Application of Electric Propulsion Maneuver Envelopes to Space Situational Awareness

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ABSTRACT

Many fundamental questions remain regarding algorithmic performance of reachable estimates for electric propulsion spacecraft in multi-revolution cases, Cartesian coordinate frames, and what additional information maneuver envelopes (an extension of reachable estimates) can provide. This paper explores these three topics using a combination of theoretical explanation and numerical simulations. We find that the algorithm performs well in multi-revolution cases but the thrust-to-weight ratio is a key driver of when the algorithm can "fail." Cartesian coordinates are, surprisingly, a good choice and performed well. Using maneuver estimates provides additional flexibility and information over reachability estimates at the expense of increased computational cost.

1. INTRODUCTION

We examine the performance of our reachability algorithm for various problems. In particular, we focus on reachability of electric propulsion spacecraft. We consider multi-revolution cases, the benefits and limitations of Cartesian coordinates, and finally the utility of maneuver envelopes.

Reachability is an important concept in astrodynamics and for a broad range of space missions. For spacecraft, the reachable set is defined by the maximal "distance" that can be obtained at a given time for a given initial condition[8]. Spacecraft reachability is difficult to estimate due to the complicated dynamics and the need to solve an optimal control problem for each point on the reachable set. Electric propulsion further complicates the problem due to the large number of controls[6]. Reachability estimation is an active area of research because it is a basic research problem that cuts across a wide range of areas from safety, space situational awareness (SSA), path planning, and others.

Reference [6] derived the reachability boundary conditions with Gaussian level set initial conditions, which supports SSA estimates. Reference [1] applied reachable sets to automate spacecraft custody and maneuver detection. This demonstrated the utility of reachability to SSA applications. References [1, 6] leveraged a continuous indirect formulation, which required solving a two-point boundary value problem[2]. The two-point boundary value problem generally requires a good initial guess, which can limit the ability to generalize this approach. Reachability is also used as a part of path planning problems[3, 4]. References [3, 4] use reachability analysis in their algorithms to solve for fuel optimal trajectories.

Reachability estimation is fundamental to many space problems. Recently, [10] developed a fast and automated reachability estimation algorithm that uses an indirect multi-stage formulation. This approach incorporated the Gaussian level set conditions found in [7] and introduced other potential boundary conditions. Reachable trajectories are also fuel-optimal trajectories. This was exploited in [11] by eliminating the distance-based cost function and using a fuel-based cost function. The result was an algorithm similar to [10], but one that recovered the maneuver envelope (fuel optimal trajectories and reachable set). The formulation provided unifying approach between fuel-optimal trajectories and reachable sets.

This paper explores three topics. The first is how well our reachable set algorithm[10] performs on multi-revolution trajectories. The second issue we explore is how well Cartesian coordinates perform and the tradeoffs with using them. Finally, we discuss the additional information gained by using maneuver envelopes[11]. The paper layout follows the issues we are investigating: multi-revolution cases, Cartesian formulation, and information gained through maneuver envelopes.

2. MULTI-REVOLUTION INVESTIGATION

Prior work[10] demonstrated good performance in estimating reachable sets for multi-body cases. However, the cases were limited to low revolution cases and the thrust-to-weight ratios were small. In this section, we examine both, the number of revolutions and the thrust-to-weight ratio, to better understand how they drive the results of the algorithm. Our analysis consists of fixing the initial conditions and changing the terminal time by a multiple of the orbital period. This allows us to observe how the algorithm performs for 1,2,3,..N revolutions. We further assume that there are 100 stages per revolution; therefore as the number of revolutions increases, so do the number of stages. Finally, we rerun this base case at different thrust levels. This approach allows us to see how number of revolutions (time) and thrust-to-weight ratio interact.

