Scargle periodogram with many non-multiple peaks. Fitting a two-dimensional Fourier series to the light curve to find P_{ψ} proved inconclusive given the relatively short observation arc relative to $P_{\bar{\phi}}$. So instead the 9 most significant Lomb-Scargle frequencies were refined by fitting a sparse Fourier series to the light curve allowing both the amplitudes and frequencies to vary. The converged solution is provided in the left plot of Figure 9.

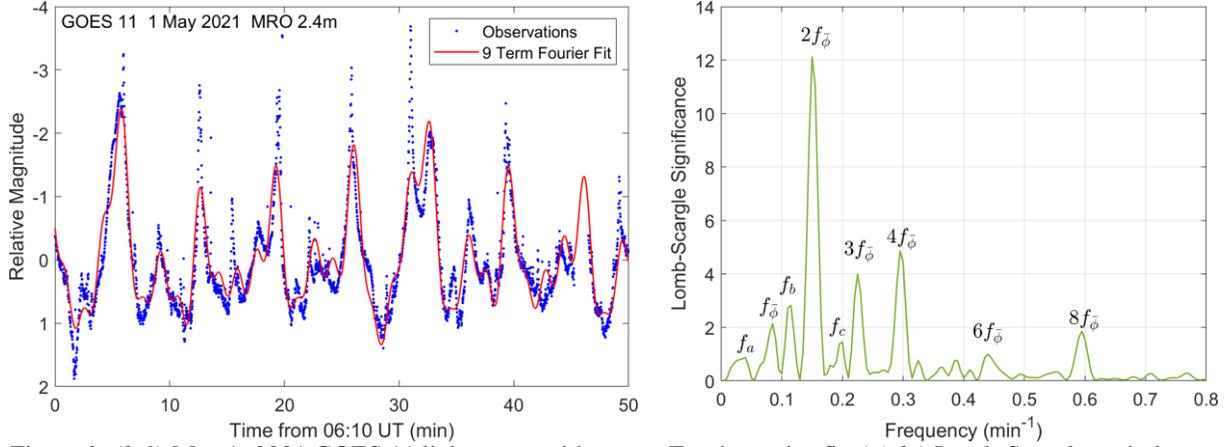


Figure 9: (left) May 1, 2021 GOES 11 light curve with sparse Fourier series fit, (right) Lomb-Scargle periodogram

In the right plot of Figure 9, the Lomb-Scargle periodogram is labeled with the known $f_{\bar{\phi}} = 0.0749 \text{ min}^{-1}$ harmonics and the three additional converged peaks $f_a = 0.030 \text{ min}^{-1}$, $f_b = 0.116 \text{ min}^{-1}$, and $f_c = 0.189 \text{ min}^{-1}$. Taking the largest amplitude frequency (f_b) and assuming different harmonic assignments, we can solve for the corresponding P_{ψ} values. The results for the most likely assignments (given simulated light curve surveys [5]) are provided in Table 1. The solutions $P_{\psi} = 24.48 \text{ min}$ and 29.37 min are dynamically less likely as their corresponding period ratios would require the satellite to be in a significantly more energetic long axis mode (LAM) tumbling state (see Ref. [5]). The period ratios for the solutions $P_{\psi} = 48.96 \text{ min}$ and 58.73 min would place the satellite near uniform rotation.

Table 1: Possible GOES 11 P_{ψ} Values on May 1, 2021

f_b Assignment	$f_{\psi} \text{ (min}^{-1}\text{)}$	$P_{\psi} \text{ (min)}$	$P_{\psi}/P_{\bar{\phi}}$
$f_{\bar{\phi}} + f_{\psi}$	0.0409	24.48	1.83
$2f_{\bar{\phi}} - f_{\psi}$	0.0340	29.37	2.20
$f_{\bar{\phi}} + 2f_{\psi}$	0.0204	48.96	3.67
$2f_{\bar{\phi}} - 2f_{\psi}$	0.0170	58.73	4.39

Candidate GOES 11 Doppler bandwidth pole solutions using observations spanning May 1-2, 2021 are provided in Figure 10.

