# Optimal Risk Mitigation Strategies for Low-Thrust Space Systems

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### ABSTRACT

With the increasing number of objects in low-Earth orbit (LEO), there is a growing need for satellites of any size and thrust capability to be able to perform collision avoidance maneuvers when the likelihood of a collision event exceeds a predetermined threshold. We present SpaceNav's optimal risk mitigation maneuver planning strategy and how it has been expanded for low-thrust spacecraft. We start by reviewing the traditional MTS approach for finding maneuvers that mitigate the risk of collision to an acceptable threshold. This uses the information from processed conjunction data messages to find the maneuver that reduces the aggregate probability of collision as required and at the same time minimizes the $\Delta v$ or duration, while still meeting a set of mission-specific constraints, like thruster limits, geodetic maneuver location, and avoiding burns during eclipse or daylight. Satellites that are only capable of performing very low thrust maneuvers might not be able to mitigate the risk of collision with a single burn, and thus here we introduce how the standard trade space framework can be expanded to find maneuver pairs that together help reduce the risk. Finally, we also show how the maneuver optimization process can also be inverted in the case of satellites undergoing orbit raising; in this case, we find the optimal time and duration to stop maneuvering in order to decrease the risk due to a conjunction event that occurs during the nominal ascent plan.

In a low thrust dual maneuver test case, with a minimum single maneuver duration of 600 seconds and a maximum maneuver duration of 3600 seconds. Testing has shown that the risk can be reduced from $P_c = 8 \times 10^{-5}$ to $P_c = 8.05 \times 10^{-8}$. With two maneuvers, the first lasting 59 minutes and the second lasting 10.001 minutes. This aligns with the projected total required duration to mitigate the high-interest event (HIE) while still being able to meet individual maneuver duration constraints.

Testing has shown that the novel approach to risk mitigation during orbit raising can reduce the risk with minimum deviations from the nominal maneuver plan. In a test case with an ascent plan of four maneuvers per day for five days and MTS limits set to 24 hours before a high-interest event with $P_c = 8.25 \times 10^{-2}$, it was found that risk was reduced to $P_c < 6 \times 10^{-8}$ by trimming the original maneuver plan by just 10 minutes. This demonstrates the capability of mitigating collision risk during ascent through small changes to the original maneuver plan. Overall, we demonstrate that no matter when close approaches occur during the lifetime of a mission or what the thrust capability of a spacecraft is, there are many different techniques available to the SSA community to mitigate collision risk.

## 1. INTRODUCTION

A more common scenario every day for any satellite operator in LEO is to encounter a conjunction event with a secondary object that violates operational safety thresholds. Some events, especially those with times of closest approach (TCA) further in the future may drop below actionable thresholds as updated primary and secondary trajectories are made available. On the other hand, some conjunction events persist or even grow in concern as the time of closest approach nears. For these events a risk mitigation maneuver is required to ensure the safety of the mission.

To determine the most optimal risk mitigation maneuver (RMM), SpaceNav creates a maneuver trade space (MTS) that encapsulates all possible maneuver times and durations before the conjunction. Each grid point in the MTS represents a potential maneuver of a given duration starting at a certain time. An ephemeris is propagated for each possible maneuver and screened against all known active conjunction events in the planning window. These events are defined as distinct close approaches with secondary objects as obtained from processing conjunction data messages. For each propagated maneuver ephemeris and each close encounter, the probability of collision and relative geometry are recalculated by propagating the secondary object to the new time of closest approach.

Once a full trade space grid has been generated, we can proceed with the maneuver optimization process. The point with the lowest collision probability is first found through a global search, followed by a local search using the Nelder–Mead optimization algorithm. In descending relative weighting, the cost function uses miss distance, collision probability and maneuver magnitude to assign a cost to each point in the MTS. Maneuver times and durations in the MTS can be further limited based on additional constraints, such as burning during eclipse or daylight, as well as geodetic location and local time of day. Eclipse constraints can be of particular importance for satellites with limited power resources as they allow for control over whether the maneuver will take place when the Sun is visible or not. For large MTS time intervals, local time constraints are beneficial if the user wishes to specify burn times when more support staff would be available, such as maneuvering only during business hours at a particular location. To prevent service interruptions to ground users, satellite constellations may want to only maneuver over the oceans or only at certain latitudes and longitudes; this can be accomplished through geodetic location constraints.

In traditional maneuver planning, if no solution is found that mitigates the risk to an acceptable level, the trade space is expanded to consider a larger span of maneuver durations. This expansion process will continue until a satisfactory solution is found or the spacecraft thruster limits are reached. However, for low-thrust space systems, the maximum possible maneuver duration will not always be enough to reduce the collision risk. In this paper, we present an approach where maneuver pairs can be constructed to allow optimization to look for an RMM in which two separate maneuvers would be performed, allowing for a larger change in trajectory while still meeting the spacecraft operational constraints. Maneuver pairing allows for specification of minimum and maximum durations for both maneuvers individually, the ratio between maneuver durations, and the number of revolutions between the maneuvers.

Conjunction events can occur at any time during the life of a satellite. In this paper, we also show how we can extend the same MTS framework to work with satellites during low-thrust orbit raising; close approaches occurring during this time present an interesting inversion of the standard RMM approach where risk can be mitigated by *not* performing a section of the nominal ascent maneuver plan. The exact time and duration to stop maneuvering will be determined by using a modified version of the MTS approach, where in this case it will minimize changes from the nominal plan instead of the RMM duration or $\Delta v$ while still mitigating the collision risk. As mentioned, the presented methodology centers around finding an optimal time and duration to stop maneuvering. The optimization can be constrained based on user input such as minimum time before the time of closest approach to modify maneuvers, as well as a maximum acceptable $\Delta v$ change from the nominal maneuver plan. Planned maneuvers that satisfy the constraints are mapped into a shifted time domain, in which all planed maneuvers occur consecutively. From here, the trade space gets generated with user-defined step sizes and axes of shifted time *vs.* shifted stop duration, where the same global search and Nelder–Mead optimization algorithm can be applied.

## 2. RISK MITIGATION PLANNING FOR A SINGLE MANEUVER

The nominal case for most satellites when a risk mitigation maneuver is deemed necessary consists in planning a single maneuver. In SpaceNav's standard RMM planning process the risk mitigating maneuver can be modeled as either finite or impulsive, although for the purposes of this work it may be beneficial to think in terms of the finite case as it lends itself more naturally to our later discussion on expanding the typical MTS approach. MTS's are central to SpaceNav's risk mitigation planning procedures. Here we will review how the creation of a MTS is triggered, how the grid points of the MTS are selected, how the MTS grid points are evaluated, and how optimization is preformed.

### 2.1 Maneuver Trade Space Creation

The very start of the RMM planning process is to determine if a maneuver is necessary. SpaceNav determines if an action must be taken based on processing conjunction data messages (CDMs). The process is always triggered by individual CDMs, *not* conjunction events. Multiple CDMs can be provided for each event—these might come from different originators or have different ephemeris sources. For each maneuver plan, all possible secondary states for a given event will be evaluated, and then only the highest collision probability data source will be considered when computing the total risk.

When new conjunction data messages are received, these CDMs are filtered to exclude untrustworthy data. Examples of data that is deemed untrustworthy are events outside of the maneuver planning horizon, stale data based on the CDM creation date, and stale secondaries based on the last observation time. Additionally, certain sources for particular secondaries are excluded, e.g., CDMs that do not model Starlink or OneWeb satellites during ascent. The CDMs that are deemed trustworthy are then evaluated against mission maneuver planning trigger thresholds. The thresholds that are evaluated are: (a) collision probability above planning threshold, e.g. $P_c \geq 10^{-5}$, (b) miss distance below planning threshold, e.g., Miss $\leq 100$ m, (c) and if an event is within a defined RIC box around the primary, e.g., $[50,\ 100, 100]$ m. When one of the aforementioned maneuver planning thresholds is violated, the next step is to generate the MTS. The MTS is defined by a grid of discrete maneuver times by discrete maneuver sizes. Maneuver sizes can be defined by either maneuver duration in seconds for finite maneuver models or by $\Delta v$ in centimeters per second for impulsive maneuver models. An example MTS with aggregate $P_c$ contours overlaid can be seen in Figure 1.