The Cartesian initial conditions we use are, $[-16371, 0, 0, 0, -4.19421, 0]$. The position coordinates are in km and velocity components are in km/s. The initial mass is equal to 1. The specific impulse is ∞. The last assumption ensures that propellant fuel consumption is 0, which helps us isolate the variables of interest. We then run this case for 1 to 16 revolutions in increments of 1 revolution. Fig. 1 shows that the algorithm is able to estimate the reachable set for the multi-revolution cases. As we increase the number of revolutions we see that the estimated reachable set remains bounded and the points are well distributed. The results in Fig. 1 illustrate that the performance of the algorithm is not simply driven by number of revolutions (or time) and that other factors play a critical role.

Our next step is to investigate how thrust levels effect the performance of the algorithm. We will use the initial conditions and revolution counts from Fig. 1 and increase the maximum thrust level of the spacecraft. The thrust values we use are $1E-6km/s^2$ and $1E-5km/s^2$ and the resulting trajectories are shown in Fig. 2 and Fig. 3, respectively. What we notice in these two figures is that the estimated reachable set becomes less well defined as it becomes larger. In particular, as the reachable set encapsulates the entire non-thrusting trajectory, we see that the ability of our algorithm to accurately map out the contours of the reachable set diminish. What these cases highlight is that the algorithm's performance is determined by a combination of duration and acceleration levels.

This raises the possibility that other coordinate systems may achieve better results. In additional testing we found that the rotating Hill Frame did not address this issue but shifting to a polar or spherical coordinate system can[5]. In discussions, the authors of [5] showed unpublished cases, which showed that for two body cases spherical coordinates alleviate this issues.

We also test multi-body interactions to help understand when the algorithm can fail in multi-body cases. During testing we observed that if the gravitational body is within the reachable set, then the algorithm fails to properly estimate the reachable set. Fig. 4 shows an Earth-centered trajectory with a lunar flyby. This is the same trajectory as shown in [10] but with a higher thrust level (0.1 N). The higher thrust level results in the reachable set encompassing the moon during the flyby. In Fig. 4 the black outline represents the first order estimate of the reachable set and the purple xs represent the non-converged results from attempting to identify the exact reachable set. These points are valid and dynamically consistent trajectories, but they do not necessarily represent trajectories on the reachable set. As we can see many of the purple xs (non-converged solution) are outside the estimated reachable set (dashed black line), which means that the estimate (dashed black line) is not valid. These two methods diverge because maximizing the reachability, in this case, involves passing through the gravitational center.

These simulations show that time combined with thrust-to-weight ratio are important factors that drive the performance of the algorithm. In particular, we see that the algorithm performs fairly well until the reachable set almost encompasses the reference trajectory or a gravitational center. Other conditions may also exist. However, at a minimum users of the algorithm from [10, 11] should cross-check the validity of the results if the reachable set is close to encapsulating the orbit or if the reachable set encompasses a gravitational center.

3. VALUE OF CARTESIAN COORDINATES

In this section we explore if our choice of Cartesian coordinates is a good/lucky choice and what are the possible alternatives. Cartesian coordinates were originally chosen [10] because they work well with the Jet Propulsion Lab's ephemerides[9]. This allows for straight forward implementation of multi-body dynamics used in Fig. 4 and [10].

We begin by reviewing the dynamical information the reachable set algorithm requires, how choice of coordinate frame influences computation, then considering use cases to better inform practitioners of the available trade space.

The reachable [10] and maneuver envelope [11] estimation algorithms require the dynamics, state transition matrix, and partials with respect to the controls. The state transition matrix and control partials are computed on the non-thrusting trajectory. For the purposes of these discussions, we limit ourselves to ideal two-body mechanics.

The state transition matrix, in Cartesian coordinates, is constantly changing over the trajectory and is not close to the identity matrix. For non-thrusting trajectories, the state transition matrix in classical orbital elements (or variants thereof) are close to the identity matrix. Usually, classical orbital elements and their variants involves only one state changing over time; thus, the state transition matrix is close to the identify matrix. Having a state transition matrix that is close to the identity matrix can be useful as it potentially simplifies calculations used to estimate reachable sets.

The second issue we need to explore is how the coordinate frame effects which use cases we can readily compute. This issue is important because the reachable sets that can be readily computed depend on the coordinate frame. This occurs because the algorithm sweeps over a subset of the co-states in order to map out the associated reachable set. Therefore, the coordinate frame and the use cases will be linked.