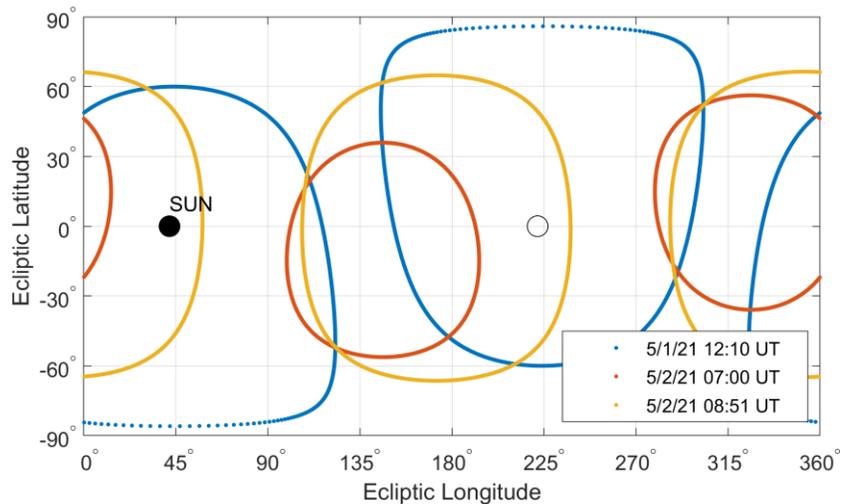


Figure 10: May 1-2, 2021 GOES 11 candidate poles in the J2000 ecliptic frame

4.4 GOES 12

GOES 12 Doppler echoes from May 8, 2021 are provided in Figure 11. The echoes indicate the satellite is in relatively rapid rotation with a minimum dispersion phase-folded $P_{\bar{\phi}} \approx 49.75$ s. As with the GOES 8 echoes in Figure 1, we can identify the magnetometer boom echoes at ± 16 Hz that lead the outermost solar sail echoes by $\sim 45^\circ$ phase. Unlike for GOES 8, these magnetometer echoes maintain the same phase and amplitude and over successive rotations, indicating the satellite is in uniform rotation.

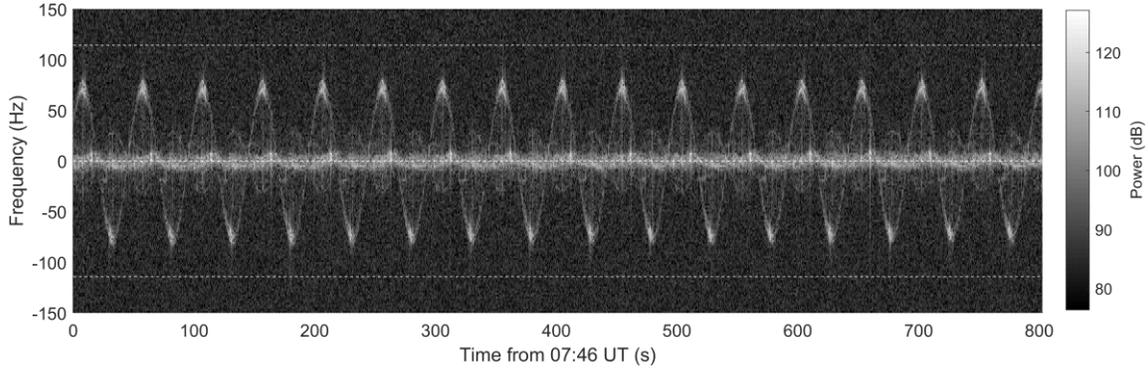


Figure 11: May 8, 2021 GOES 12 Doppler echoes ($P_{\bar{\phi}} \approx 49.75$ s)

Candidate pole solutions for GOES 12 on May 9, 2021 are provided in Figure 12. There is clear consistency between the Doppler bandwidth and 2D RPD solutions.