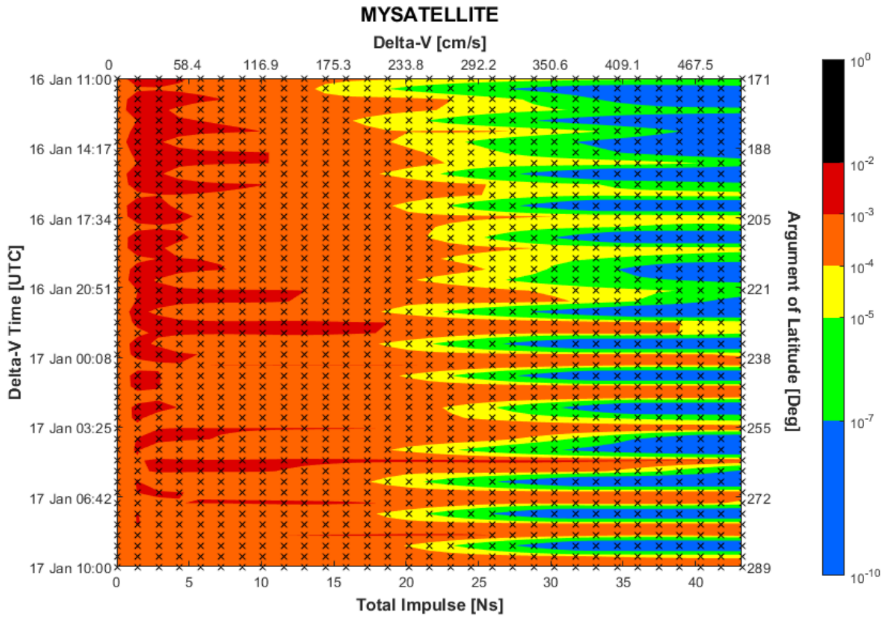



Fig. 1: Example MTS for impulsive maneuver with grid points shown and aggregate $P_c$ contours overlaid.

For each grid point in the MTS, a new primary trajectory modeling a different risk mitigation maneuver is propagated using high-fidelity numerical models. This new ephemeris is then screened against all known active conjunctions. Note that the secondary states with covariance from previously ingested CDMs are propagated to the new time of closest approach. Additionally, all of the latest, non-stale CDMs from each originator and secondary source are considered in the screening. As and example with CDMs from CSpOC, both the ASW and O/O CDMs are used if available. From the filtered CDMs, aggregate risk is computed. Aggregate collision probability is calculated using the highest $P_c$ CDM for each event. Aggregate miss is calculated using the minimum miss distance CDM for each event. SpaceNav evaluates aggregate risk and minimum miss instead of looking at individual events in order to avoid generating risk mitigation plans that induce additional risk by maneuvering into a third object. As an example of the importance of using aggregate $P_c$, we can imagine that risk mitigation planning was done using only the upper left $P_c$ contour shown in Figure 2. In this case, it would be reasonable to think that selecting an RMM that falls in the upper right hand corner of the MTS would be safe. However, upon looking at the aggregate collision probability contour, it becomes evident that a maneuver falling in the upper right hand corner of the MTS would be a poor choice as it induces a new high risk conjunction event.

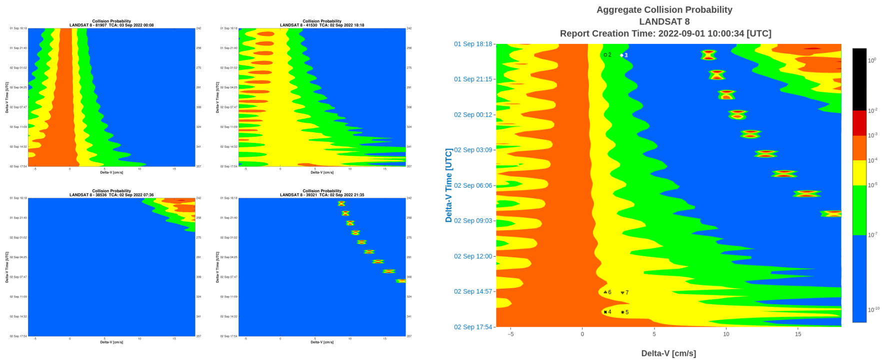



Fig. 2: Four individual $P_c$ contours and how they combine into a singular aggregate $P_c$ contour.

### 2.2 Optimization

Once the MTS grid points have been generated, propagated and evaluated for aggregate risk and miss, the next step is to find a optimal maneuver within the MTS. In order to find an optimal solution, first a cost function to optimize must be defined. The cost function that SpaceNav uses takes into account the risk of the maneuver, as well as the maneuver time and magnitude. The goal of SpaceNav's cost function is to reduce the risk to an acceptable mission threshold while at the same time minimizing the size of the maneuver. Once the cost has been computed for each grid point in the MTS, three seeds are selected and the Nelder–Mead optimization algorithm is used to find a minimum cost maneuver.

SpaceNav's cost function, Equation (1), has been designed to penalize large maneuvers and trajectories that exceed the configurable target aggregate $P_c$ and Miss thresholds. For finite maneuvers, the equivalent $\Delta v$ is used for the purposes of optimization.

$$J = |\Delta v| + \alpha_1 \left| \min\left( \log_{10}(P_{c_{\text{target}}}) - \log_{10}(P_{c_{\text{maneuver}}}),\ 0 \right) \right| + \alpha_2 \left| \max\left( \text{Miss}_{\text{target}} - \text{Miss}_{\text{maneuver}}, 0 \right) + \sum_{i=1}^{n_c} J_{c_i} \right. \quad (1)$$

where $J_{c_i}$ are the maneuver time and magnitude constraints, and $\alpha_1$, $\alpha_2$ are weighting constants. For long duration maneuvers, time constraints are based on when the maneuver ends, i.e, maneuvers must be completed by a certain time. When using the maneuver stop time to evaluate timing constraints, a non-rectangular MTS is created. Instead of manually selecting the trade space times for optimization, these are often selected based on physical constraints. Supported constraints in SpaceNav's *Risk Mitigation Service* are the time offset or number of revolutions from the TCA of the HIE, fixed maneuver time, sunlight or eclipse constraints, specific location (latitude/longitude), and local time, i.e., business hours. Cost function constraints $J_{c_i}$ are weighted such that minimums cannot occur in an area where the constraint is violated if there exists a point in the MTS where all time and magnitude constraints are met.

In order to find the minimum cost point in the MTS a global search of all MTS grid points is preformed to find the minimum $P_c$ grid point. This grid point is then fed into SpaceNav's seed selection algorithm to generate three seeds that will be used as the initial simplex in the Nelder–Mead optimization algorithm. The Nelder–Mead optimization algorithm is a direct search optimization algorithm that is well-suited for nonlinear problems. First an initial simplex (2D triangle) is selected from the MTS grid, each simplex vertices has its cost evaluated and the highest cost vertex is reflected through the midpoint of the two other vertices. Depending on the relative value of the cost at this reflected point and the initial three vertices the simplex is evolved through either a reflection, extension, contraction, or shrink. SpaceNav evaluates convergence based on the minimum cost vertex or the relative change in simplex size after each iteration. The Nelder–Mead algorithm is well suited for this problem as it does not require the computation of cost function gradients and the number of function evaluations required is small, relative to other optimization algorithms. This reduction in the number of cost function evaluations is of importance since for each evaluation a new maneuver plan must be generated and propagated. An example of how the Nelder–Mead algorithm finds minimums can be seen in Figure 3; observe how the simplex continues to get smaller and lower in the average cost across all vertices. For some contrived functions the Nelder–Mead algorithm can fail to converge, however this is extraordinarily unlikely to occur in practical applications. See [1] for a more in depth explanation of the intricacies of the Nelder–Mead algorithm.