The first use cases we consider is positional reachability. In this use case we care about the maximal reach of the spacecraft and do not care about the velocity. This use case maps to SSA or conjunction analysis. In this case, we want to use current or prior spacecraft location information to predict where it might be in the future. In Cartesian coordinates, polar coordinates, or spherical coordinates[5] positional reachability is straightforward to compute as we can randomly draw the co-states associated with x, y, and z coordinates from the unit ball to generate the positional reachable set. The mapping between positions and classical orbital elements is highly nonlinear. This means we do not have a trivial method for directly computing the positional reachable set for classical orbital elements (or variants).

The second possible use case is orbital reachability, where we care about both position and velocity. This use case maps to orbit reconstruction or path planning. In these cases, we want to map out the maneuver set. In Cartesian coordinates this involves sweeping over the position and velocity co-states. For other coordinate systems we can similarly sweep over the non-mass co-states to generate the maneuver set. For this case, the reachable set should be invariant to choice of coordinate frame.

The choice of coordinate frame depends on the use case. If positional reachability is required, then a coordinate frame that has distinct position and velocity states is required. For these cases, Cartesian, spherical, or polar coordinates are a good choice. If only maneuver sets are needed then classical orbital elements or similar coordinates can potentially provide performance gains. If both cases may be required then Cartesian, spherical, or polar coordinates can simplify code development and testing.

4. INFORMATION GAINED IN MANEUVER ENVELOPES

We now explore possible ways that the additional information contained in maneuver sets can be used. Maneuver envelopes include the reachable set and non-reachable feasible trajectories. As part of the estimate of maneuver envelopes we also obtain the near-optimal control laws[11].

We begin by using the same initial conditions and thrust level as case 2 from [10]. We then use the algorithm from [11] which allows us to generate the maneuver envelope, shown in Fig. 5. In Fig. 5, the maneuver envelope is a series of concentric shells where each shell represents the positions that can be reached for a fixed fuel costs. The center represents zero fuel costs and the outer points are the reachable set, where the thruster is always on. Therefore, as we move from the center to the outer edge, the fuel consumption increases in a monotonic manner that can be readily extracted from the algorithm. This highlights that state observations can be used to estimate fuel consumption.

5. CONCLUSIONS

In this paper we examined how time (orbital revolutions) and thrust-to-weight ratios effect the performance of our reachable set algorithms. We see that multi-revolution cases are not a limitation in and of themselves. In numerical testing we saw that the algorithm failed when the reachable set encompassed the orbit, which was driven by the thrust to weight ratio and time. This suggests that reference frames with a coordinate that measures along track changes might be better suited for these cases. We will explore this in future work. In multi-body cases the algorithm can generate poor estimates of the reachable set when the set encompasses a gravitational center. We also find that Cartesian coordinates work well for many use cases and that other coordinate frames may offer benefits for special use cases but are less generalizable. Finally, the use of maneuver envelopes can allow users to estimate fuel usage based on spacecraft position/velocity observations.

6. REFERENCES

[1] Daniel Aguilar Marsillach and Marcus J Holzinger. Spacecraft custody maintenance and maneuver detection using robotic telescopes and reachable sets. *Journal of Guidance, Control, and Dynamics*, 44(4):667–683, 2021.

[2] Arthur E Bryson and Yu-Chi Ho. *Applied optimal control: optimization, estimation, and control*. Routledge, 2018.

[3] Qi Chen, Dong Qiao, and Changxuan Wen. Minimum-fuel low-thrust trajectory optimization via reachability analysis and convex programming. *Journal of Guidance, Control, and Dynamics*, 44(5):1036–1043, 2021.

[4] Christian M Chilan and Bruce A Conway. A reachable set analysis method for generating near-optimal trajectories of constrained multiphase systems. *Journal of Optimization Theory and Applications*, 167(1):161–194, 2015.

