

# A study of measuring beam wander from stars for ground-based laser illumination

Nazim Ali Bharmal

n.a.bharmal@durham.ac.uk

Durham University, Palatine Centre, South Road, Durham DH1 3LE

## ABSTRACT

The issue of estimating beam wander from stars inspired by astronomical techniques is studied in this work to evaluate whether it is not only possible but also realistic. The optical characterisation of orbiting objects, be they debris (so poorly known a priori) or satellites (better knowledge) not only improves the current awareness of objects in their orbital plane but also permits the parameters of the object to be more fully understood. The application of astronomical tomographic wavefront sensing is assessed for angle-of-arrival measurements. Using a realistic situation, for moderate seeing it is shown that the probability of reducing wander from 1 arcsecond by 50—90% using 1 guide-star (NGS) ranges from 20—33%, and for 2 NGS 1—20%. For these non-negligible probabilities, the 1-sigma motion at 1000km would be reduced from ca. 5m to 0.5—2.5m.

## 1. MOTIVATION

Space Surveillance and Tracking of orbiting objects using optical wavelengths, be they debris (so poorly known a priori) or catalogued satellites (better knowledge) not only improves the current awareness of objects in their predicted orbit but also permits the dynamical parameters of the object, such as rotation, to be more robustly understood. The advantages of such knowledge of course aid SSA but will also aid measures to understand satellite lifetimes and prior knowledge for the nascent in-orbit servicing market. The critical aspect of using optical wavelengths is the ability to measure the *phase* which considerably improves measurement precision compared to *intensity* or *polarisation* because phase records *where* light is scattered from.

A clear example of the information from phase versus intensity alone is shown in Figure 1. Using solar illumination alone, although the illuminating flux is high (approximately 1.2kW/m<sup>2</sup>), observations are restricted to when satellites are not within the umbra and there is no ability to modify the illumination parameters spatially or in time. The alternative is to use active illumination which will be from a ground-based laser. Such illumination is utilised in range finding to geodetic satellites where the phase is ignored and so the key information is distance timing to an object. An alternative approach is to project an intensity with known spatial variation, for example a well-defined beam profile, so that the angular position of a targeted object can be known to within the uncertainty of the projected beam.



Figure 1 The phase from the Fourier transform of the right hand half of the flower is combined with the amplitude from the left hand half, and then the inverse transform of the combination is shown on the right. As imaging is essentially a Fourier transform process, this demonstration motivates us to investigate new ways to compensate for phase distortions, or infer them in a more robust way.

The standard method for compensating optical wavelength propagation through the atmosphere is to use adaptive optics[1], wherein a measurement is made of a reference target whose properties are known and so the aberrations can be deduced and then the opposite wavefront is applied. For the purposes of this presentation, it is necessary to study the topic of measuring the aberrations when a reference target is not necessarily available. To consider this more fully, the goal of this study is to understand how to accurately project a laser beam from the ground that can target an object by overcoming *beam wander* induced by the atmosphere using measurements from stars that are in the field-of-view.

## 2. ASTRONOMICAL TECHNIQUES

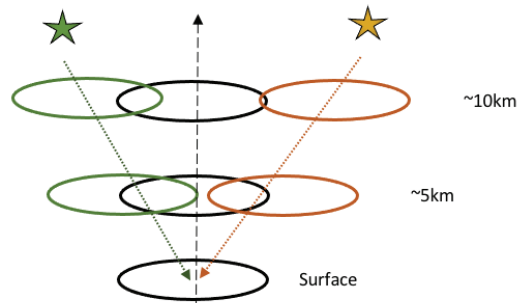


Figure 2 A figurative description of using off-axis stars (green and yellow) to measure their angle-of-arrival and then project these measurements into the on-axis direction (black) for estimating angle-of-arrival for compensation of beam wander. The overlap between projected telescope aperture at the two altitudes in this diagram (5km and 10km) is key to the quality of the measurement: while it is always exact for aberrations induced at the surface, it becomes less exact with altitude.

Beam wander from projecting a laser beam through the atmosphere is the equivalent of measuring the angle-of-arrival of light which accumulate due to the layered nature of turbulence that exists between the surface and, approximately, the tropopause. The angle-of-arrival is altered by aberrations that are not isotropic and so cannot be measured for any arbitrary direction and applied to another. Instead for objects that are insufficiently bright to be measured, we must

consider that the only other suitable reference targets to measure the angle-of-arrival from are stars, which limits measurements to near the sun during the day (with the associated issues) or during the night. The methodology applied in astronomy to measure induced aberrations for directions without an associated reference is figuratively described in Figure 2 where two stars measurements are projected into an arbitrary direction.

