Orbit Refinement for Doppler Removal using Observations from Multiple Frequencies, Multiple Ground Sites, and Multiple Overpasses

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August 1, 2022

ABSTRACT

Geometric phase shifts affect signals received from satellites. This paper addresses the problem of estimating and removing geometric phase. Our approach is based on calculating orbital range using orbit propagation software which depends on so called ”orbital elements” for inputs. We adjust the orbital elements to achieve the best base-banding and develop a time-domain metric to measure quality. We iteratively adjust orbital elements to optimize the metric. The novelty of our work is our inclusion of observations of signals received at multiple sites, at multiple frequencies, and on multiple overpasses. We show that estimating the orbital elements and predicting the satellite path on its next overpass are dramatically improved by exploiting geometric and temporal diversity available in multiple ground sites and multiple overpasses.

1. INTRODUCTION

Outer space around Earth is becoming increasingly important real-estate for commercial and military applications [9]. The trend to deploy smaller, less expensive satellites is increasing congestion in Earth's orbital space. Control of and communication with satellites in orbit relies upon the ability to accurately predict their trajectories. This is usually done using orbit propagation algorithms/software such as SGP4 [4]. To predict the path followed by a particular satellite, parameters called two-line elements or “TLEs” describing the orbit must be input to SGP4. These TLEs are computed and updated by the Joint Space Operations Center (JSOC), a component of the US Military. Due to the small size of some spacecraft (such as cubesats) and the increasing congestion in space, estimating TLEs is an increasingly error prone process. Thus there is a need to investigate techniques that can improve the quality of TLE estimates.

This paper proposes to estimate satellite TLE directly from observations accessible at ground sites. This reduces dependence on JSOC TLE, which are used here only as initial conditions. The method we offer estimates and re-estimates the TLE on each overpass, maintaining a low-error track over time. The current version of the method, does require a TLE estimate be provided to seed the search in the initialization phase, but it adapts its own estimate of the TLE over time. The method could be modified to perform an initial wide-area search over TLE space to find an initial TLE, but that study is beyond the scope of this paper.

The price paid for automatic maintenance of accurate TLE over time is that the satellite must transmit a known beacon signal. That requires hardware onboard the satellite (SWAP) and it consumes power, which can be real concerns in small satellites. The method for TLE estimate proposed in this paper is based on information contained in the Doppler shift exhibited in the received beacon signal observed at multiple ground sites over multiple overpasses.

We note that Riesing and Cahoy [8] propose a method for TLE prediction in which the current TLE is predicted in a batch least-squares sense from prior TLEs. TLEs are converted to position and velocity vectors. The prediction is carried out in an open-loop fashion in the parametric space of the TLEs. The estimation is not based on the physics of satellite motion. Our approach differs by incorporating direct observations of a Doppler shifted RF tone that encodes information about the true path of satellite motion during an overpass. By incorporating data from multiple overpasses and observations from multiple ground sites, we obtain low error estimates of TLE that produce orbital paths that agree with the actual path when viewed by the Doppler frequency produced.

2. DOPPLER ESTIMATION

Suppose an RF carrier signal $\cos(2\pi f_0 t)$ is transmitted from a satellite in orbit. The signal received at the ground station is

$$\cos\left(2\pi f_0 t + \frac{2\pi f_0 R(t)}{c} + \varphi(t)\right),$$

where $R(t)$ is the time varying range to the satellite, c is the speed of light, and $\varphi(t)$ is possible scintillation arising from refraction and scattering in the Ionosphere. This model changes in amplitude due to propagation and scintillation.

The addition of phase terms due to satellite range and scintillation can be seen as either a blessing or a curse. The introduction of range into the received phase makes global positioning possible. The introduction of scintillation opens the possibility for scientific exploration into the mechanisms at work in the Ionosphere. For the purpose of communication data links, however, both geometric (range-based) phase and scintillation are viewed as impairments to be compensated or mitigated. The geometric phase leads to a shift in the frequency (Doppler effect) of the received signal. To see this recall that the instantaneous frequency $f_i(t)$ of a sinusoidal signal is the time derivative of the total phase