[5] Adam Evans, Claudio Bombardelli, Roberto Armellin, and Laura Pirovano. Analytical state transition matrix approach to low-thrust reachability computation. Number AAS 23-206. AAS/AIAA, August 2023.

[6] Marcus J Holzinger and Daniel J Scheeres. Reachability results for nonlinear systems with ellipsoidal initial sets. *IEEE transactions on aerospace and electronic systems*, 48(2):1583–1600, 2012.

[7] Marcus J Holzinger, Daniel J Scheeres, and R Scott Erwin. On-orbit operational range computation using gauss's variational equations with j 2 perturbations. *Journal of Guidance, Control, and Dynamics*, 37(2):608–622, 2014.

[8] Alexander B Kurzhanski and Pravin Varaiya. Dynamic optimization for reachability problems. *Journal of optimization theory and applications*, 108(2):227–251, 2001.

[9] Ryan S Park, William M Folkner, James G Williams, and Dale H Boggs. The jpl planetary and lunar ephemerides de440 and de441. *The Astronomical Journal*, 161(3):105, 2021.

[10] Prashant Patel and Daniel Scheeres. Rapidly and automatically estimating the reachability of electric propulsion spacecraft. In *Advanced Maui Optical and Space Surveillance (AMOS) Technologies Conference*, page 571, 2022.

[11] Prashant Patel and Daniel Scheeres. No initial guess required: Rapidly computing the feasible set of fuel-optional electric propulsion trajectories. Number AAS 23-114. AAS/AIAA, January 2023.