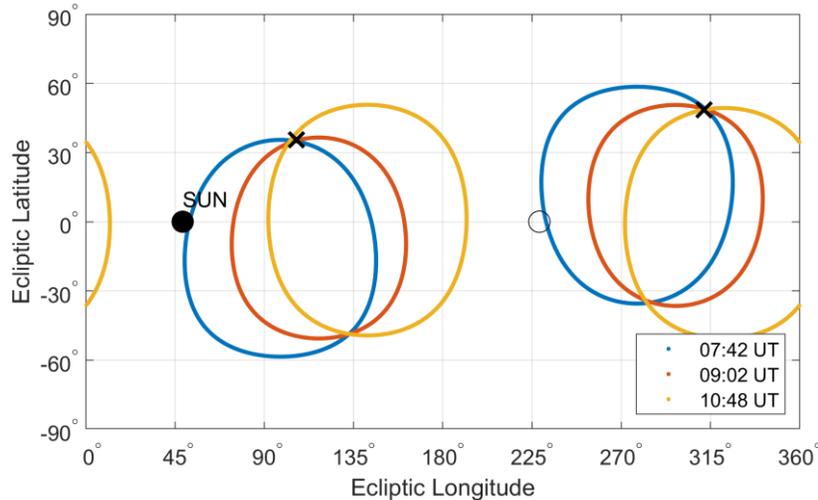


Figure 12: May 9, 2021 GOES 12 candidate poles in J2000 ecliptic frame

4.5 Titan 3C Transtage (02770, 1967-040F)

On May 1, 2021 we observed the Titan 3C Transtage rocket body (NORAD ID: 02770) at a range of $\sim 118,000$ km. This upper stage is in a highly eccentric orbit with perigee and apogee radii of ~ 4000 km and $\sim 122,000$ km respectively. The resulting echoes are provided in Figure 13. Analysis of the echoes indicates $P_{\bar{\phi}} \approx 8.9$ s. As with the GOES satellites, this rocket body's center of mass is not aligned with its center of figure given the differences in Doppler bandwidth from one side of the object to the other. This is not unexpected given that much of the mass is likely concentrated near the engines at the upper stage's base. With b_1 and b_2 denoting the observed Doppler bandwidths for opposite sides of the body which are rotating at equal angular rates, we have the relationship $b_1/r_1 = b_2/r_2$ where r_1 and r_2 are the corresponding radial extents relative to the center of mass, and the overall length $d = r_1 + r_2$. For the May 1 echoes, $b_1 \approx 22$ Hz and $b_2 \approx 63$ Hz. This yields $r_1/d \approx 0.26$ and $r_2/d \approx 0.74$. So the rocket body center of mass is shifted roughly 25% of the overall length from the center of figure, likely

towards the base. The dashed lines in Figure 13 denote the estimated maximum Doppler bandwidth assuming an overall length d of 4.57 m [11]. The relatively small observed bandwidth in Figure 13 indicates a somewhat pole-on view.

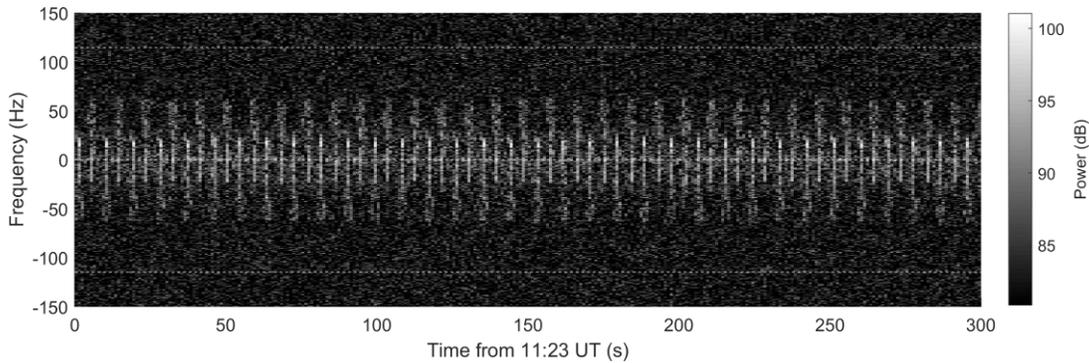


Figure 13: May 1, 2021 Titan 3C Transtage Doppler echoes ($P_{\bar{\phi}} \approx 8.9$ s)

