Parametric Generation of Whistler Waves in the Ionosphere

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Abstract

This paper explores generation of electromagnetic whistler waves with frequencies close to the electron cyclotron frequency. We evaluate the wave amplitude of whistler waves excited by parametric interaction of two high frequency (HF) waves launched into the ionosphere from the ground. Then, we apply a quasi-linear mechanism of cyclotron heating of ionospheric electrons by the parametrically excited spectrum of whistler waves. It is demonstrated that this mechanism can produce considerable amount of suprathermal electrons with energies greatly exceeding the ambient electron thermal energy.

I. Introduction

We will analyze parametric excitation of whistler waves with frequencies close the electron cyclotron frequency in the F-region ionosphere. The excitation mechanism is parametric interaction of two high frequency (HF) waves at frequencies shifted by the frequency of about the electron cyclotron frequency and propagating at different angles to the geomagnetic field. Conditions under which resonance interaction can take place will be established and amplitudes of the excited electromagnetic whistler waves will be estimated. In the third section, a quasi-linear mechanism of electron cyclotron heating by electromagnetic whistler waves will be applied to estimate the energy of accelerated electrons. The problem of electron cyclotron heating by intense whistler waves is important for many applications. Among those are thermonuclear fusion and different aspects of space plasma physics[1-4]. We consider waves at frequencies close to the electron cyclotron frequency. In the case when the spectrum of phase velocities is broad enough, the heating can be described using the framework of quasilinear theory. The problem in many respects is similar to that of the absorption of a Langmuir wave packet. In the absence of collisions the wave-particle interaction leads to the formation of a “plateau” on the electron distribution function along the diffusion lines and supresses the wave damping [5-6].

Finally the summary of obtained results will be presented in section four followed by the list of references.

II. Parametric excitation of whistler waves via beat wave process

In this section we will analyze excitation of electromagnetic whistler waves propagation at small angles towards the geomagnetic field. We will conseder HF waves participating in a parametric excitation process with frequencies considerably larger then an electron cyclotron frequency and will use the following dispersion relation for these waves:

$$\omega^2_{\mathbf{k}_{1,2}} = k^2_{1,2}c^2 + \omega^2_{pe} \tag{1}$$

In (1) ω and k are the frequency and wave vector of HF wave, indices 1 and 2 correspond to the first and second wave, c is the speed of light, and $\omega_{pe} = (4\pi e^2 n_0 / m_e)^{1/2}$ is an electron plasma frequency. For simplicity we suggest that HF waves are monochromatic and propagate in y-z plane. The geomagnetic field is directed along the z axis. The angles between wave vectors, $\mathbf{k}_1, \mathbf{k}_2$, and the geomagnetic field are θ_1 and θ_2, respectively. We also suggest that the frequencies of monochromatic HF pump waves satisfy the following conditions:

$$\omega_{\mathbf{k}_{1,2}} > \omega_{pe} >> \omega_{ce} \geq \omega_{\mathbf{k}_1} - \omega_{\mathbf{k}_2} \tag{2}$$

In (2), $\omega_{ce} = eB_0 / m_e c$ is the electron cyclotron frequency. We consider excitation of whistler waves due to parametric interaction of HF waves when they can satisfy the resonance condition for excitation, i.e.:

$$\mathbf{k}_3 = \mathbf{k}_1 - \mathbf{k}_2 \tag{3}$$

$$\omega_{\mathbf{k}_3} = \omega_{\mathbf{k}_1} - \omega_{\mathbf{k}_2} \tag{4}$$

where $\omega_{\mathbf{k}_3}$ satisfies a dispersion relation for the whistler waves:

$$\omega = \omega_{ce} \frac{k_z}{k} \frac{1}{1 + \omega_{pe}^2 / (c^2 k^2)} \tag{5}$$

In (5), k_z is the wavenumber of a whistler wave along the direction of an external magnetic field.

The resonance conditions (3) – (4), necessary for efficient beat wave excitation for given values of external magnetic field, B_0, and plasma density, n_0, can be satisfied only for certain values of propagation angles, θ_1 and θ_2, of the HF pump waves. As an example, Figure 1 presents the dependence of the angle θ_2 on θ_1 and plasma density n_0 with magnetic field $B_0 = 0.5$ G. To make this plot, the following parameters were used: $\omega_1 = 2\pi \cdot 9 \cdot 10^6$ rad/s, $\omega_2 = \omega_1 - \omega_3$ rad/s, $\omega_3 = 0.8\omega_{ce}$. In Fig. 2, the dependence of the ratio k_z / k on the angle θ_2 for the different values of θ_1 is presented.