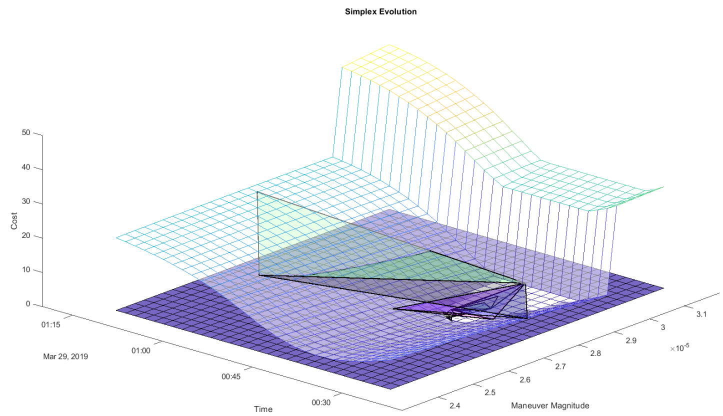



Fig. 3: Example of Nelder–Mead simplex evolution towards minimum cost value. Differently colored triangles represent subsequent simplex iterations. The cost function can be seen as mesh grid, note the large jump in cost function as maneuver magnitude increases, reflects where the magnitude constraint is violated.

## 3. RISK MITIGATION PLANNING FOR A DUAL MANEUVER

The goal of this extension of the MTS approach is to find optimal maneuver pairs when a maximal single maneuver does not meet operational safety thresholds. An example case can be seen in Figure 4, where unconstrained optimization finds a solution that mitigates the risk of a future HIE but violates the maximum maneuver duration. Dual maneuver planning provides a optimal combination of two maneuvers which satisfy maneuver size constraints and are separated by a set number of revolutions. There are many ways in which to search for an optimal dual maneuver, here we will outline SpaceNav's general approach to dual maneuvers and then highlight the configurations that SpaceNav supports.

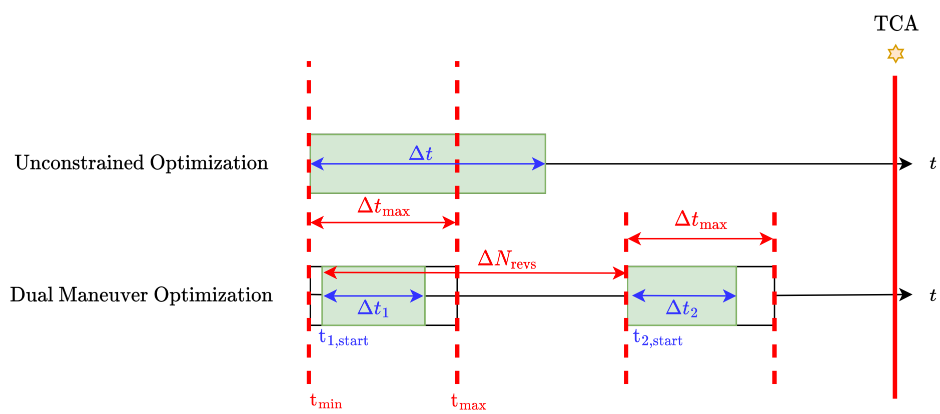



Fig. 4: Singular RMM that violates maneuver size constraints compared to a potential dual RMM plan where both maneuvers meet maneuver size constraints. Note that variables in red are constraints while variables in blue are the outputs of the optimization process.

### 3.1 Dual Maneuver Trade Space Creation

The MTS as used by SpaceNav is subject to the constraint that there are only two axes to optimize over. In the nominal case, these are maneuver time and maneuver size. Problematically, for the dual maneuver case, there are four variables that must be selected. As seen in Figure 4, optimization must find the start time and duration of both the first and second maneuvers. Through the incorporation of additional constraints and basic linear algebra, we can reduce these four variables to two. We will focus on the dual maneuver case here, but it should be noted that this approach can easily be extended to an arbitrary number of maneuvers.

The first step in creating the dual MTS is to introduce a new constraint that defines the number of revolutions between planned maneuvers. This constraint can be seen in Figure 4 as $\Delta N_{\text{revs}}$. By fixing the number of revolutions between maneuvers, all time variables can be defined by the start time of the first maneuver and the number of revolutions until the next maneuver, this relationship can be seen in Equation (2). Observe that by changing the start time of the first maneuver, the start time of the second maneuver will also change. What remains constant is the number of revolutions between maneuvers.

$$t_{2,\text{ start}} = t_{1,\text{ start}} + \Delta N_{\text{revs}} \tag{2}$$

As the start time of the first maneuver and the constraint on the number of revolutions between maneuvers fully defines all maneuver start times, the start time of the first maneuver will be used as one axis in the MTS. Next, a maneuver duration axis must be defined that can allow for the duration of maneuver one and two to vary as desired. Let $\mathbb{A} = \{\text{All linear combinations of } t_{1,\text{start}},\, \Delta t_1,\, \Delta t_2\}$; this represents any point in the space defined by the axes shown in Figure 5. Let $\mathbb{D}$ be a subset of $\mathbb{A}$ such that all timing and magnitude constraints are satisfied. In Figure 5, $\mathbb{D}$ is represented as all points within the red dashed box.

$$\mathbb{D} \subset \mathbb{A} = \left\{ (t_{1,\text{ start}}, \Delta t_1, \Delta t_2) \,\middle|\, \begin{array}{l} t_{\min} \leq t_{1,\text{ start}} \leq t_{\max} - \Delta t_{\min} \\ \Delta t_{\min} \leq \Delta t_1 \leq \Delta t_{\max} \\ \Delta t_{\min} \leq \Delta t_2 \leq \Delta t_{\max} \end{array} \right\} \tag{3}$$

By imposing the duration and magnitude constraints, the problem space, $\mathbb{A}$, is greatly reduced. However, even the subset of the problem space that satisfies all constraints, $\mathbb{D}$, still does not result in a two-dimensional space that can be used as the MTS. It is clear by inspection of Figure 5 that $\mathbb{D}$ is three-dimensional. In order to resolve this we will define two additional constraints that will create a two-dimensional subset of $\mathbb{D}$ and will set the maneuver magnitude combinations that will be considered. The first additional constraint will be called $\beta$ and it defines an additional offset in duration from $\Delta t_{\min}$ for the first maneuver. The second additional constraint, called $\vec{v}$, is a unit vector that lies in the $\Delta t_1$–$\Delta t_2$ plane, with an origin located at $(\Delta t_{\min} + \beta,\ \Delta t_{\min})$. The points in $\mathbb{D}$ that fall within the span of $\vec{v}$ and $t_{1,\text{ start}}$ will comprise the MTS. Different definitions for $\vec{v}$ can be used to change the type of dual maneuver combinations that are considered. In Figure 5, $\vec{v}$ is defined as the unit vector pointing from the minimum magnitude constraint to the maximum magnitude constraint.