This technique is called tomography[2] and can be applied on a point-by-point basis within the telescope aperture or by describing the aberrations measured using a modal basis, such as Zernike polynomials[3]. For the purposes of angle-of-arrival, the only modes required are the two linear terms that produce image motion or beam wander along orthogonal axes. The usual requirement for tomography to be considered successful is that the projected apertures from objects being used as references overlap so that the altitudes that aberrations are induced at can be correctly identified. However, for the purposes of estimating the angle-of-arrival in a direction away from the measurement reference(s), the only requirement is that a sufficient correlation[4] exists in the angle-of-arrival between the projected telescope pupil in these directions. This correlation of angle-of-arrival is shown for a situation where the seeing is 1 arcsecond and the telescope has an aperture of 1m diameter. The important aspect is that even for situations where the projected apertures have no overlap, a significant correlation exists, at least for transverse aperture separation with respect to the direction of angle-of-arrival.

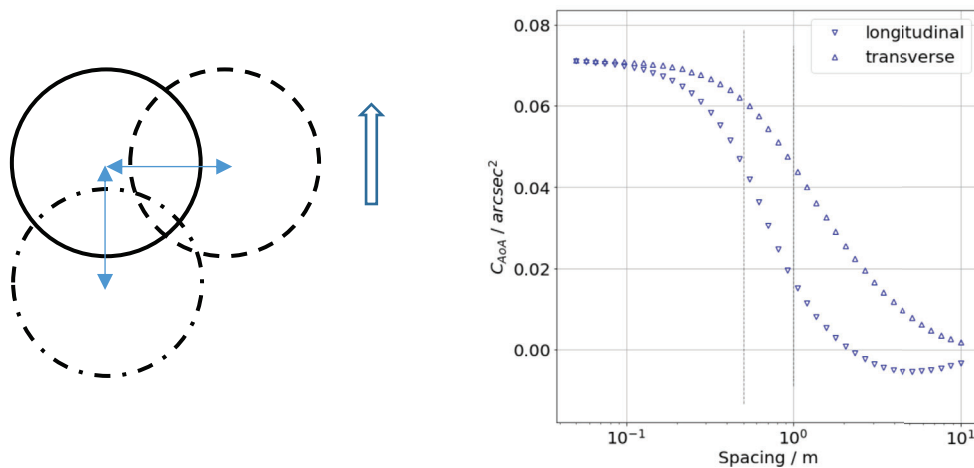


Figure 3 The covariance of angle-of-arrival for the two directions, longitudinal (parallel to the direction of separation of projected apertures) and transverse (perpendicular), for integrated seeing of 1", an aperture of 1m diameter. The vertical dashed lines represent half and the entire diameter of separation of the aperture.

The consequence is that stars within a relatively wide radius can be useful used to estimate the angle-of-arrival for beam wander. This is more particularly true for SST, since the non-sidereal tracking of targets at velocities ca.150—15 arcsec/sec for LEO or GEO orbits implies that for the 1m telescope aperture and turbulence up-to a distance of 24km (for zenith angles of 60° and a turbulence region inducing aberrations at a height of 12km) a separation of upto 3.5m which is equivalent to approximately 30". Therefore, stars will be present within the usable field of view for beam wander estimation for between 0.2 and 2 seconds, implying that any detection of angle of arrival has to be flexible to accommodate fast variations in the constellations of stars available for detection.

### 3. MINIMUM MEAN SQUARE ESTIMATOR

An optimized estimator for projecting measured angles of arrival is a least squares method with priors based on measured covariances[5]. The methodology for computing the vector of length two of estimated angles-of-arrival,  $s_{on}$ , from a vector of off-axis measurements,  $s_{off}$ , of length 2 times the number of stars used for measurements, is based on estimated covariance matrices,

$$s_{on} = C_{on,off} C_{off,off}^{-1} s_{off} = T_{MMSE} s_{off}, \quad (1)$$

where  $C_{off,off}$  is a fitted or calculated covariance matrix based on numerical models of the measured covariances of angle-of-arrival,  $C_{on,off}$  is a calculated covariance matrix on the expectation of the estimated and measured angles of arrival, and their combination to form the estimator is  $T_{MMSE}$ . The components are more specifically,

$$\begin{aligned} C_{off,off} &= \langle s_{off}^T \cdot s_{off} \rangle, \text{ and} \\ C_{on,off} &= \langle s_{on}^T \cdot s_{off} \rangle. \end{aligned} \quad (2)$$

To compute these matrices, either measurements must be made to form approximations and then parameter fitting with models applied to produce synthetic versions, or the models applied with *prior* information on those parameters to produce synthetic alternatives. In this paper, we do not explicitly address this problem as we define the parameters and then use these values to produce an exact MMSE estimator.