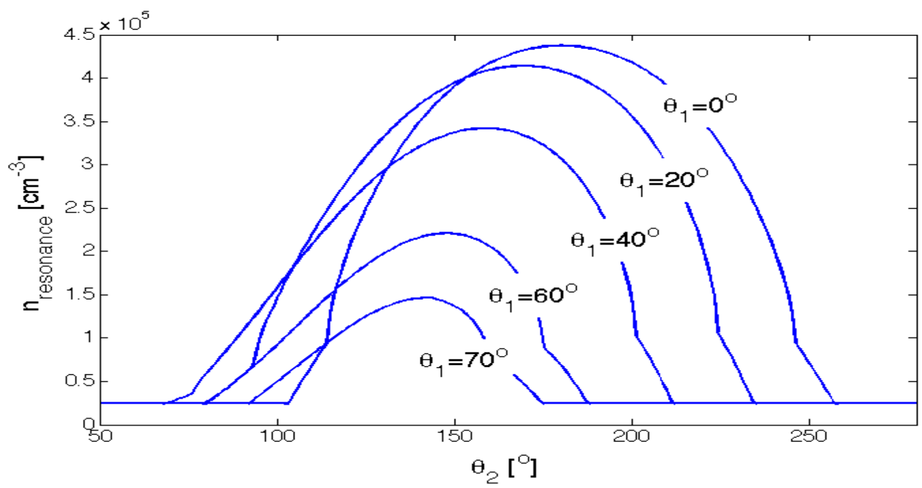


Fig. 1. Relation between the electron density and the angle θ_2 (the angle between the wave vector of HF wave 2 and the geomagnetic field) for different values of the angle θ_1 (the angle between the wave vector of HF wave 1 and the geomagnetic field) to satisfy the resonance conditions (3) – (4) for $\omega_3 / \omega_{He} = 0.8$ and $B_0 = 0.5$ G.