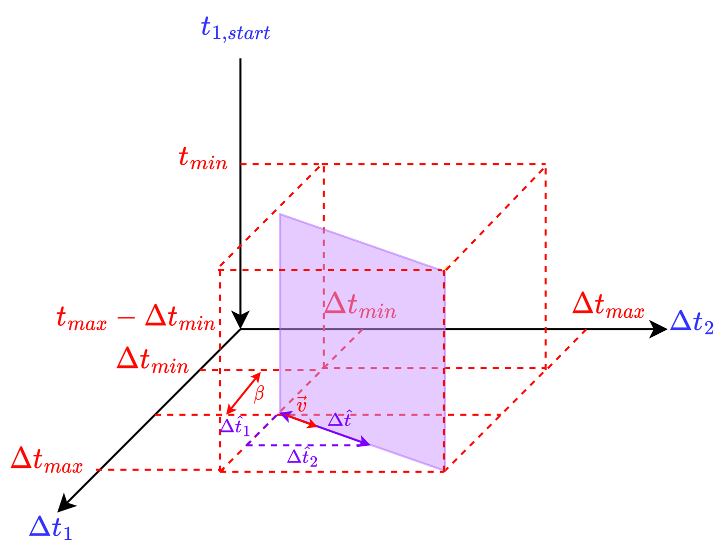



Fig. 5: Representation of all possible dual maneuvers that satisfy maneuver timing and magnitude constraints (red box) and a potential dual maneuver MTS using the maximum duration configuration (purple rhombus).

When evaluating points in the MTS they must be mapped back to real values for $t_{1,\text{ start}}$, $t_{2,\text{ start}}$, $\Delta t_1$ and $\Delta t_2$. The calculation for $t_{1,\text{ start}}$, $t_{2,\text{ start}}$ is defined by Equation (2), calculating $\Delta t_1$ and $\Delta t_2$ can be done by adding up the components in each direction.

$$\Delta t_1 = \Delta t_{min} + \beta + \hat{t}_1 \tag{4}$$

$$\Delta t_2 = \Delta t_{min} + \hat{t}_2 \tag{5}$$

As $\Delta t_{min}$ and $\beta$ are hard set, they can easily be accounted for. On the other hand the projections of the MTS duration value onto the maneuver duration axes, $\hat{t}_1$ and $\hat{t}_2$, will vary as $\vec{v}$ changes. In order to simplify this calculation the MTS will be shifted to be centered on the origin and hence make the span of $\vec{v}$ and $t_{1,\text{ start}}$ a vector space, this shifted space can be seen in Figure 6. In this shifted space simple scalar projection can be used to calculate $\Delta\hat{t}_1$ and $\Delta\hat{t}_2$.

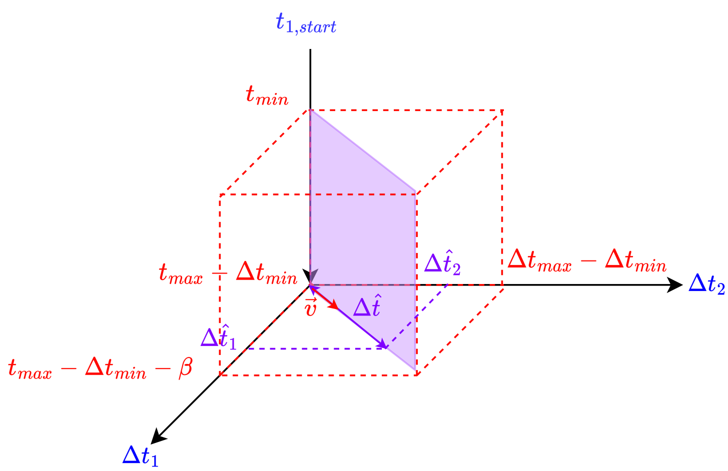



Fig. 6: Representation of all possible dual maneuvers that satisfy maneuver timing and magnitude constraints after being shifted such that $\vec{v}$ is centered on the origin (red box).

By defining maneuver constraints in the shifted space as depicted in Figure 6, we can formally define the MTS and its mapping to real maneuver plans. Let $\mathbb{B}$ be the standard basis for a three-dimensional space. Let the first, second, and third dimension correspond to $\Delta t_1$, $\Delta t_2$, and $t_{1,\text{ start}}$ respectively, i.e.,

$$\mathbb{B} = \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \tag{6}$$

.

The problem space, $\mathbb{P}$, which encompasses all maneuver times and magnitudes can be defined as the span of $\mathbb{B}$,

$$\mathbb{P} = \text{span}(\mathbb{B}) \tag{7}$$

.

Finally, let $\vec{v}$ be a unit vector which lies entirely in the span of $\mathbb{B}(1)$ and $\mathbb{B}(2)$,

$$\vec{v} = \begin{bmatrix}c_1\\c_2\\0\end{bmatrix}, \quad \|\vec{v}\| = 1, \quad c_1,\ c_2 \in \mathbb{R}^+ \tag{8}$$

.

The MTS, $\mathbb{M}$, will be the points in the span of $\vec{v}$ and $\mathbb{B}(3)$ that satisfy all maneuver timing and magnitude constraints:

$$\mathbb{M} \subset \mathbb{P} = \left\{ \text{span}(\mathbb{B}(3), \vec{v}) \;\middle|\; \begin{array}{l} t_{\text{min}} \leq t_{1,\text{ start}} \leq t_{\text{max}} - \Delta t_{\text{min}} \\ 0 \leq \Delta\hat{t}_1 = \text{Sproj}_{\mathbb{B}(1)} \left\| \Delta\hat{t} \right\| \vec{v} \leq \Delta t_{\text{max}} - \Delta t_{\text{min}} - \beta \\ 0 \leq \Delta\hat{t}_2 = \text{Sproj}_{\mathbb{B}(2)} \left\| \Delta\hat{t} \right\| \vec{v} \leq \Delta t_{\text{max}} - \Delta t_{\text{min}} \end{array} \right\} \tag{9}$$

where $\hat{t}$ corresponds to the maneuver duration in the MTS and $\text{Sproj}_{\vec{x}}\vec{y}$ corresponds to the scalar projection of $\vec{y}$ onto $\vec{x}$.

$$\text{Sproj}_{\vec{x}}\vec{y} = \frac{\vec{y} \cdot \vec{x}}{||\vec{x}||} \tag{10}$$

To go from the MTS magnitude value $\Delta\hat{t}$ to individual maneuver durations we simply have to project $\Delta\hat{t}$ onto the axis in question and add back in the minimum maneuver constraint for that axis.

$$\Delta t_1 = \text{Sproj}_{\mathbb{B}(1)} \left\| \Delta\hat{t} \right\| \vec{v} + \beta + \Delta t_{\text{min}} = \frac{\left\| \Delta\hat{t} \right\| \vec{v} \cdot \mathbb{B}(1)}{\left\| \mathbb{B}(1) \right\|} + \beta + \Delta t_{\text{min}} = \left\| \Delta\hat{t} \right\| \begin{bmatrix} c_1 \\ c_2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \beta + \Delta t_{\text{min}} = c_1 \left\| \Delta\hat{t} \right\| + \beta + \Delta t_{\text{min}} \tag{11}$$

$$\Delta t_2 = \text{Sproj}_{\mathbb{B}(2)} \left\| \Delta\hat{t} \right\| \vec{v} + \Delta t_{\text{min}} = \frac{\left\| \Delta\hat{t} \right\| \vec{v} \cdot \mathbb{B}(2)}{\left\| \mathbb{B}(2) \right\|} + \Delta t_{\text{min}} = \left\| \Delta\hat{t} \right\| \begin{bmatrix} c_1 \\ c_2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \Delta t_{\text{min}} = c_2 \left\| \Delta\hat{t} \right\| + \Delta t_{\text{min}} \tag{12}$$