### 4. AVERAGING ESTIMATOR

A naïve approach to producing an estimator for angle-of-arrival from off-axis measurements is to simply average those measurements. Clearly this approach is only suitable if there is a high-degree of correlation between off-axis measurements and does not provide for any consideration of the noise in the measurements, which is outside the scope of this study. The estimator is then written as,

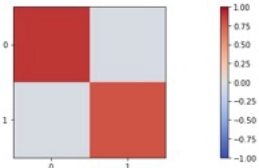
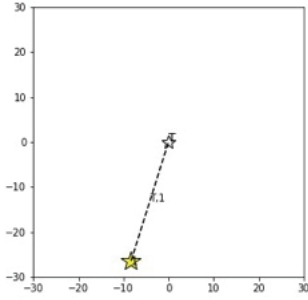
$$s_{on} = T_{average} s_{off}, \quad (3)$$

where  $T_{average}$  simply computes a mean of each direction's angle-of-arrival from the  $N$  stars being observed. Clearly this is an estimator that can be rapidly implemented for varying numbers of stars algorithmically, and the potential to add compensation for varying measurement noise is also straightforward.

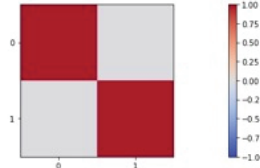
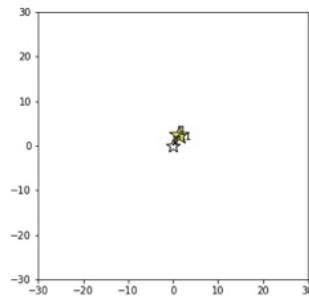
### 5. RESULTS OF MONTE CARLO SIMULATION

With our methodology in hand, using off-axis stars's angle-of-arrivals to estimate the on-axis angle-of-arrival, the results of comparing the estimation using either MMSE or simple averaging is compared in a situation where no measurement noise is present. The study concentrates on the effect of varying star numbers and their positions to understand the feasibility of compensating beam wander from rapidly moving targets.

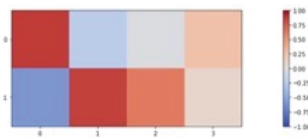
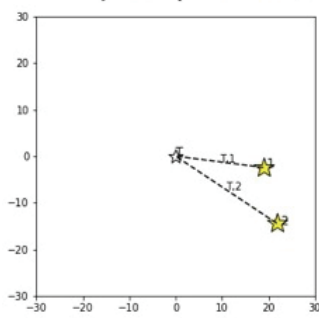
A(I)



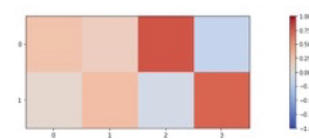
A(II)



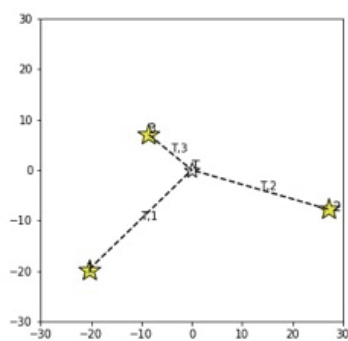
B(I)



B(II)



C(I)



C(II)

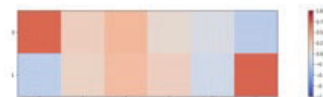
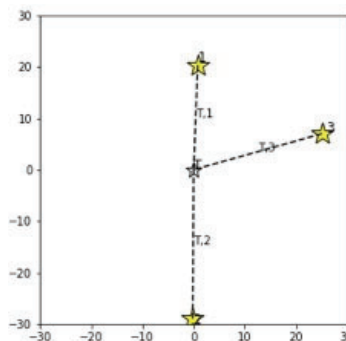


Figure 4 The left-hand columns (category I) represent stars (yellow) unfavourably placed within a 30'' by 30'' region around the target (white) while the right-hand columns (category II) represent a favourable distribution. The MMSE estimator is below each stellar constellation and demonstrate how the measured angles-of-arrivals are, essentially, scaled and even inverted, in order to improve the on-axis estimate.