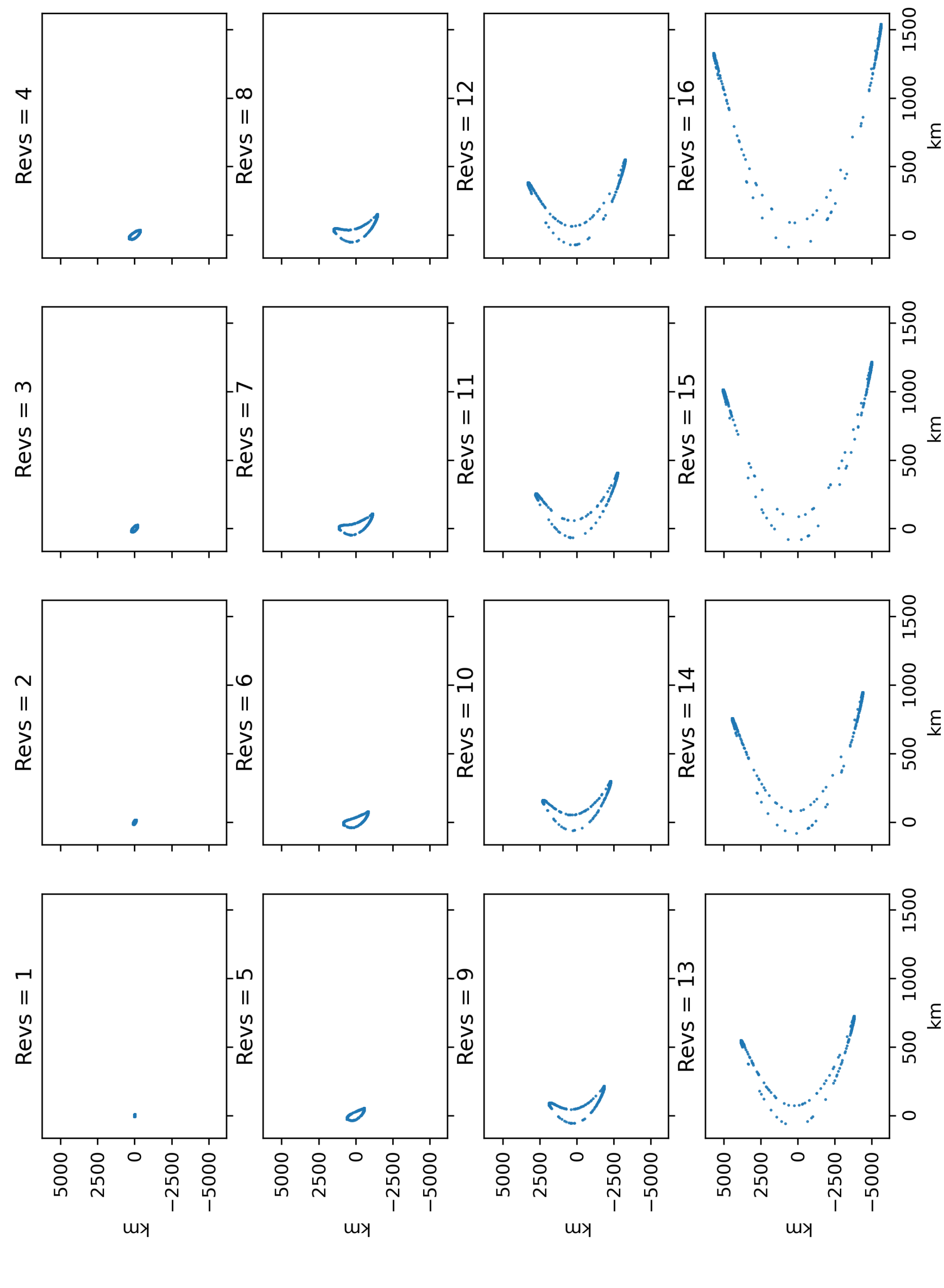

Fig. 1: This represents a ‘low thrust case’ with max acceleration = 1e-7 km/s^2, which is equivalent to 50 mN of thrust for a 500 kg spacecraft. The figure shows that over 16 revolutions the reachable set remains well defined and estimated by the algorithm.

