# **Comparing Traditional and Admissible-Region Schemes For Angles-Only Initial Orbit**

## Determination

Utkarsh R. Mishra Texas A&M University Suman Chakravorty Texas A&M University

> Islam I. Hussein Trusted Space

Weston Faber L3Harris Technologies

Siamak Hesar Kayhan Space Corporation

**Benjamin Sunderland** *Kayhan Space Corporation* 

### ABSTRACT

Probabilistic Admissible Region (PAR) is a technique to initialize the probability density function (pdf) of the states of a Resident Space Object (RSO). It combines apriori information about a few of the orbital elements, in the form of postulated statistics, and a single partial-state observation to initialize the pdf of the states of an RSO. This paper presents a comparison between Gauss's method IOD and PAR for angles-only observations and another comparison between PAR and Constrained Admissible Region (CAR) for angle and angle rates observation. It is shown that the PAR generates a more informative initial pdf which leads to better tracking performance downstream. Finally, solution to a multiple closely spaced RSO tracking problem with PAR initializations is presented.

## 1. INTRODUCTION

Initial Orbit Determination (IOD) techniques are a fundamental element of Space Domain Awareness (SDA) solutions. In the SDA context, IOD involves determining the initial state of a Resident Space Object (RSO). A single short-arc measurement from a space surveillance sensor like a radar or a telescope provides only partial-state information. A single partial-state measurement and even multiple closely spaced measurements are insufficient for initializing the state of a Resident Space Object (RSO).

In an ideal single object tracking scenario, IOD [7] schemes piece together multiple partial state measurements from a single object, over time, and try to fit a state vector. This method assumes that the series of observations came from the same Resident Space Object (RSO). But in the Multiple Object Tracking (MOT) scenarios, with multiple observations in each frame, there would be combinatorial growth in the possible 'hard' observation-to-observation (obs-to-obs) associations[6]. It is also considerably easier to mathematically evaluate the likelihood of an observation associated with a cataloged object compared to keeping track and evaluating the likelihood of associating each observation (among many) in one pass to each observation (among many) in a second pass.

A 'target' is defined as an RSO whose pdf is available in the space object catalog. The problem of finding the correct obs-to-obs association can be avoided if it can be posed as a target-to-observation (sometimes also called track-to-observation, or track-to-obs) association problem. This can be done by initializing the pdf of the states of the RSO



Space of postulated states

Fig. 1: Mapping samples from measurements and postulated states to the state space

that generated the observation, forward propagating the said pdf using the dynamics equations, and calculating the measurement likelihood of the measurements at the following passes. The initialization can be done using techniques like Constrained Admissible Region or Probabilistic Admissible Region algorithms.

The admissible region is the set of physically acceptable orbits that can be constrained even further if additional constraints on some orbital parameters like semi-major axis, eccentricity, etc, are present [10, 13, 14, 15]. This results in the Constrained Admissible Region (CAR) [4, 5, 3]. If hard constraints are replaced, based on known statistics of the measurement process, with a probabilistic representation of the admissible region, it results in the probabilistic admissible region (PAR) [9]. PAR can be used for orbit initiation in Bayesian tracking [8].

The three contributions of this work are as follows:

- Compare IOD and tracking performance for Gauss's method initialization and PGM filter recursive update versus PAR initialization and PGM filter recursive update.
- Present the use of PAR in a real Multi-Target Tracking scenario with heavy clutter.

## 2. METHODS FOR INITIALIZATION AND RECURSIVE UPDATE

## 2.1 Probabilistic Admissible Region (PAR)

The objective of PAR based algorithms is to get a probabilistic characterization of the uncertainty in the states given knowledge of the statistics of the measurement process and some statistics on the orbital parameters. The statistics on the orbital parameters could come from apriori knowledge, or be postulated based on the physics of the RSO population. In the angles-only case, the right ascension and declination ( $\alpha$ ,  $\delta$ ) observation and apriori knowledge of semi-major axis, a, eccentricity 2, inclination i, and right ascension of ascending node  $\Omega$  are used to characterize the PAR.

The pdf of the measurements can be denoted by  $p(\alpha, \delta)$ . The distribution over  $(a, e, i, \Omega)$ , with slight abuse of notation, be denoted by  $p(a, e, i, \Omega)$ . For simplicity and without loss of generality, assume that these parameters are independent of each other, and from  $p(\alpha, \delta)$ . The joint distribution in  $(a, e, i, \Omega, \alpha, \delta)$  can be written as:

$$p(a, e, i, \Omega, \alpha, \delta) = p(a)p(e)p(i)p(\Omega)p(\alpha, \delta)$$
(1)

Now, samples are drawn from the joint distribution over  $(a, e, i, \Omega, \alpha, \delta)$ . Each sample is then mapped to state space to get a particle cloud representing the initial uncertainty in the state. Extensive details can be found in

### 2.2 Constrained Admissible Region (CAR)

Constrained admissible regions (CAR) based track initialization in space object tracking involves the use of predefined constraints to determine the feasible region for initializing object tracks. This approach focuses on selecting initial track states that not only adhere to physical and geometric constraints but also satisfy admissibility criteria such as kinematic consistency and observability. By defining these constrained admissible regions, the track initialization process becomes more robust and reliable, enhancing the accuracy and efficiency of subsequent tracking algorithms. CAR-based track initialization facilitates better tracking performance by ensuring that the initial estimates are within feasible and meaningful regions of the state space, leading to improved overall tracking capabilities in the context of space object surveillance and monitoring.