With the MTS now properly defined and the mapping to real maneuver plans established, the next problem to solve is finding the minimum and maximum maneuver magnitudes, $\Delta\hat{t}_{\text{min}}$ and $\Delta\hat{t}_{\text{max}}$, that define the MTS. By definition of the MTS, we know that it is centered on the origin, as such we can say $\Delta\hat{t}_{\text{min}} = 0$. Because $\vec{v}$ plays a role in defining the MTS it must be taken into account when finding $\Delta\hat{t}_{\text{max}}$. Note that in Figure 7 which constraint is violated first can differ depending on the definition of $\vec{v}$. Additionally, observe that $\Delta\hat{t}_{\text{max}}$ will always be equal to the minimum value of $\Delta\hat{t}$ that violates a constraint. Continuing with this idea, to solve for $\Delta\hat{t}_{\text{max}}$, we can calculate the value of $\Delta\hat{t}$ that violates each constraint and then select the minimum $\Delta\hat{t}$.

$$\Delta\hat{t}_1 = \Delta t_{\text{max}} - \Delta t_{\text{min}} - \beta = c_1 \left\| \Delta\hat{t} \right\|, \quad \left\| \Delta\hat{t} \right\| = \frac{\Delta t_{\text{max}} - \Delta t_{\text{min}} - \beta}{c_1} \tag{13}$$

$$\Delta\hat{t}_2 = \Delta t_{\text{max}} - \Delta t_{\text{min}} = c_2 \left\| \Delta\hat{t} \right\|, \quad \left\| \Delta\hat{t} \right\| = \frac{\Delta t_{\text{max}} - \Delta t_{\text{min}}}{c_2} \tag{14}$$

$$\Delta\hat{t}_{\text{max}} = \min\left( \frac{\Delta t_{\text{max}} - \Delta t_{\text{min}} - \beta}{c_1}, \frac{\Delta t_{\text{max}} - \Delta t_{\text{min}}}{c_2} \right) \tag{15}$$

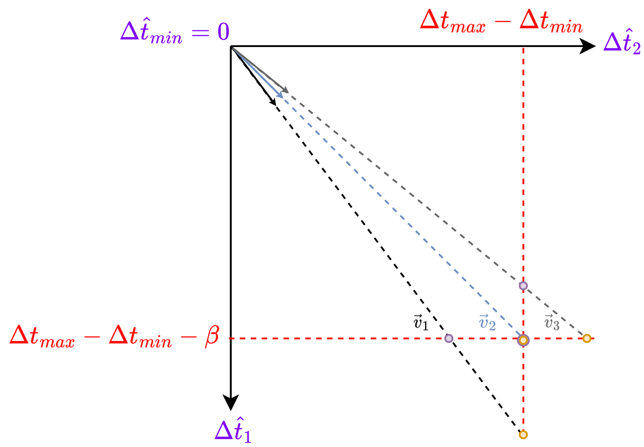



Fig. 7: Representation of different definitions for $\vec{v}$ can change the maximum acceptable magnitude for $\Delta\hat{t}$. Note $\Delta\hat{t}$ shown as purple dot.

The MTS duration grid points are then selected as the linearly spaced points from $\Delta\hat{t}_{\text{min}}$ to $\Delta\hat{t}_{\text{max}}$. Additional points may be added to ensure that both the minimum and maximum values exist within the trade space. Similarly, the MTS maneuver times are selected as linearly spaced times from $t_{\text{min}}$ to $t_{\text{max}} - \Delta t_{\text{min}}$, with end points included.

It is clear that the selection of $\vec{v}$ will change the MTS selected, and therefore will change the maneuver pairs that are considered. One methodology for selecting $\vec{v}$ is to ensure it points towards the maximum value constraint. This can be seen as $\vec{v}_2$ in Figure 7; by setting $\vec{v}$ in this manner it ensures that the minimum and maximum maneuver pair plan is in the MTS. Alternatively, if a particular maneuver ratio is desired, $\vec{v}$ can easily be selected by inputting the desired ratio and dividing by the magnitude. As an example, one might want to do 85 % of the total burn duration in the first maneuver and 15 % in the second maneuver. This could be achieved by specifying, $\vec{v} = \frac{[.85,\ .15]}{\|[.85,\ .15]\|}$.

A final configuration for the MTS that should be considered is how to expand from the standard single maneuver MTS to a dual maneuver MTS. In this case the standard MTS will be run as described in section §2, with the distinction that there is no expansion limits on the maneuver size. The maneuver magnitude constraint is passed in as a single maneuver magnitude constraint. When the MTS contains a maneuver duration and size that satisfies operational thresholds, the maneuver size is examined to determine if it can be accomplished in a single maneuver. If the required maneuver size is not achievable with a single maneuver, a dual MTS will be generated with a fixed first maneuver size and a variable second maneuver size. This is achieved by setting $\vec{v} = [0,\ 1]$ and solving for the desired first maneuver size according to Equation (16). Where $\text{Mag}_{\text{total}}$ represents the minimum magnitude value that meets operational constraints, found in the initial run of the nominal MTS. $C \in [0,\ 1]$, represents a new additional input that determines how close to the maximum possible value the first maneuver magnitude is set. An example of how this MTS would look can be seen in Figure 8.

$$\text{Mag}_1 = \min\left(\text{Mag}_{\text{total}} - (2 - C(\Delta t_{\text{min}})), \Delta t_{\text{max}} - \Delta t_{\text{min}}\right) \tag{16}$$

$$\beta = \text{Mag}_1 - \Delta t_{\text{min}} \tag{17}$$

By solving for $\text{Mag}_1$ in this manner, it can be ensured that the second maneuver is no larger than is required to meet operation thresholds. As an example, if the single maneuver MTS finds a burn duration of 3800 seconds is required to mitigate the risk and the single maneuver duration constraint is 400 s to 3600 s. It would not be optimal to select the first maneuver duration as 3600 s because that would set the minimum total duration across both maneuvers in the MTS to be 4000 s. A better selection for the first maneuver size would be $3400\,\text{s} = 3800\,\text{s} - 400\,\text{s}$, ensuring that the minimum total duration across both maneuvers is equal to 3800 s. Furthermore, because the dual maneuver ephemeris and the single maneuver ephemeris will differ, it is possible that a dual maneuver combination, of total duration less than 3800 s, could mitigate the risk. In order to account for this possibility, the $C$ parameter in Equation (16) was introduced. By setting this to a value less than one, the minimum total duration in the dual maneuver MTS, can be reduced. Continuing with the current example if we choose $C = 0.90$ and plug into Equation (16) we get, $\text{Mag}_1 = 3800 - (1.1 \times 400) = 3360\,\text{s}$.

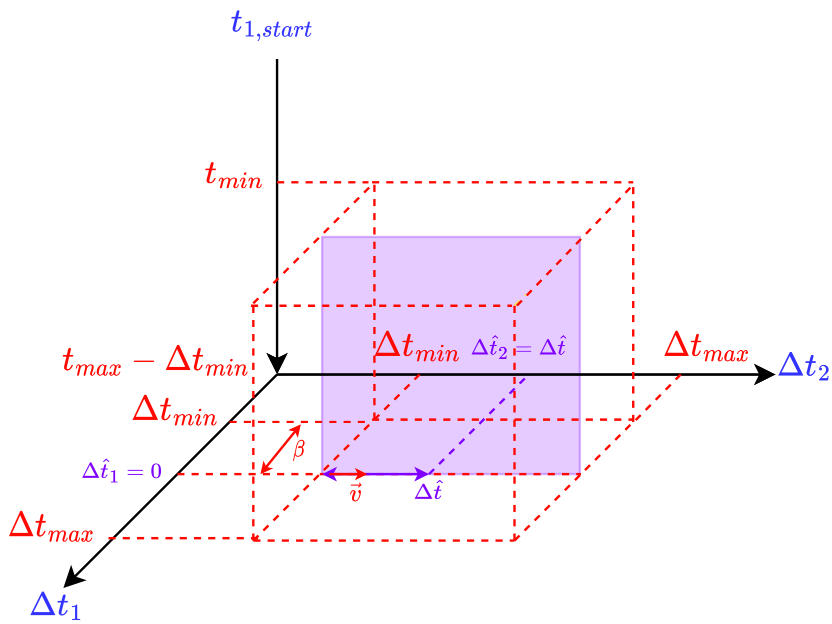



Fig. 8: Example of MTS selection when going from single maneuver MTS to dual maneuver MTS. Observe that $\Delta t_1$ will be constant in this case and $\Delta t_2$ will vary as $\Delta \hat{t}$ changes.