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi}\frac{d}{dt}\left(2\pi f_0 t + \frac{2\pi f_0 R(t)}{c}\right) \\ &= f_0 + \frac{f_0}{c}\dot{R}(t), \end{aligned}$$

where $\dot{R}(t) = \frac{d}{dt}R(t)$ which is also called the range-rate. We see that the Doppler frequency shift arises from time-varying range. To get a sense of the magnitude of Doppler frequency, consider an example where orbital speed is $v = 7000$ m/s and a carrier frequency of $f_0 = 2.4$ GHz (S-band), the maximum Doppler shift is approximately $f_{d,\max} = f_0 v/c \approx 56$ kHz. Communication receivers often use phase locked loops (PLL) to synchronize with the phase in the received signal. PLLs are phase estimators. A PLL with sufficiently wide bandwith to pull in a 56 kHz Doppler offset comes at the cost of high variance phase estimation.

An alternative to PLL-based Doppler compensation is to calculate the time-varying phase due to changing range $R(t)$ during an overpass and use this to demodulate the Doppler offset. In practice, the phase of local oscillators is adjusted by the geometric phase to track the actual phase in the received signal. This approach was followed in [3] and the present paper may be viewed as an extension of that work.

The range $R(t)$ is deterministic and can be calculated. In the absence of maneuvers, satellites follow orbital trajectories that can be modeled mathematically and computed numerically using orbit propagation software. In this work, we used the SGP4 propagator [4] built into the Skyfield [6] Python library.

Orbit propagators require three things as inputs: (1) coordinates of the ground station where the signal is observed, (2) the time of observation, and (3) parameters called orbital elements that describe the satellite orbit. One widely used format for specifying orbital elements is the two-line element or TLE [1]. Given this information, the orbit propagator calculates estimates of the range $R(t)$ over the time window of an overpass. This range estimate can be incorporated into the signal processing involved in downconverting the received signal to baseband. We note that SGP4 produces several other outputs including range rate $\dot{R}(t)$, latitude, longitude, altitude, azimuth, and elevation.

This scheme seems reasonable and it often works quite well. However, the models used by orbit propagators cannot account for all the forces experienced by a satellite and the orbital elements or TLEs can have errors and grow stale over timescales as short as 24 hours. Thus, methods relying on orbit propagation and TLEs can produce erroneous range estimates. Let's look at an example. Three beacon signals (150, 400, and 1066 MHz) were received from a CERTO instrument aboard the Cassiope satellite. Spectrograms of the received signal shown in Fig. 1 present the expected Doppler frequency shift. SGP4 computed range $R(t)$ over the overpass was used to correct the geometric phase and bring the signals to baseband. Spectrograms $P(t, f)$ of basebanded signals show how energy is distributed over time and frequency. If the basebanding operation was perfect, the baseband spectrograms should show a spectral line running down the center of the plot at zero-frequency. The baseband spectrograms in Fig. **??** show that the basebanding operation is not perfect. Instead of a straight line down the middle of the spectrograms, each one exhibits a "wiggle". This indicates an error in the range estimate computed using SGP4 and the given TLEs. Up close inspection of the 150

MHz spectrogram in Fig. 3 illustrates how sever is the residual geometric phase/Doppler shift. Note also the presence of scintillation on this carrier manifesting itself as the "fuzziness" in the middle of the spectrogram. Clearly using SGP4+TLE reduces the amount of work that a PLL must do to compensate for frequency offset in the received signal, but the accuracy of this approach ultimately depends on the quality of the TLEs provided to the SGP4 orbit propagator.