4.6 Atlas Agena D (07964, 1975-055B)

We observed an Atlas Agena D upper stage (NORAD ID: 07964) near GEO on May 2, 2021 and the resulting Doppler echoes are provided in Figure 14. Phase-folding indicated $P_{\bar{\phi}} \approx 56.5$ s. The asymmetric Doppler bandwidth indicates this rocket body also has a center of mass offset with $r_1/d \approx 0.4$ and $r_2/d \approx 0.6$. This is a less significant offset than the Titan 3C Transtage above. With $d \approx 6.48$ m [12], the observed Doppler bandwidth is very near the maximum estimated value in Figure 14, indicating a roughly side-on view of the pole.

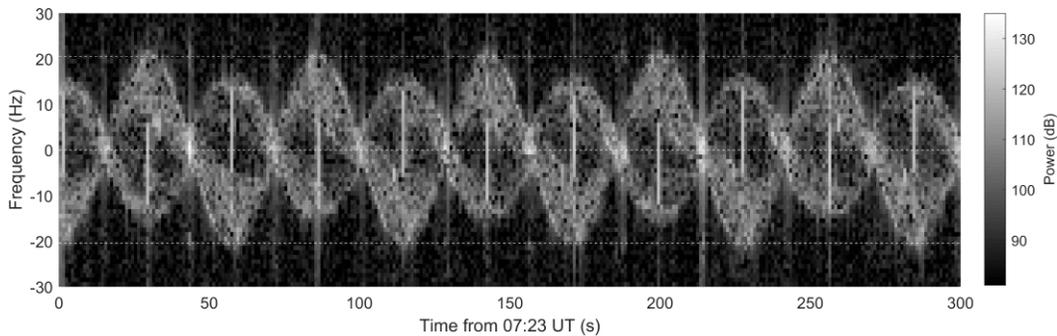


Figure 14: May 2, 2021 Atlas Agena D Doppler echoes ($P_{\bar{\phi}} \approx 56.5$ s)

5. DISCUSSION

The radar-derived $P_{\bar{\phi}}$ estimates for the GOES 8-12 satellites spanning late 2019 – mid 2021 are provided in Table 2 (see Ref. [8] for details on the Dec. 2019 and Feb. 2020 observations). There is significant diversity in the satellite spin periods with values ranging from less than 60 s to >10 min. Also, GOES 8, 9, and 11 were in non-principal axis tumbling while GOES 10 and 12 were in uniform rotation. The combined radar and optical data show clear indication that GOES 8 was locked in a 5:1 tumbling period resonance in Feb. 2020 and late Apr./May 2021. It appears that the satellite was in this resonance in Apr. 2018 as well [5]. Table 2 shows that GOES 8's precession period varied significantly while captured in this resonance. This is consistent with dynamical modeling of the solar radiation torques (i.e. the YORP effect), where the two tumbling periods $P_{\bar{\phi}}$ and P_{ψ} remain in lock step while the overall spin rate changes [9].

Table 2 also illustrates GOES 12's rapid increase in spin rate from Dec. 2019 to May 2021. The satellite was tumbling in 2014 [2] and at some point returned to uniform rotation before spinning up. This mirrors GOES 8's

rapid uniform spin down in 2014 and subsequent transition to tumbling [5]. So it is possible that GOES 12 will soon spin back down and return to the tumbling regime.

Table 2: Summary of Radar-Derived GOES $P_{\bar{\phi}}$ Estimates (* denotes tumbling)

Date	GOES 8	GOES 9	GOES 10	GOES 11	GOES 12
Dec. 6, 2019	*353 s (5.9 min)	-	-	*775 s (12.9 min)	882 s (14.7 min)
Feb. 18, 2020	*215 s (3.6 min)	-	30.5 s	-	462 s (7.7 min)
Feb. 20, 2020	*216 s (3.6 min)	-	30.5 s	-	454 s (7.6 min)
May 1, 2021	*318 s (5.3 min)	-	-	*799 s (13.4 min)	-
May 6, 2021	*333 s (5.6 min)	*724 s (12.1 min)	-	-	-
May 9, 2021	*342 s (5.7 min)	*715 s (11.9 min)	-	-	49.75 s

Figure 15 summarizes the radar-derived candidate pole directions for GOES 8, 9, and 12 on May 9, 2021. The combined Doppler bandwidth and 2D RPD solutions are denoted by x's and the tentative GOES 9 3D RPD solution is denoted by the red diamond. Interestingly, the two tumbling satellites GOES 8 and 9 have similar pole directions near the sun and anti-sun directions. Also, comparing GOES 8's candidate poles in Feb. 2020 [8] and May 2021, they are in roughly the same directions relative to the sun and anti-sun directions.