### 3.2 Dual Maneuver Optimization

The dual maneuver MTS creation has been completed after propagation and evaluation of aggregate miss and $P_c$ for each grid point. The next step is to use the Nelder–Mead optimization process, as described in section §2, to find an optimal solution. In order to use the Nelder–Mead optimization algorithm, a cost must be assigned to each grid point. The cost function used for the dual maneuver MTS is identical to the cost function described in section §2, with the key distinction that the maneuvers are evaluated individually and the total cost is equal to the sum of the individual maneuver costs. Note that when the second maneuver is evaluated, it is assigned an infinite miss value and a zero $P_c$ value. This is done in order to not double count these values in the cost function. If this was not done, the weight associated with the miss and $P_c$ component of the cost function would effectively double.

$$J = J_1 + J_2 \tag{18}$$

$$J_1 = |\Delta v_1| + \alpha_1 \left| \min\left( \log_{10}(P_{c_{\text{target}}}) - \log_{10}(P_{c_{\text{dual maneuver}}}),\ 0 \right) \right| + \alpha_2 \left| \max\left( \text{Miss}_{\text{target}} - \text{Miss}_{\text{dual maneuver}}, 0 \right) \right. + \sum_{i=1}^{n_c} J1_{c_i} \tag{19}$$

$$J_2 = |\Delta v_2| + \sum_{i=1}^{n_c} J2_{c_i} \tag{20}$$

## 4. RISK MITIGATION PLANNING DURING ORBIT RAISE

The goal of this extension of the MTS approach is to modify (trim) a nominal ascent maneuver plan to mitigate risk while keeping as much of the original plan as possible. The mechanism for how this optimal modification will be found is to vary what we will call a *stopping maneuver*. A stopping maneuver represents the section of the nominal maneuver plan that is *not* preformed. An example of a stopping maneuver and how it modifies the ascent maneuver plan can be seen in Figure 9, where the top row shows a nominal ascent maneuver plan with $n$ maneuvers and the bottom row shows one possible modification to the ascent maneuver plan with the section shown in red representing the stopping maneuver, i.e., the section of the original maneuver plan that will *not* be preformed. Additionally, note that *stop time* will refer to the start of the stopping maneuver and represents the first time in the nominal maneuver plan that is modified. The *resume time* will be used to denote the end of the stopping maneuver and represents the time that the nominal maneuver plan is resumed. The *resume time* minus the *stop time* is equal to the *stop duration* shown as $\Delta t_{\text{stop}}$ in Figure 9.

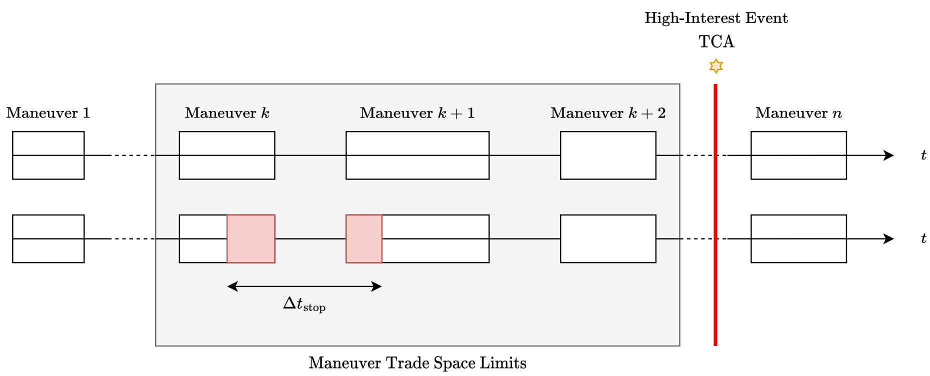



Fig. 9: Nominal ascent maneuver plan (top) and modified maneuver plan (bottom).

### 4.1 Finite Stop Maneuver Trade Space Creation

The *finite stop MTS* will be the MTS that encompasses all stopping maneuvers to be considered and optimized over. The axes of the *real time* finite stop MTS are stop time and stop duration. By varying these two values, the section of the nominal maneuver plan that the stopping maneuver trims is changed. In order to find an optimal stopping maneuver the same approach shown in section §2 can be used, however some care must be taken with how we set up the MTS to ensure each point is unique and that the Nelder–Mead optimization algorithm can be applied. To avoid these unwanted repetitive areas a *shifted* finite stop MTS is created.

The inputs that are required to define the finite stop MTS are minimum maneuver time, maximum maneuver time, maneuver time step, minimum maneuver duration, and maximum maneuver duration. One may note that these inputs are very similar to the nominal MTS case, however the interpretation of these values and how they create a finite stop MTS is different. To highlight these differences we will now walk through an illustrative example of how the finite stop MTS is generated.

Consider the ascent maneuver plan shown in Figure 9, which has $n$ maneuvers with $k$, $k+1$, $k+2$ being the final three planned maneuvers before the TCA of an HIE. These three maneuvers fall in the gray box, which is determined by the minimum maneuver time and the maximum maneuver time. Any planned maneuvers occurring within this window can potentially be trimmed. There are many ways one might select these minimum and maximum maneuver times; a sensible approach is to set them as a minimum and maximum time before an HIE.

For this example we will choose minimum and maximum maneuver times as shown in Figure 9 such that maneuvers $k$, $k+1$, $k+2$ are selected to generate the MTS. To clarify, these three maneuvers are used to generate the finite stop MTS, which represents all possible modifications to the nominal maneuver plan that will be considered. When the modified maneuver plan is evaluated, the entirety of the maneuver plan is still propagated.

Moving forward, we will refer to maneuvers $k$, $k+1$, $k+2$ as maneuver one, two, and three three, respectively. In Figure 10, maneuver one, two, and three can be seen on the right hand side—note how they align with the time to stop planned maneuvering. Observe that the blue hatched areas of the real time finite stop MTS are areas of the MTS that represent stopping times and durations that result in repetitive modified maneuver plans. The horizontal blue hatched areas represent stopping maneuvers that fall in between planned maneuver times and hence do not change the nominal maneuver plan. Diagonal blue hatched areas represent stopping maneuvers that result in the same section of the maneuver plan being trimmed. The red hatched areas represent stopping times and durations that have resume times after the end of maneuver three and hence are not allowed based on the definition of the MTS.

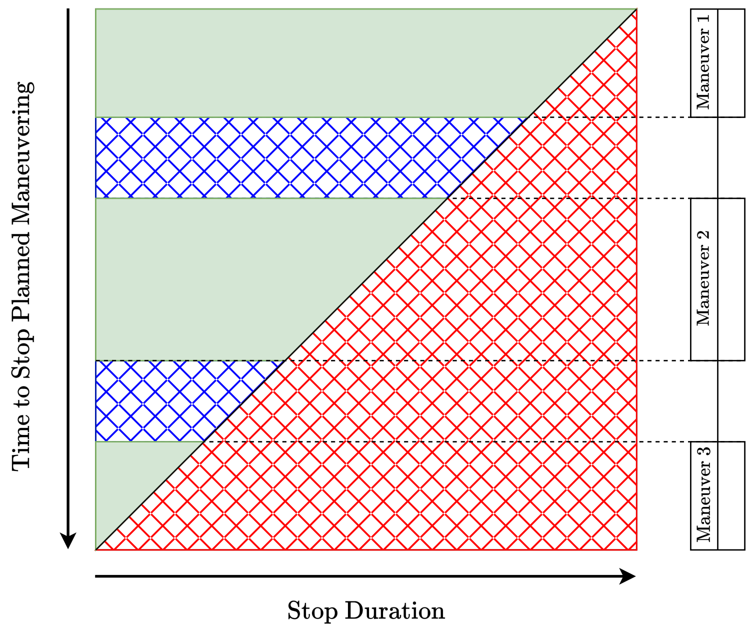



Fig. 10: Real time scale finite stop MTS.