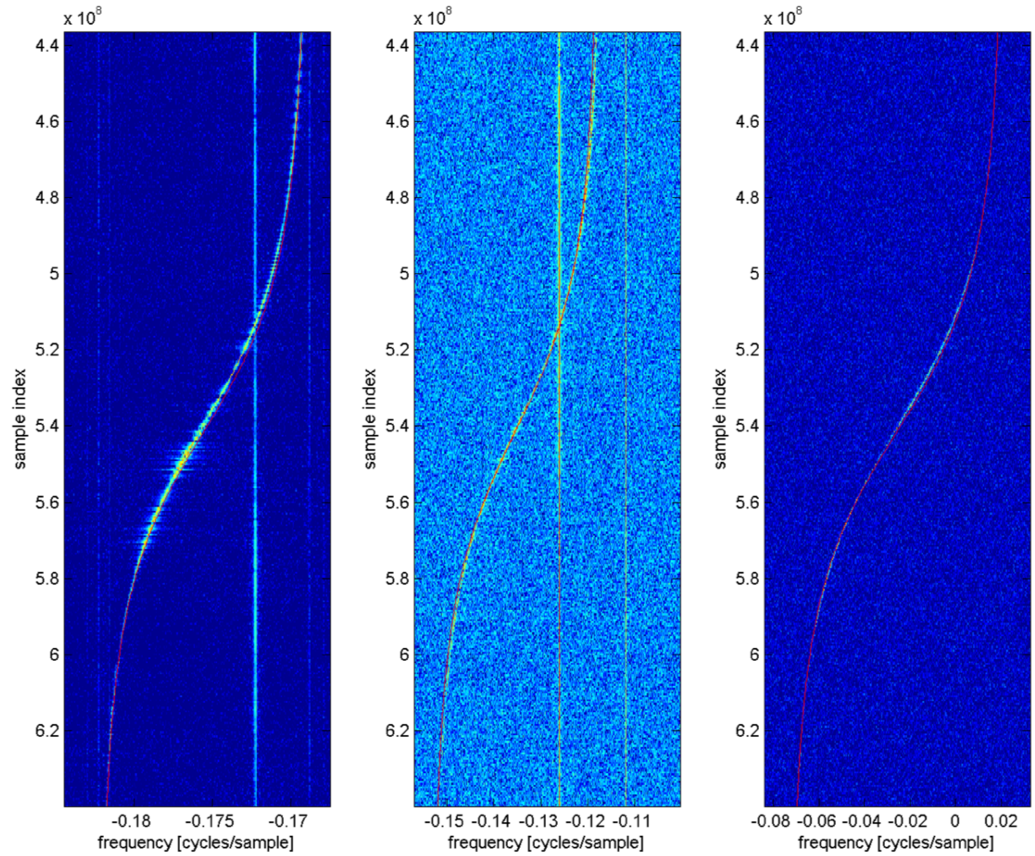

Fig. 1: Spectrograms of three beacons at 150, 400, and 1066 MHz. The actual Doppler frequency shift experienced by the received signal is observable. Also shown as the overlaid red line is Doppler frequency shift calculated using SGP4 and the TLEs available at the time of the overpass. The agreement between predicted and observed Doppler shift is good but imperfect.

What is needed is a method to improve the accuracy of range estimation. We build upon the existing architecture described above and the prior work in [3] that uses TLEs and orbit propagators. Our approach assumes that the orbit propagator is perfect and all the range error is due to errors in the input TLEs. We adjusts the TLEs to fit the predicted Doppler profile to the observed Doppler profile. What is novel about our work compared to [3] is that we incorporate observations from multiple overpasses and at multiple ground sites.

In [3] an adaptive scheme was presented which adjusts the TLE parameters to find an orbit which provides a good frequency match to the observed Doppler information. This was achieved by extracting frequency information using a chirp Z transform. Differences in frequency between the satellite signal and its SGP4-generated model were minimized using a Nelder-Mead minimization algorithm [2]. Nelder-Mead is a derivative-free descent method that uses only function evaluations. It only requires an initial starting point for its search and a box to search within. The initial value for TLEs is obtained from a NORAD database of TLEs [5].

The present work extends [3] to incorporate multiple beacon frequencies, observations from multiple ground sites, and observations from multiple overpasses. These additional observations lead to significantly improved estimation of