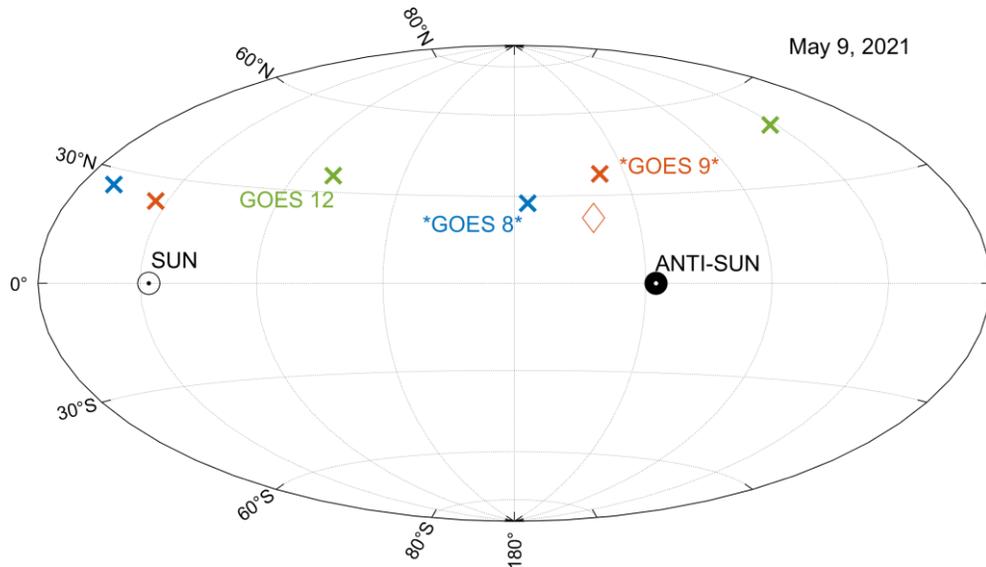


Figure 15: Radar-derived candidate poles for GOES 8, 9, and 12 on May 9, 2021 in the J2000 ecliptic frame (* denotes tumbling). The x's are Doppler bandwidth/2D RPD solutions and the diamond is a tentative 3D RPD solution.

The radar-derived precession periods for GOES 8, 9, and 11 greatly aided with interpretation of the associated light curves. For all three satellites, $2f_{\bar{\phi}}$ was the most dominant frequency light curve frequency and other $f_{\bar{\phi}}$ harmonics were also present. This strong trend further validates findings from prior simulated light curve surveys [5] and can be leveraged when analyzing other tumbling GOES light curve observations.

6. CONCLUSIONS

In this work, Doppler radar and optical light curve observations were used to estimate the spin periods and pole directions of both uniformly rotating and tumbling debris objects near and above GEO. Doppler echoes obtained for the Titan 3C Transtage at a range of 118,000 km demonstrate that the 34 m Deep Space Network antennas can observe sizeable targets in cislunar space. These observations further demonstrate the spin state diversity and continued evolution of the defunct GOES 8-12 satellites. Most notably, GOES 8 remains captured in a 5:1 tumbling

period resonance with an evolving spin rate, while GOES 12 spun up rapidly from Dec. 2019 to May 2020. These findings are consistent with the GOES satellite spin states being driven primarily by solar radiation torques.

ACKNOWLEDGEMENTS

This work was supported by the National Science Foundation Graduate Research Fellowship Program and a Jet Propulsion Laboratory R&TD innovative spontaneous concepts proposal. The authors would also like to thank NASA Goldstone Deep Space Communications Complex for providing antenna observing time.

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