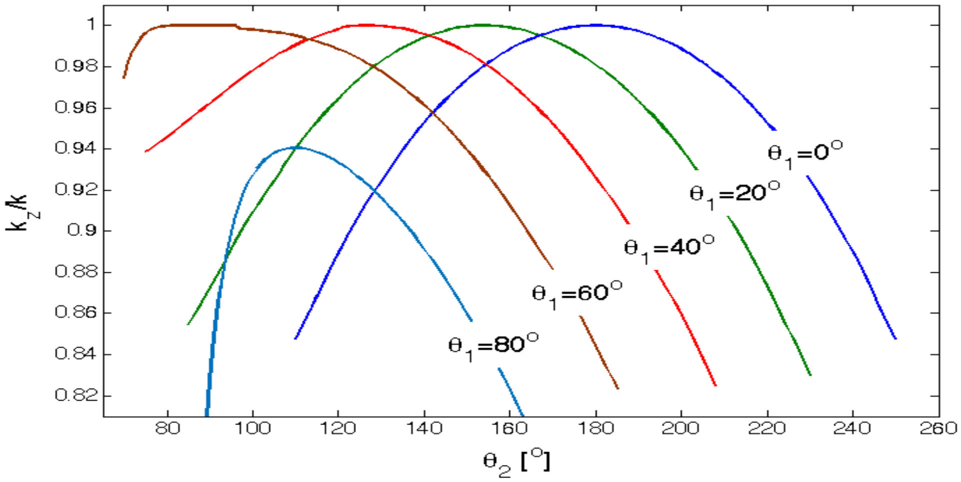


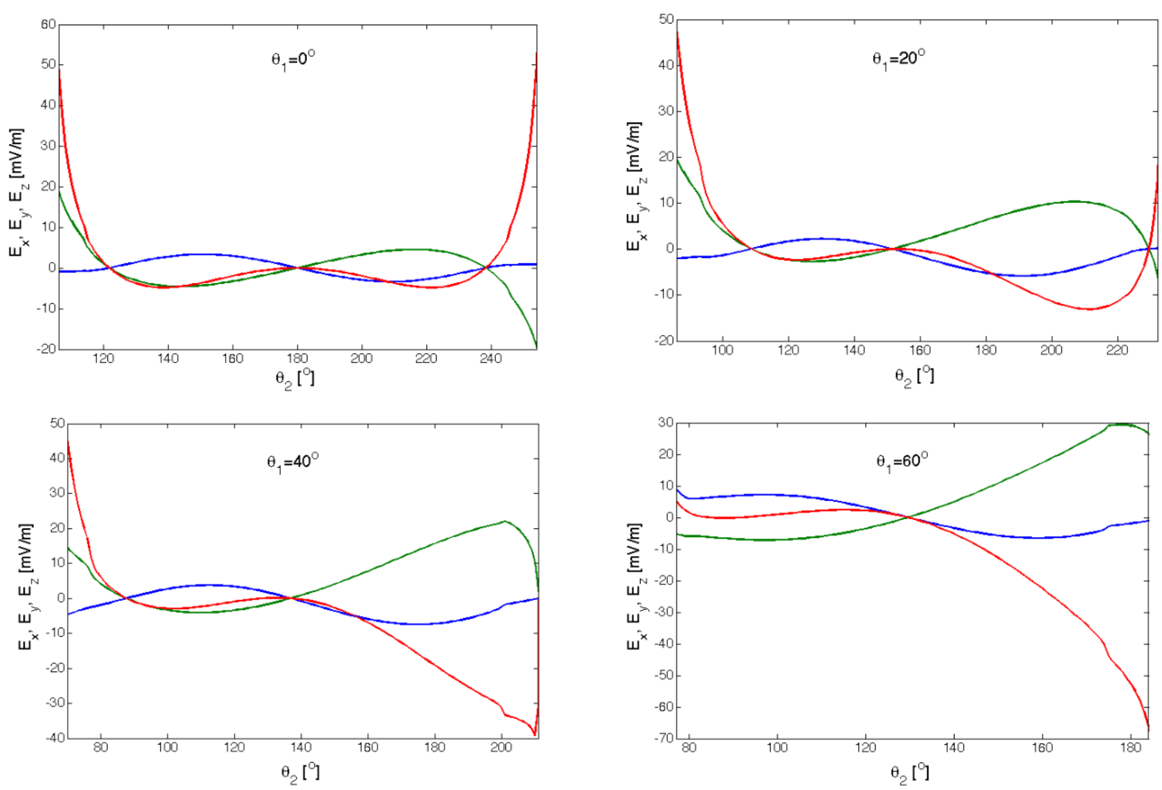
Fig. 2. Dependence of the ratio k_{3z} / k_3 in the dispersion relation for a whistler wave (5) on the propagation angle θ_2 for different values of θ_1.

Analyzing results presented in Fig. 1 and Fig. 2 allows us to conclude that for typical values of the magnetic field in the F layer for densities between 4×10^4 to 4×10^5 cm^{-3}, it is always possible to find values of angles q_1 and q_2 that satisfy the resonance conditions (4) – (5).

The larger is the angle q_1, the shorter is the range of the allowable angles θ_2 satisfying the resonance conditions and the smaller is the required plasma density. From the results presented in Figure 2 it also follows that the angle between the wave vector of a generated whistler wave and the geomagnetic field is small because the ratio $k_z / k £ 1$, i.e., the generated whistler wave propagates in a small cone around the geomagnetic field direction.

Next, we can estimate amplitudes of parametrically excited electromagnetic whistler eigenmodes. We will omit process of derivation of nonlinear equations that were used to obtain the excited whistler wave amplitudes. In the case when resonance conditions (4) – (5) are not satisfied, the amplitude of excited whistler type waves that are not plasma eigenmodes, does not exceed 0.01 mV/m. To correctly estimate amplitude of parametrically excited whistler wave, it is necessary to include real or artificial (in the form of effective collision frequency) collisions into the model. For our estimates, we will use electron neutral collisions. This will allow us to avoid singularity in the expression for wave amplitudes. The electron-neutral collision frequency is $n_{en} = n_n V_{Te} s_e$, where n_n is the density of the neutral molecules, $V_{Te} = (T_e / m_e)^{1/2}$ is the thermal electron velocity, T_e is the electron temperature, and

σ_e is the scattering cross section of electrons colliding with neutrals. Below we will present numerical results of the beat whistler wave excitation by two HF pump waves with the following plasma parameters: $\sigma_e = 1.5 \cdot 10^{-15}$ cm^2, $n_n = 10^{10}$ cm^3, $T_e = 3000^o K$. The resulting electron collision frequency used to obtain numerical solution can be estimated as $\nu_{en} \approx 6.4 \times 10^2$ 1/s. In Figure 3, the components of the excited whistler wave electric field are presented as a function of an angle θ_2 of the second pump wave for different values of θ_1. As one can see, the amplitude of the electric field components transverse to the geomagnetic field is in the range 10 – 20 mV/m. The amplitude of the parallel to the geomagnetic field component is about 10-40 mV/m. Wave amplitudes sharply increase near the ends of the region where the resonance conditions can be satisfied. This increase can be attributed to the fact that near the end points the electron plasma frequency approaches the electron cyclotron frequency, $\omega_{pe} \to \omega_{ce}$ and our approximation is not applicable. Note that the amplitude in the resonance case is about three orders of magnitude larger than in the case of non-resonant excitation.