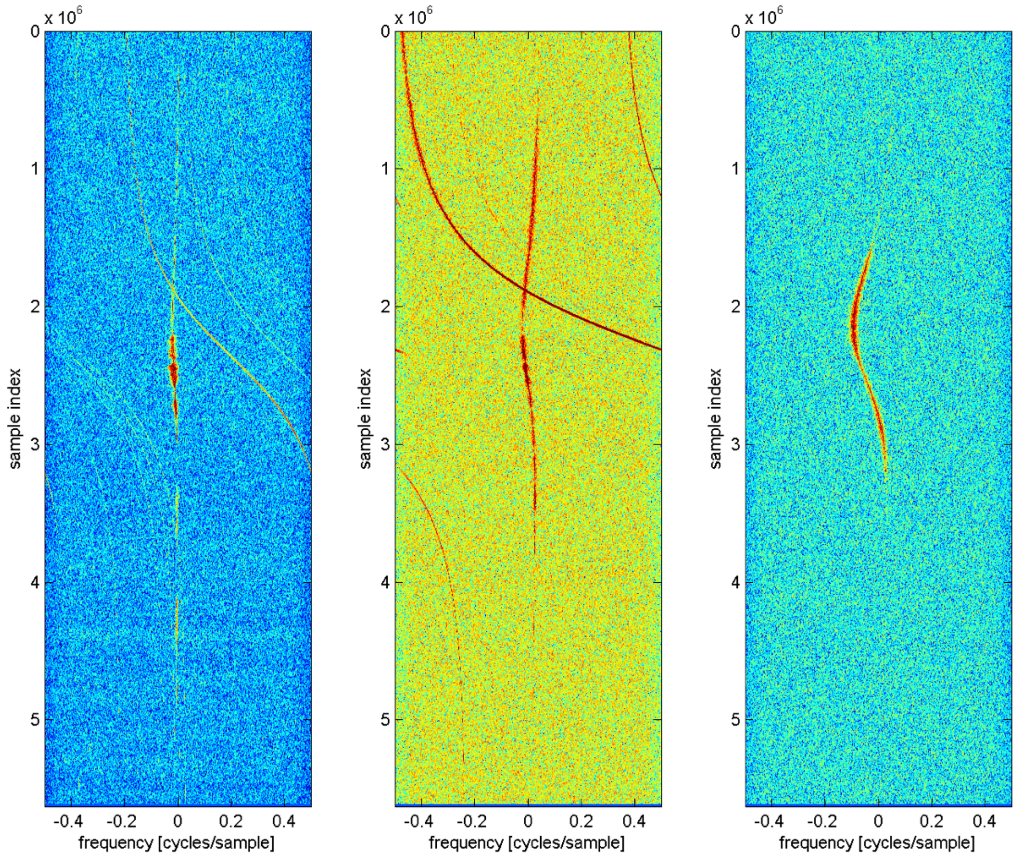

Fig. 2: Spectrograms of basebanded beacon signals from Fig. 1 are shown. The SGP4 predicted range is used to baseband the signals. Range error leaves a “wiggle” in the spectral energy over time.

TLE parameters resulting in improved compensation of Doppler due to satellite motion. A further extension is using a time-domain metric to measure the quality of the TLE estimates. This avoids the computational expense of frequency domain transformations employed in prior work, resulting in a more efficient algorithm.

A metric that measures the amount of range error is the following

$$\sum_t \sum_f |f| P(t,f).$$

This metric multiplies each timeslice of the spectrogram by the weighting function $|f|$. When all the energy in $P(t,f)$ is at zero frequency, the weighting function is zero, the product is zero, and the accumulated value of $|f|P(t,f)$ results in a zero value for the metric. If the spectrogram $P(t,f)$ presents any wiggle, energy away from zero frequency is magnified by the magnitude of frequency and leads to a positive value for the metric with the value increasing the greater is the residual frequency offset or range-rate error. Thus, the metric is sensitive to errors in range during an overpass. We use this metric evaluated on signals basebanded using SGP4-produced range $R(t)$ to measure the accuracy of the input TLEs. Our algorithm keeps adjusting the TLEs (orbital elements) until the metric is minimized. At this point, the predicted range matches as closely as possible using SGP4 the actual range to the satellite. By incorporating data from multiple ground sites (geometric diversity) and from multiple overpasses (temporal diversity), the accuracy of TLE estimates is improved.