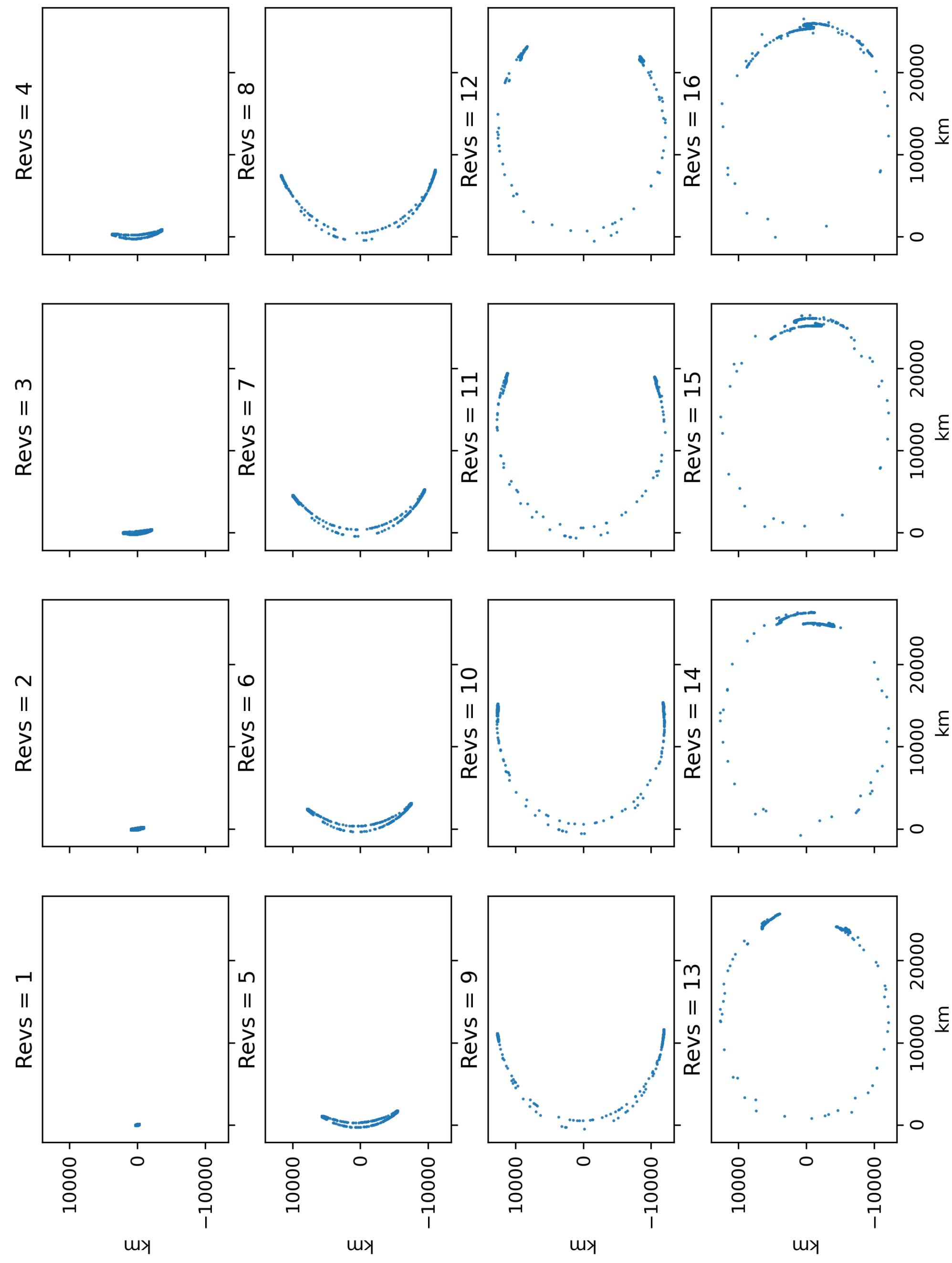

Fig. 2: This represents a 'medium thrust case' with max acceleration = 1e-6 km/s^2, which is equivalent to 500 mN of thrust for a 500 kg spacecraft. The figure shows that for 13-14 revolutions the reachable set remains well defined and estimated by the algorithm. On revolution 15 it becomes less well defined.