Figure 11 shows how two distinct stopping maneuvers can result in the same modification to the maneuver plan. Observe that although $\Delta t_{\text{stop}_\text{A}}$ and $\Delta t_{\text{stop}_\text{B}}$ are of different durations because $\Delta t_{stop_B}$ extends into the time between planned maneuvers, it does not result in any additional trimming of the nominal ascent maneuver plan. In order for the optimization process to function as intended, it is required that each point in the MTS represent a unique solution.; as such, these repetitive stop times and stop durations prove problematic.

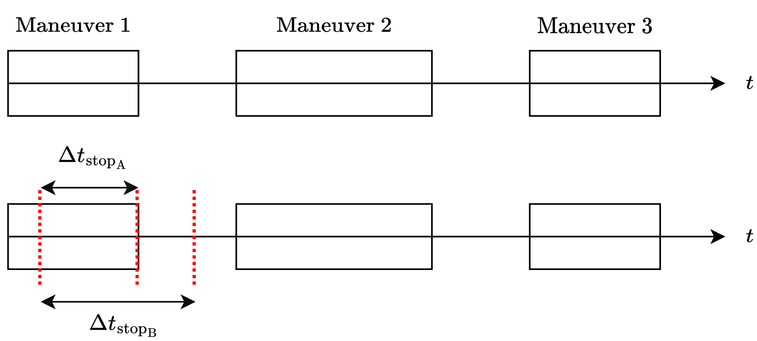



Fig. 11: Two distinct stopping maneuvers in that result in an equivalent modified maneuver plan.

To address this issue, we define a rescaled finite stop MTS in which all maneuvers occur consecutively. By optimizing in a shifted time domain where all maneuvers occur consecutively, we can remove all of the blue hatched areas from the MTS, as seen in Figure 12. Now that all possible points in the MTS correspond to uniquely modified maneuver plans, MTS grid point selection can begin. A grid of $N$ by $M$ rescaled stop maneuver times and rescaled stop maneuver durations will be generated. The rescaled stop maneuver times go from the first time within the MTS time limits that contains a maneuver to the last time within the MTS time limits that contains a maneuver with a fixed timestep size. Additional shifted stop times will be added to ensure that the shifted start time of each maneuver within the MTS time limits is included in the grid. The rescaled stop durations are set based on a fixed step size from the minimum stop duration to the maximum stop duration.

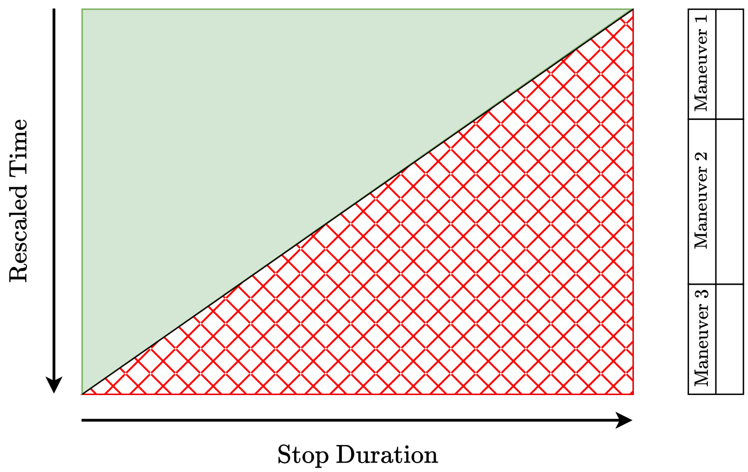



Fig. 12: rescaled finite stop MTS.

Converting from the shifted time and stop duration domain to the real stop time and stop duration domain is a simple process of adding back in the corresponding amount of time between maneuvers. This is done by first calculating the shifted resume time, which is equal to the shifted stop time plus the shifted stop duration. The shifted stop and resume times are then compared to the shifted start and stop times of each planned maneuver to find which maneuver the shifted stop time and resume time fall in. This process is summarized by equations (21), (22), and (23), where $L$ represents the maneuver number that the shifted stop time and shifted start time fall in. Maneuver number one corresponds to the first maneuver in the MTS. The time durations between planned maneuvers is saved as $\Delta t_{\text{Shift,i}}$, $\Delta t_{\text{shift, 1}}$ represents the time duration from the end of the first maneuver in the MTS and the start of the second maneuver.

$$\text{Stop Time} = \text{Shifted Stop Time} + \Delta t_{\text{shift}} \tag{21}$$

$$\text{Resume Time} = \text{Shifted Resume Time} + \Delta t_{\text{shift}} \tag{22}$$

$$\Delta t_{\text{shift}} = \sum_{i=1}^{L-1} \Delta t_{\text{shift, i}} \tag{23}$$

### 4.2 Finite Stop Maneuver Optimization

Once each point in the MTS is mapped to a unique maneuver plan, these maneuver plans will be propagated and evaluated for aggregate miss and $P_c$ as previously discussed in section §2. Next, each grid point needs to be assigned a cost value in order for optimization to occur. The cost function remains generally unchanged from its description in section §2 with the key distinction that the maneuver duration that is being minimized is the rescaled stop duration. By minimizing this value a modified maneuver plan that meets operational constraints and has minimal changes from the nominal ascent maneuver plan can be found. Note that the rescaled stop duration value is used rather than the stop duration value. This ensures that stopping maneuvers which span multiple maneuvers are not overly penalized in the cost function. As an example, imagine a stopping maneuver that trims the end and start of two consecutive maneuvers by two minutes. If these maneuvers are scheduled five hours apart, this stopping maneuver would have a total stopping duration of five hours and four minutes. Compare this to a stopping maneuver that trims ten minutes from the end of one maneuver. This second stopping maneuver would only have a total stopping duration of ten minutes while actually trimming more of the maneuver plan. Once a cost value has been assigned to each grid point, seed selection and Nelder–Mead optimization can be preformed exactly the same as seen in section §2.

### 4.3 Stop Time *vs.* Resume Time Maneuver Trade Space

In order to more easily interpret the results that optimization finds, a new output graph was created with axes of maneuver stop time *vs.* maneuver resume time. An illustrative example of this new MTS representation can be seen in Figure 13. Contours for both aggregate $P_c$ and aggregate miss will be plotted with the axes shown. Observe how by looking down the vertical axis one can see where the stopping maneuver begins and by looking at the horizontal axis one can see where the stopping maneuver ends. The red hatched areas represent the nonsensical combination of ending the stopping maneuver before it begins and will not have any contour data associated with it. The blue hatched areas once again represent repetitive stopping maneuvers and will also not have any contour data associated with it. The benefit of not plotting contours for blue hatched areas is that it allows the reader to more easily tell the starting and ending times of each maneuver in the nominal ascent maneuver plan. Points 1, 2, 3, 4 shown in Figure 13 represent different possible stopping maneuvers. One can see how these stopping maneuvers modify the nominal ascent maneuver plan in Figure 14.