In practice, all the spectrogram computations are less efficient. Therefore, we propose an alternative means to evaluate the same function using time-domain processing. This alternative approach recognizes the fact that the magnitude

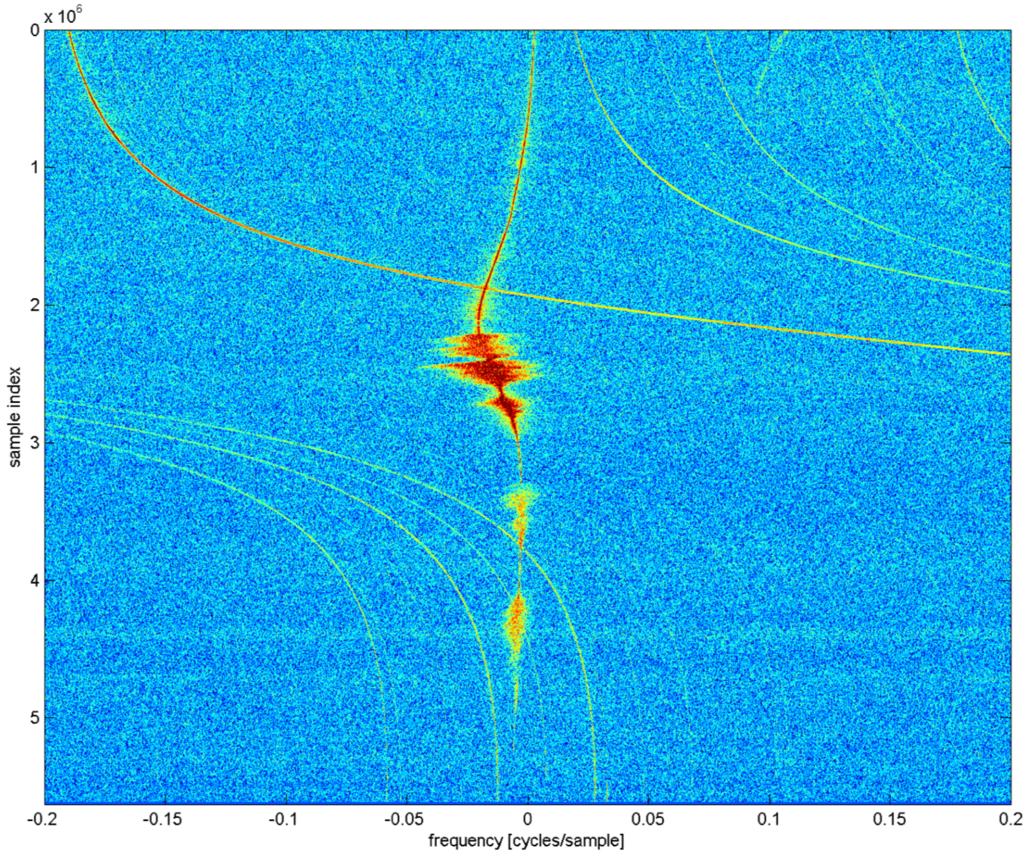

Fig. 3: Zooming in on the 150 MHz carrier at baseband illustrates the effect ("wiggle") arising from range error. Note that the "fuzziness" arises from scintillation.

response of a differentiation filter is $|f|$. Therefore, the metric may be evaluated by differentiating the received signals. We use a band-limited differentiation filter [7]. The metric we use passes the downconverted signal through a bandlimited differentiator and measures the output energy,

$$J_2 = \sum_n |\mathrm{DIFF}[s_{\mathrm{downconverted}}(n)]|^2$$

where DIFF represents discrete-time bandlimited differentiation A perfectly downconverted signal would provide a zero-frequency signal, i.e. a constant, so that the output of a differentiator would have no energy.

The advantage of the time-domain approach proposed here is that it avoids the cost of repeated Fourier transform (spectrogram) calculations and frequency domain peak search during the iterations of the algorithm. Because the proposed cost function (signal energy) is measured at the output of a differentiator (a time-domain filter), fast convolution may be applied. Also, since the objective function measures the energy at the output of the differentiator, Parseval's relation may be exploited to avoid the reconstruction of the time-domain signal because signal energy may be computed in the frequency domain after multiplying by the differentiator frequency response. This leads to a very efficient algorithm.

Figure 4 shows the basic architecture of our estimator. The current estimates of the TLEs are distributed to each ground site where they are used with an SGP4 orbit propagator to estimate the range to the satellite on each overpass in a set of overpasses. I/Q data at possibly multiple frequencies are demodulated to baseband and fed into the metric calculator. The fitness metric is applied at each beacon frequency and at each ground site location. The metrics, not the raw data, are communicated to a central location and combined there. Then the Nelder-Mead algorithm is used to adjust the TLEs to lower the value of the metric. Nelder-Mead is a derivative-free descent method that only needs function evaluations. Note that while the diagram illustrates that multiple carrier frequencies and multiple ground sites are involved in the optimization, the framework may include data recorded on multiple overpasses. This loop can be performed on recorded receiver data in an off-line manner to estimate optimal TLEs at the time the data were recorded. Or the loop can be iterated on streaming data and the TLEs improve over time and maintain accuracy over time.