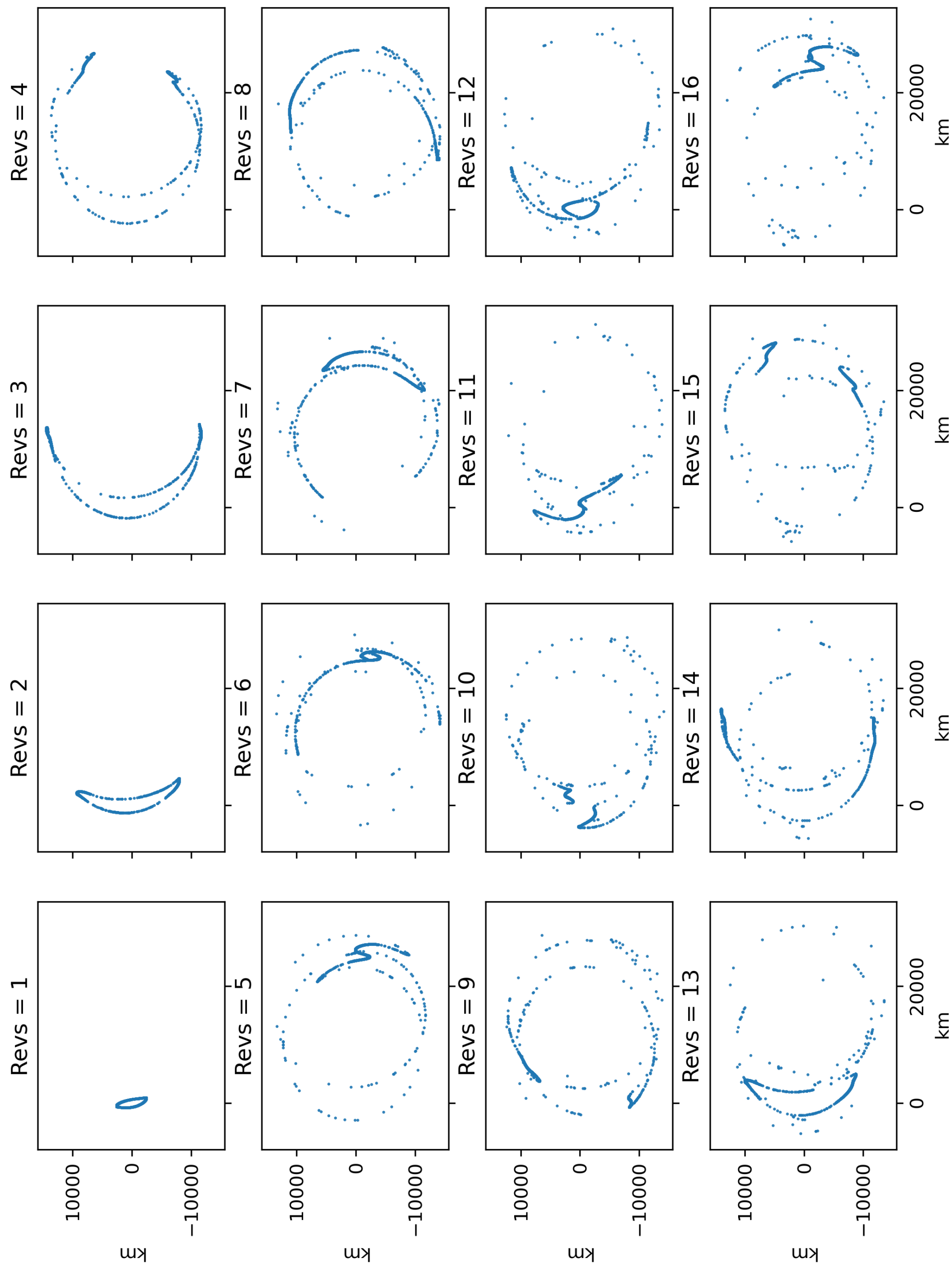

Fig. 3: This represents a 'medium thrust case' with max acceleration = 1e-5 km/s^2, which is equivalent to 5 N of thrust for a 500 kg spacecraft. The figure shows that for 4 revolutions the reachable set remains well defined and estimated by the algorithm. On revolution 4 it becomes less well defined.