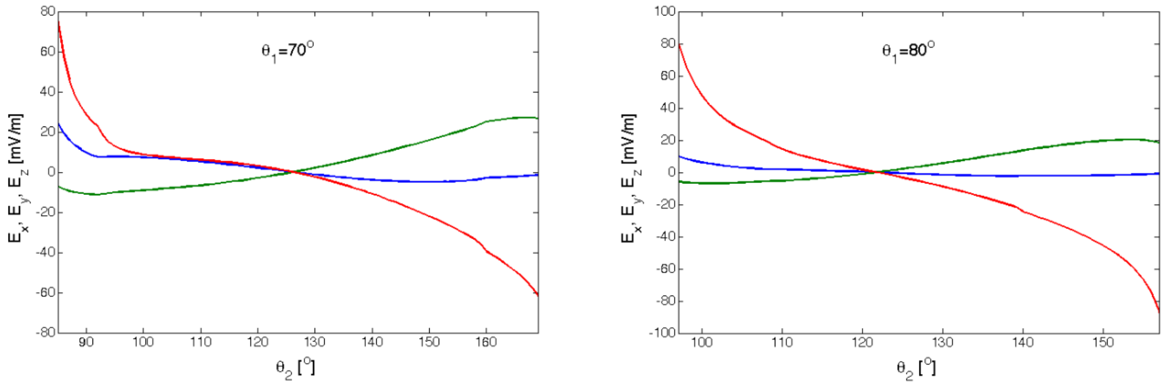


Fig.3. Electric field components of a whistler wave parametrically generated due to beating of two pump HF waves with amplitude 1 V/m as a function of an angle of a second pump wave θ_2 for different values of θ_1.

III. Electron cyclotron heating by electromagnetic whistler waves

In this section, we will investigate the plasma electron heating by electromagnetic whistler waves with frequencies close to the electron cyclotron frequency. We will analyze the case with a broad wave spectrum of whistler waves. Quasilinear diffusion equation will be used for description of evolution of the electron distribution function in the presence of the excited whistler wave spectrum. Heating of electrons in this case can be described using the framework of quasilinear theory. In the absence of collisions, the wave-particle interactions lead to the formation of a “plateau” on the electron distribution function along the diffusion lines and precludes the wave damping. If collisions are considered, this will tend to thermalize the distribution function and hence counteract the “plateau” formation caused by the wave-particle interaction and thus leading to continuous wave damping and particle heating. Below we will focus on the quasilinear mechanism of electron heating by electromagnetic whistler waves in the absence of collisions that can be neglected in the top ionosphere.

Let us consider a 1D packet of electromagnetic electron cyclotron waves propagating along the external magnetic field:

$$\mathbf{E} = \sum_{k_z} \mathbf{E}_{k_z} \exp[i(\omega_{k_z} t - k_z z)] \qquad (6)$$

Electrons in resonance with the waves experience quasilinear diffusion along the characteristics of the kinetic equation [5-6]:

$$\frac{\partial f}{\partial t}=\sum_{k_z}\left[\frac{\partial}{\partial v_z}\frac{k_z v_\perp}{\omega_{k_z}}+\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}v_\perp\left(1-\frac{k_z v_z}{\omega_{k_z}}\right)\right]\left\{D_{k_z}\left[\left(1-\frac{k_z v_z}{\omega_{k_z}}\right)\frac{\partial f}{\partial v_\perp}+\frac{k_z v_\perp}{\omega_{k_z}}\frac{\partial f}{\partial v_z}\right]\right\} \qquad (7)$$

In (7) f (**v**, t) is the distribution function of resonance electrons in velocity space and the diffusion coefficient is given by:

$$D_{k_z}=\frac{\pi e^2}{2m_e}E_{k_z}^2\,\delta(\omega_{k_z}-k_z v_z-\omega_{ce}) \qquad (8)$$