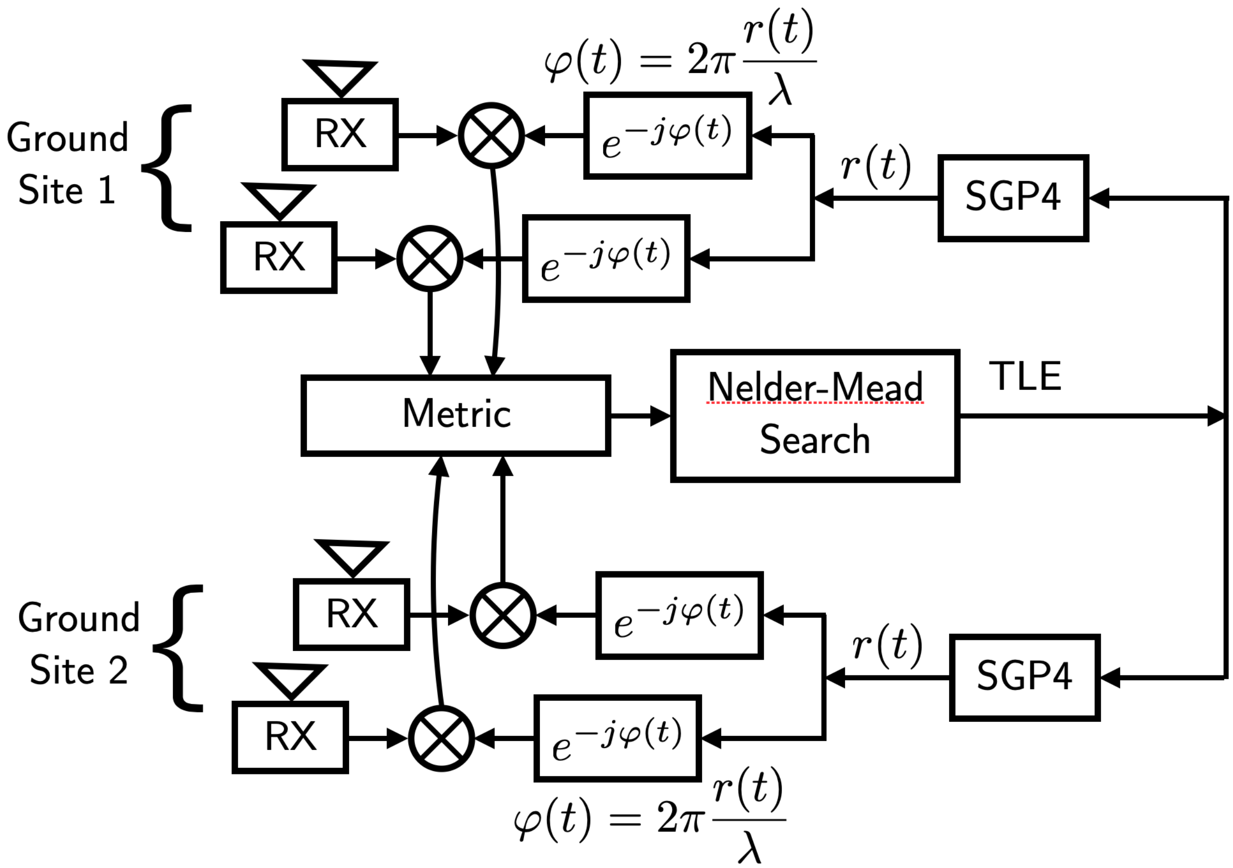

Fig. 4: Block diagram of orbit refinement system. This system adjusts orbital elements in the TLE to find a best fit to data observed during one or several satellite overpass. The system may include multiple ground sites and multiple receivers at each site. Each receiver detects a different beacon frequency.