#### 2.3 Gauss's method, Traditional Initial Orbit Determination

Gauss's method is employed for the preliminary determination of an orbit using a minimum of three observations, although additional observations enhance the accuracy of the determined orbit. The initial steps involve vector addition to compute the position vector of the orbiting body at each observation time. By applying the principles of angular momentum conservation and the Keplerian orbit model, which assumes the orbit lies in a two-dimensional plane within three-dimensional space, a linear combination of these position vectors is established. Additionally, the relationship between a body's position and velocity vectors, expressed through Lagrange coefficients, is utilized. Through vector manipulation and algebraic operations, a set of equations is derived to facilitate the subsequent computations and refinement of the orbit determination. For detailed derivation, refer to Curtis[2].

Particle Gaussian Mixture Filter has been used in the subsequent examples to update the pdf of the state with new measurements.

#### 2.4 Particle Gaussian Mixture Filter

Particle Gaussian Filter (PGM)[11, 12] employs an ensemble of possible state realizations for the propagation of the state probability density. A Gaussian mixture model (GMM) of the propagated uncertainty is then recovered by clustering the ensemble. Subsequently, the posterior density is obtained through a Kalman measurement update of the mixture modes.

## Algorithm 1 Particle Gaussian Mixture Filter

- 1: Given  $\pi_0(x_0) = \sum_{i=0}^{M(0)} w_i(0) \mathscr{G}_i(x_0; \mu_i(0), P_i(0))$ , transition density kernel  $p_n(x|x'), n = 1$
- 2: Sample  $N_p$  particles  $X^{(i)}$  from  $\pi_{n-1}$  and the transition kernel  $p_n(x|x')$  as follows: Sample  $X^{(i)'}$  from  $\pi_{n-1}(.)$ , Sample  $X^{(i)}$  from  $p(.|X^{(i)'})$ .
- 3: Use a clustering algorithm  $\mathscr{C}$  to cluster the set of particles  $X^{(i)}$  into  $M^{-}(n)$  Gaussian clusters with weights, mean and covariance given by  $\{w_i^{-}(n), \mu_i^{-}(n), P_i^{-}(n)\}$
- 4: Update the mixture weights and mixture means and covariance to  $\{w_i(n), \mu_i(n), P_i(n)\}$ , given the observation  $z_n$ , utilizing the Kalman update (Equations 2,3)
- 5: n = n + 1, go to Step 2.

Consider the measurement update, given that the prior component is Gaussian, and if the update is approximated using the Kalman/linear minimum mean squared error (LMMSE) update, we have:

$$\mu_i(n) = \mu_i^{-}(n) + P_{i,zx}^{T}(n)P_{i,zz}^{-1}(n)(z_n - E_i[h(X)]),$$
(2)

$$P_{i}(n) = P_{i}^{-}(n) - P_{i,zx}^{T}(n)P_{i,zx}^{-1}(n)P_{i,zx}(n),$$
(3)

where

$$P_{i,zx}(n) = E_i \Big[ \big( h(X) - E_i(h(X)) \big) \big( X - E_i(X) \big)^T \Big],$$
(4)

$$P_{i,zz}(n) = E_i \Big[ \big( h(X) - E_i(h(X)) \big) \big( h(X) - E_i(h(X)) \big)^T \Big],$$
(5)

#### 2.5 Nonlinear Batch Least Squares

Batch Least Squares aims to minimize the sum of squared differences between observed and predicted values by iteratively adjusting model parameters to find the best-fit solution. Algorithm 2 details the steps in Nonlinear Batch Least Squares Estimation[1].



## 3. ANGLES ONLY MEASUREMENTS



Fig. 2: Comparison between PAR and Gauss's method IOD followed by PGM filtering to process 11 measurements

### 3.1 Sensor Model

Modern SDA telescopes take measurements of the right ascension ( $\alpha$ ) and declination ( $\delta$ ) of the RSOs. For simulations presented in this paper, instead of this frame a coordinate frame whose axes are always parallel to the ECI reference frame, but is always centered on the sensor site, is being used on both the simulation (to generate measurements) and model (for measurement model that gets called by the filter). This has been done only for ease of implementation and reproduction of the results presented here and requires trivial modifications when using Local tangent plane coordinates like ENU. The measurement model can be written as follows:

$$\mathbf{y} = h(\mathbf{x}) + \mathbf{v} \tag{6}$$

$$h(\mathbf{x}) = [\boldsymbol{\alpha}, \boldsymbol{\delta}]^T \tag{7}$$

where  $\alpha$ , and  $\delta$  are a function of the range vector  $\rho$ , which can be found by:

$$\boldsymbol{\rho} = \mathbf{r} - \mathbf{q} \tag{8}$$

where  $\mathbf{r}$  is the inertial position vector of the RSO and  $\mathbf{q}$  is the inertial position vector of the sensor site.

$$\hat{\boldsymbol{\rho}} = [\cos(\alpha)\cos(\delta), \sin(\alpha)\cos(\delta), \sin(\delta)]^T$$
(9)

which gives:

$$\alpha = tan^{-1} \left( \frac{\hat{\rho}(2)}{\hat{\rho}(1)} \right) \tag{10}$$

$$\delta = tan^{-1} \left( \frac{\hat{\rho}(3)}{\hat{\rho}(1) \times (1/cos(\alpha))} \right)$$
(11)

Equation 11 is written in a somewhat unconventional form as a hint to use four quadrant inverse (atan2) when implementing these equations on a computer.