In Figure 4 pairs of configuration of either one, two or three (A,B, or C) stars in a 1 arcminute field-of-view are shown placed around an on-axis target and used for a Monte Carlo simulation of 10,000 random instances of an atmosphere with a three layer atmospheric model. The distribution of the layers sums to 1" seeing with 60% within the surface layer at the telescope pupil, with 20% at 5km and a further 20% at 10km. The telescope aperture is 2m. The angle-of-arrivals from the stars are used when both estimators are used. The configurations (constellations) of stars are either "favourable" or "unfavourable" (categories I and II respectively) with respect to the measurement of angles of arrival.

Table 1 The input and input less estimate standard deviations for the six scenarios shown in Figure 4., where all units are arcseconds. The input is identical in all cases therefore the residuals show the *potential* quality of reduction in beam wander.

A(I)	Axis	Input	Average - Input	MMSE - Input	A(II)	Axis	Input	Average - Input	MMSE - Input
	H	1.09	0.43	0.42		H	1.09	0.07	0.07
	V	1.08	0.66	0.63		V	1.08	0.1	0.1
B(I)	Axis	Input	Average - Input	MMSE - Input	B(II)	Axis	Input	Average - Input	MMSE - Input
	H	1.09	0.59	0.54		H	1.09	0.25	0.16
	V	1.08	0.42	0.33		V	1.08	0.17	0.14
C(I)	Axis	Input	Average - Input	MMSE - Input	C(II)	Axis	Input	Average - Input	MMSE - Input
	H	1.09	0.38	0.29		H	1.09	0.36	0.27
	V	1.08	0.30	0.21		V	1.08	0.44	0.39

In Table 1, the results of using the two estimators for the six cases for computing potential residuals when compensating for beam wander. It is clear that in most cases, the MMSE estimator, despite always being better, is not necessarily favourable even in this zero measurement noise scenario. When it is, is due to particular constellation around the target direction e.g. C(II), which is the least likely scenario as the constellation position will rapidly change. Therefore it seems clear that for this study, using a simple average will be sufficient to reduce beam-wander by at least 50% and up-to 90% using up-to 3 stars.

The probability of having sufficiently bright stars must now be considered, where we briefly introduce the measurement noise for angle-of-arrival.

## 6. PROBABILITY OF HAVING SUFFICIENT BRIGHT STARS

The Cramér-Rao Bound[6] is closely adhered to by the centre-of-mass estimator to measure angle-of-arrival, from simulation, with a the MMSE propagates the error with a (effectively) unitary factor. Thus, the CRB for any star's angle-of-arrival is a good estimate of the uncertainty for the target direction. From the results in the previous figures, it is sensible therefore to ask what flux will result in a CRB of 0.1". If we assume a (nearly) photon-noise limited detector

(e.g. sCMOS or similar) then each NGS requires a brightness of 13 photons which translates to a  $m_V = 17.7$  target.

Using estimates of galactic stellar density, the number of targets of brightness  $m_V \leq 18$  is between 600 and 4000 per square degree (for galactic pole and galactic centre pointing directions). In simulation field-of-view this is a density of 0.18 to 1 star respectively. For the lower limit, this can be interpreted as a 18% chance there is a sufficiently bright NGS in the field of view and then the sample results from Table 1 Row A apply: a reduction in the motion of the DLR laser pulse by 50—90%. There is a 1% chance that 2 stars are present in the field of view and it is vanishingly small for 3. For the upper limit, there is approximately 1/3<sup>rd</sup> chance to find 1 star, 1/5<sup>th</sup> for 2 stars, and 6% for 3 stars of sufficient brightness. Therefore, it can be expected that a reduction in the beam wander of at least 2/3<sup>rd</sup>s from using stars to estimate beam wander is achievable.

## 7. CONCLUSION

The study of using stars to estimate angle-of-arrival for predicting, and therefore compensating for, beam wander of projected laser illumination for enhancing SST is found to be practicable and realistic for a reduction of at least half, for observation directions away from the galactic pole 50% of the time. This is the worst case scenario and therefore suggests that this approach to beam-wander compensation is not only feasible but will also be practical for many observations. The main conclusion I draw regarding how to implement the estimation is that simply averaging measurements (or using them directly if only one star is sufficiently close) is effective. The next steps are to understand how to utilise the noise from any star to produce a strategy that can take into account both bright and faint stars.

## 8. REFERENCES

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