3. RESULTS

To test the performance of the proposed method, we devised a scheme whereby the true satellite TLE would be known so that error may be computed. A satellite TLE is selected at random from a TLE catalog available from Celestrak [5]. This is considered to be the true TLE of the satellite. Error in TLE is simulated by perturbing the true TLE by drawing errors uniformly at random.

The proposed method produces an estimated TLE which should be close to the true TLE. The true and estimated orbital paths are obtained by propagating using SGP4 and the true and estimated TLEs. On an overpass of a given ground station, the range $R(t)$ between the satellite and the ground station may be computed. Three types of errors are computed to assess the quality of the estimated TLE. The first assesses the range accuracy of the estimated TLE. The range error is important because the range is directly proportional to the phase of the Doppler-shifted carrier signal. The derivative of range (range rate) is directly proportional to the Doppler frequency. It is the range and not the range rate that is used to downconvert the beacon carrier signal.

The mean-squared range error (MSRE) between the ground site and the true and estimated satellite paths is computed as

$$MSRE = \sum_t |\varepsilon(t)|^2$$
$$\varepsilon(t) = R_{\text{true}}(t) - R_{\text{estimated}}(t).$$

We compute the MSRE in two cases. The first is the MSRE for the current overpass for which thet TLE estimate is updated. The second is the MSRE for the next overpass for which no data has yet been observed. This is the "predicted" MSRE and is the important figure of merit for the proposed method because it is the error associated with the concept of operations of the method. As an example, the user would update the TLE on the current overpass and use the updated TLE to compute the time of the next overpass and to determine the azimuth and elevation angles for pointing a tracking antenna for the next overpass. The last error is simply the two-norm of the error between the true and estimated TLEs.

We used a series of ground sites spread out on land and islands around the Earth in the equatorial region. We performed simulations using 1, 3, 5, and 7 ground sites and using 1, 3, and 5 overpasses. This gave 12 different scenarios.

Figures 5 and 6 show the estimated (current) and predicted MSRE averaged over 30 different trials. The results are tabulated based on the number of ground sites and the number of overpasses used for the estimation/prediction. When data from a single overpass (the current overpass) is used to refine the TLE estimate, both estimation error and prediction error show a significant reduction (by orders of magnitude) as the number of ground stations increases. The same general trend is true when multiple overpasses are used. The TLE error, shown in Fig. 7 also shows dramatic improvement with the number of ground stations for the single overpass case. Both geometric diversity of multiple ground sites and temporal diversity of multiple overpasses appear to offer advantages for estimating TLEs. Range error approaching zero can be obtained as either the number of ground stations or the number of overpasses increase. By the time five ground sites or five overpasses is reached, the range error is essentially zero. The TLE error also approaches zero as the numbers of ground sites and overpasses increases.

Range Rate Error Estimation		Number of Ground Sites			
		1	3	5	7
Number of Overpasses	1	0.0010077	0.0000031	0.0000097	0.0000000
	3	0.0473306	0.0035210	0.0112312	0.0000000
	5	0.0000000	0.0079842	0.0000000	0.0000000

Fig. 5: Mean square range error for estimated TLE using data from the current overpass.

4. CONCLUSION

The orbit estimation method presented in this paper estimates the TLE parameters by fitting the SGP4 propagated range between a ground site or sites and the satellite on one or more overpasses. The geometric diversity provided by multiple ground sites and multiplie overpasses provide a dramatic improvement in ability to accurately predict range. This paper has demonstrated the superiority of multiple overpasses and/or multiple ground sites over the single overpass and single ground site case.

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Range Rate Error Prediction		Number of Ground Sites			
		1	3	5	7
Number of Overpasses	1	0.2473497	0.0000188	0.0000041	0.0000000
	3	0.0604092	0.0066289	0.0111890	0.0000000
	5	0.0000000	0.0049741	0.0000000	0.0000000

Fig. 6: Mean square range error computed for next overpass using TLE estimated from data on the current overpass.

TLE Parameter Error		Number of Ground Sites			
		1	3	5	7
Number of Overpasses	1	1.6560570	0.0002330	0.0001360	0.0000054
	3	1.1052699	0.1181236	0.5276010	0.0000070
	5	0.0000279	0.4801311	0.0000072	0.0000066

Fig. 7: Error between the true and estimated TLEs.

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