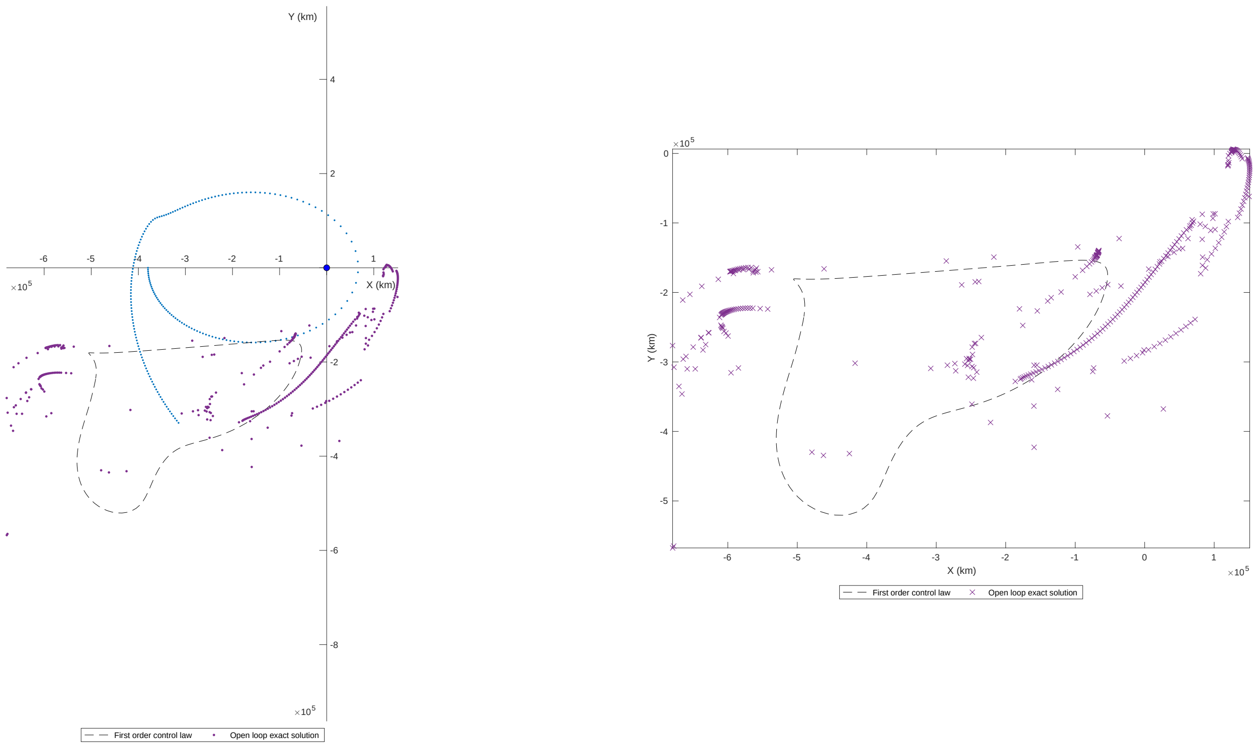

Fig. 4: Reachable set encompasses a gravitational body during transit which causes a subsequent failure in the algorithm.

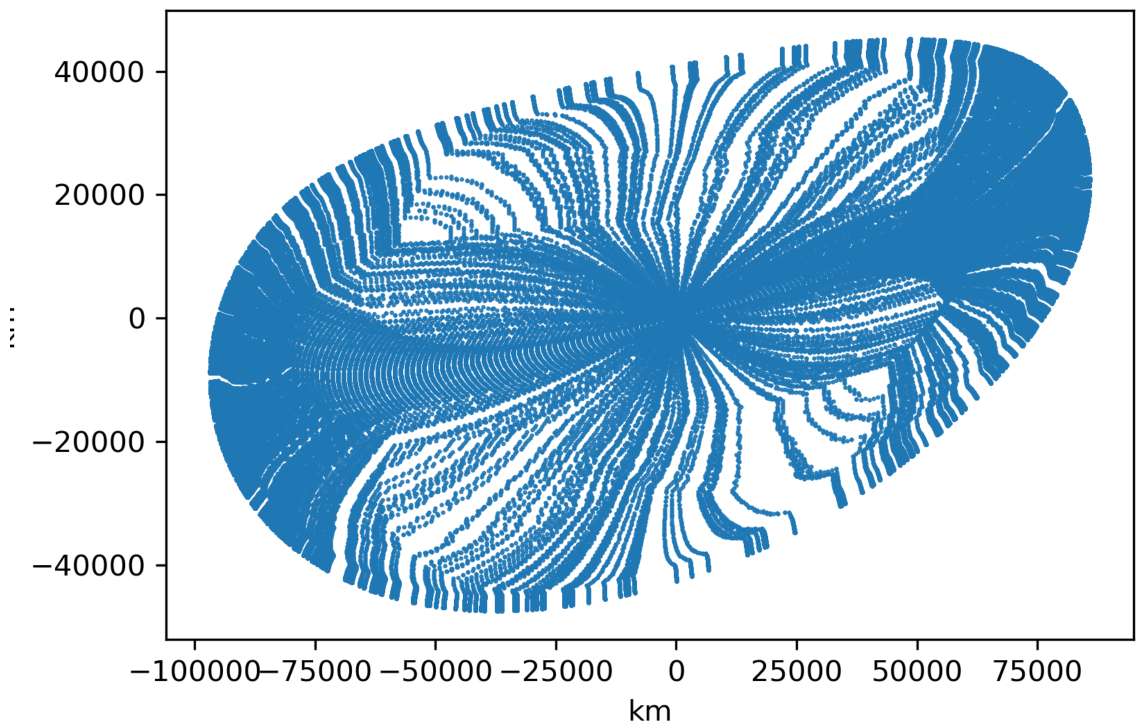

Fig. 5: Fuel optimal positional maneuver envelope.