As it follows from (7), particle interaction with these waves leads to diffusion of the resonant particles along the diffusion lines given by:

$$\frac{1}{2}v_\perp^2+\frac{1}{2}v_z^2-\int_0^{v_z}dv_z'\,v_{ph}(v_z')=const \qquad (9)$$

where $v_\perp$ is the electron velocity perpendicular to geomagnetic field, $v_z=\frac{\omega-\omega_{He}}{k_z}$ is the longitudinal velocity component in the normal Doppler cyclotron resonance with the wave at a frequency ω and wavenumber k_z, and $v_{ph}=\omega/k_z$ is the phase velocity of this wave. The dispersion relation of these waves has a form (see e.g. Akhiezer et al, 1974)

$$\omega=\frac{\omega_{ce}}{1+\omega_{pe}^2/(k_z c)^2} \qquad (10)$$

Using this dispersion law the electron velocity can be expressed in the form

$$v_z=-v_a\frac{(\omega_{pe}/k_z c)^3}{1+\omega_{pe}^2/(k_z c)^2} \qquad (11)$$

where $v_a=c\frac{\omega_{ce}}{\omega_{pe}}$ is the electron Alfven velocity. For ionosphere plasma conditions this velocity in a case if $\omega_{pe}<<kc$ which is when the wave frequency is close to the electron cyclotron frequency $\Delta\omega=\omega_{ce}-\omega<<\omega_{ce}$. In this case, the following relation expressing the resonant velocity through the wave number can be obtained from equations (10) and (11):

$$k_z \approx \frac{\omega_{pe}}{c}\left(\frac{v_a}{v_{res}}\right)^{1/3} \text{ or } v_{res} \approx -v_a(\Delta\omega/\omega)^{3/2},\ v_{res}\equiv v_{z,res} \qquad (12)$$

Then the phase velocity can be written in the form

$$v_{ph} = - v_{res}^{1/3} v_a^{2/3} \text{ or } v_{ph} = - v_a (\Delta\omega / \omega)^{1/2} \tag{13}$$

It follows from the equation (9) that interaction with the cyclotron waves leads to the diffusion of the electrons along the lines:

$$w = \frac{1}{2} v_\perp^2 + \frac{3}{4} v_z^{4/3} v_a^{2/3} = const \tag{14}$$

and predominant transverse heating of resonant electron occurs. The evolution of the distribution function is shown in Fig. 4.

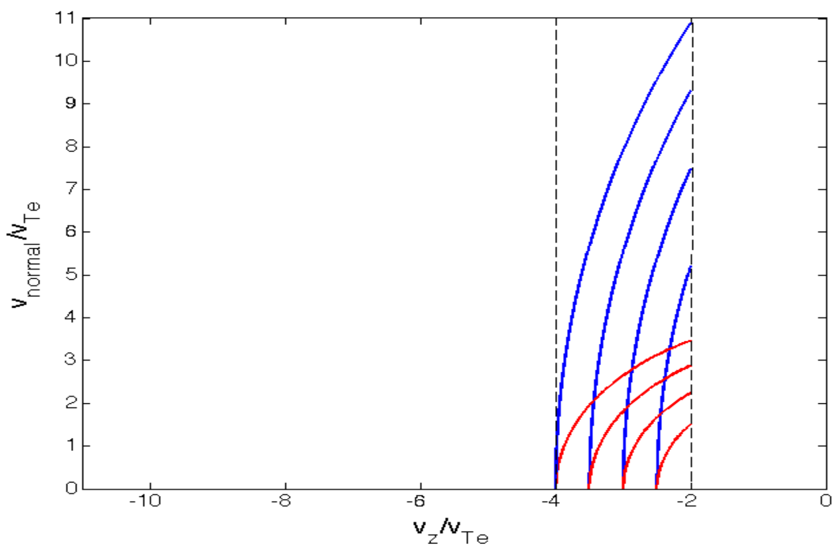


Fig. 4. Evolution of electron distribution function: 1) for Maxwell distribution (red) and 2) after quasilinear diffusion (blue).

Fig. 4 clearly shows that diffusion of electrons along the diffusion lines lead to considerable increase in their energy in the perpendicular to the geomagnetic field. Wave-particle interactions tend to form a “plateau” along diffusion lines given by equation (14), while collisions will tend to thermalize the resonant part of the distribution function, reorganizing particle motion between diffusion lines which results in enhanced transverse heating over the case when no collision are present. Influence of collisions on the quasilinear heating process will be analyzed in a future work.

IV. Conclusions

In this work we considered the problem of excitation of intense electromagnetic whistler waves in the ionosphere by two HF pump waves via beat wave mechanism. The necessary conditions for efficient wave excitation, when resonance conditions for wave frequencies and wave vectors can be satisfied, were determined. It was shown that excited whistler waves propagate almost parallel to the geomagnetic field. We also examined quasilinear electron cyclotron heating by the excited broadband spectrum of whistler waves with frequencies near the electron cyclotron frequency. It was demonstrated that quasilinear heating mechanism can strongly increase the perpendicular energy of electrons above the ionization energy of the neutral constituents in the ionosphere. These results are important for many applications in space and fusion plasmas, in particular in the ionospheric heating by transmitters launched into space.

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