#### 3.2 Gauss's Method vs PAR for angles only observation

Figure 2 gives details of comparing PAR and Monte carlo over Gauss's method to process 11 angle only measurements. PAR used the first measurement to initialize the pdf of states and used the subsequent measurements to update the said pdf. In Gauss's method, the first three measurements were jittered based on the measurement error statistics, and 1000 triplets were made. Then 1000 Gauss's initializations were carried out to get an initial particle cloud. This was followed by recursive updates for the following eight-time steps using PGM.

It can be seen in Figure 2 that the final particle cloud for PAR is far more compact in both position and velocity space. This can be quantitatively seen in the entropy and 'volume' (the determinant of the covariance matrix) plots where the PAR + PGM combination clearly beats the Gauss's method + PGM in having a more informative initial as well as a more informative final pdf. Figure 3 shows the output of Gauss's initial particle cloud processed using the Batch Nonlinear Least Squares algorithm. The discontinuous nature and the extremely large size of the particle cloud shows that Gauss with Nonlinear Batch Least Squares is not a suitable candidate when the uncertainties in measurements are considered.

#### 4. MULTIPLE OBJECT TRACKING WITH PAR INITIALIZATION

PAR, due to its ability to do single-measurement initialization, seamlessly integrates into multiple object tracking filters like the Cardinalized Probability Hypothesis Density (CPHD) filter and variations of the Multi Hypothesis Tracking (MHT) filter. This is demonstrated with an example using real SDA telescope data. Eleven pictures of the AMAZONAS satellites in Geostationary orbit were taken from San Jose, California (37.3414, -121.6429). The measurement cadence, albeit not constant, was about 5 seconds. The RA/Dec are in the J2000 coordinate system and are the apparent RA/Dec from the telescope. These were derived from the background stars in the images using code from astrometry.net. The star positions are used to compute a conversion from pixel coordinates to RA/Dec. The RA/Dec coordinates of the objects in the image are then passed through the Multi-target algorithm and tracks are computed.

Very briefly, the following steps were followed:



Fig. 3: Particle cloud in position (left) and velocity (right) space after processing first eleven measurements. The first three measurements were used for IOD, and the following eight were used for nonlinear batch least squares.

- Frames were processed sequentially. In the first time step, sixty-five PAR pdfs were initialized using the following apriori statistics: *a* ~ *U*(41,000,45,000), *e* ~ *U*(0,0.1), *i* ~ *U*(0°,0.5°), Ω ~ *U*(0°,360°).
- The pdfs were forward propagated using simple two-body dynamics.
- A data association matrix, populated with the likelihood of associating an object (initialized at time step one) with a measurement at time step two. A cutoff (e-3) is enforced on the minimum likelihood to be considered.
- The top data association is found using Munkres algorithm on the log of the Data Association Matrix. The mean
  and covariance of objects are updated if they were associated with some observation with a non-zero likelihood.
- If an object consistently (> 3 time steps) does not associate with any measurements, it is culled.
- The propagate, associate, update cycle is repeated.



Fig. 4: Left: Ten frames of right ascension and declination measurements for AMAZONAS satellites in GEO. Right: Tracks that survived in Multi-target tracking filter after processing all the frames.

Right-hand side of Figure 5 shows the four tracks that are identified. Note that the only apriori information that was used was the very poor guesses on  $(a, e, i, \Omega)$ . No clustering scheme was used to identify clutter and real tracks. Figure 4 shows the 65 PAR particle clouds initialized after the very first time step. On the right hand side is the particle cloud



Fig. 5: Left: Particle clouds for sixty-five objects, initialized using PAR on each measurement in the first frame, in position space. Right: Particle cloud for one of the initialized objects in velocity space.



Fig. 6: Data Association Matrix at time step two.

for one of the objects in velocity space. Figure 6 shows the Data Association Matrix populated with the likelihood of associating a particular measurement (columns) with a target (rows).

### 5. CONCLUSION

A comparison between PAR and Gauss IOD was performed and it was shown that PAR produces a more compact initial pdf and PAR + PGM beats Gauss's method + PGM as well as CAR+PGM by giving a more informative posterior pdf.

Future work would focus on improving the clustering schemes used in PGM, coming up with analog of showcasing tracking performance by plotting errors in estimated states vs ground truth with +/- 3-sigma bounds for multi-modal PGM, and implementing more realistic multi-target tracking simulations.

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