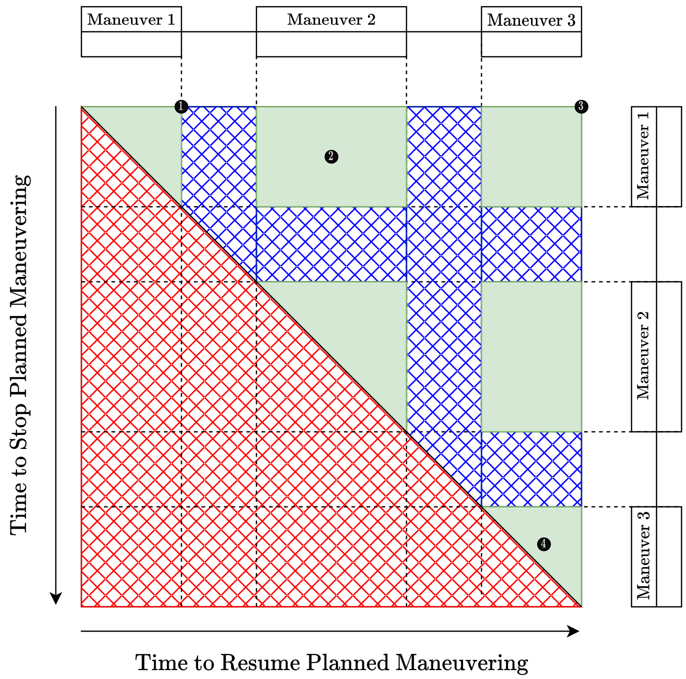



Fig. 13: Finite Stop MTS mapped to resume time *vs.* Stop time axes, with example optimal maneuver shown.

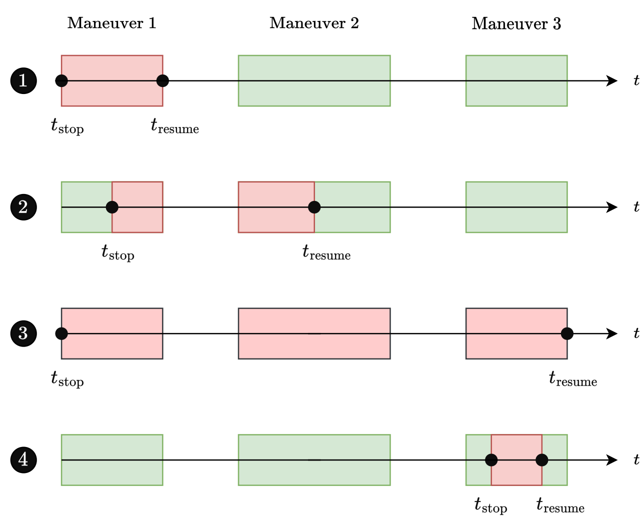


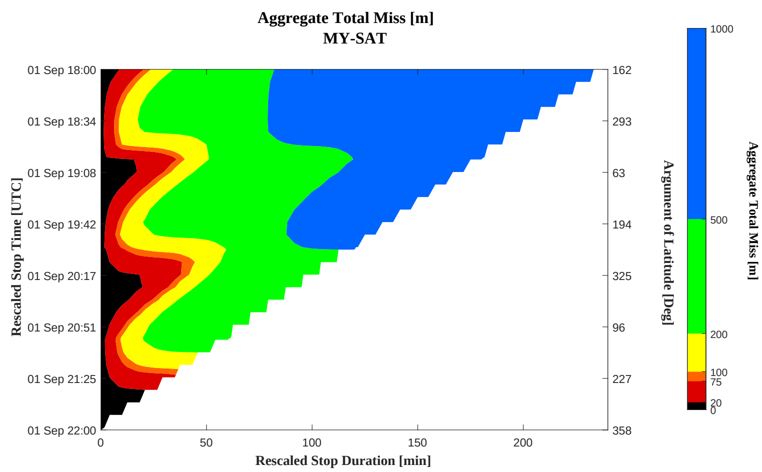

Fig. 14: Resulting maneuver plan modifications from points shown in Figure 13

## 5. RESULTS

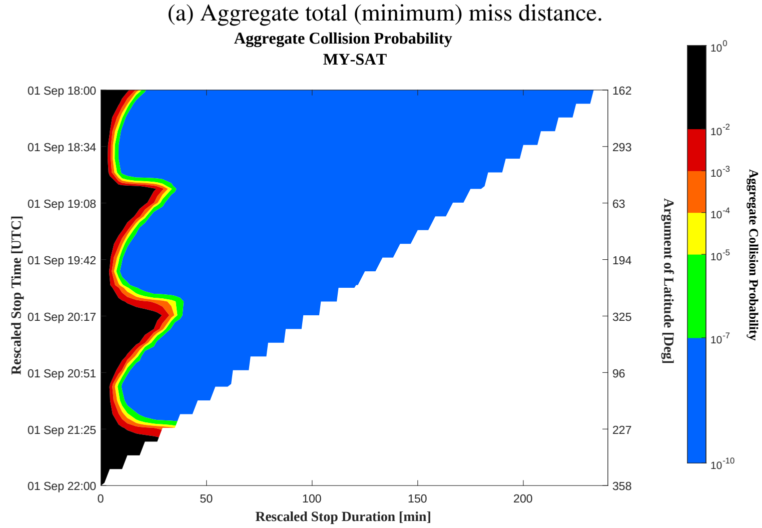
In order to demonstrate the power of these new MTS designs, test cases were developed where these approaches could be applied. The test case for risk mitigation during ascent was set up for a low thrust satellite preforming a long duration, low-thrust orbit raise, in which an HIE arises. The dual maneuver test case was contrived to reflect a non-maneuvering low-thrust satellite that has an HIE arise, for which the maximal single maneuver burn does not meet mission safety thresholds.

### 5.1 Risk Mitigation During Ascent Test Case

The finite stop maneuver optimization was tested with a nominal ascent trajectory starting at 500 km with 4 maneuver per day for an ephemeris duration of 5 days. Individual maneuvers were 1 h long with a 5 h gap between planned maneuvers. The MTS time limits were defined to be 24 hours (4 maneuvers) before a HIE with $P_c = 8.25 \times 10^{-2}$ and Miss $= 7.5$ m. The resulting shifted MTS and stop time *vs.* resume time MTS can be seen below, with aggregate miss and $P_c$ contours plotted.

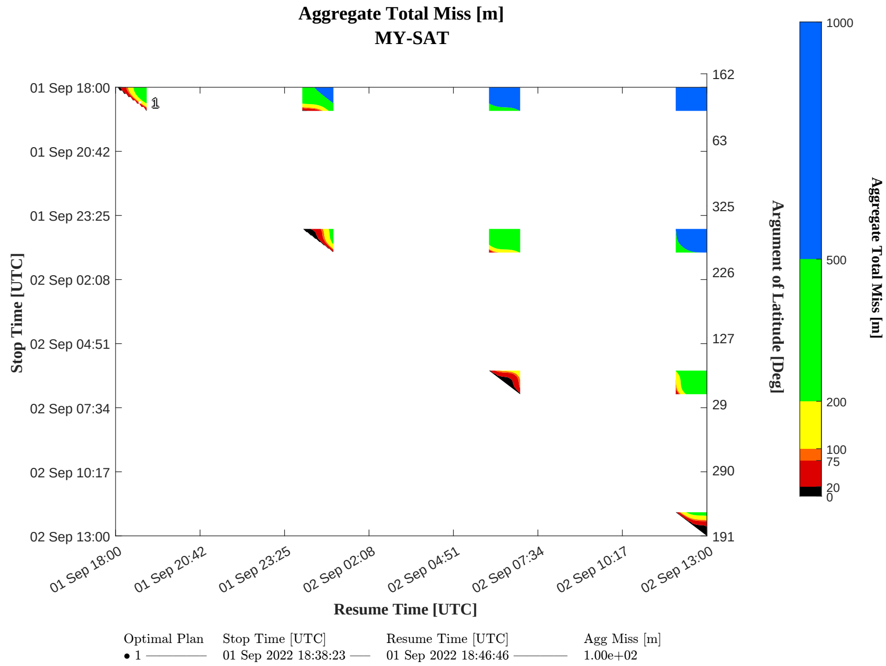



(a) Aggregate total (minimum) miss distance.

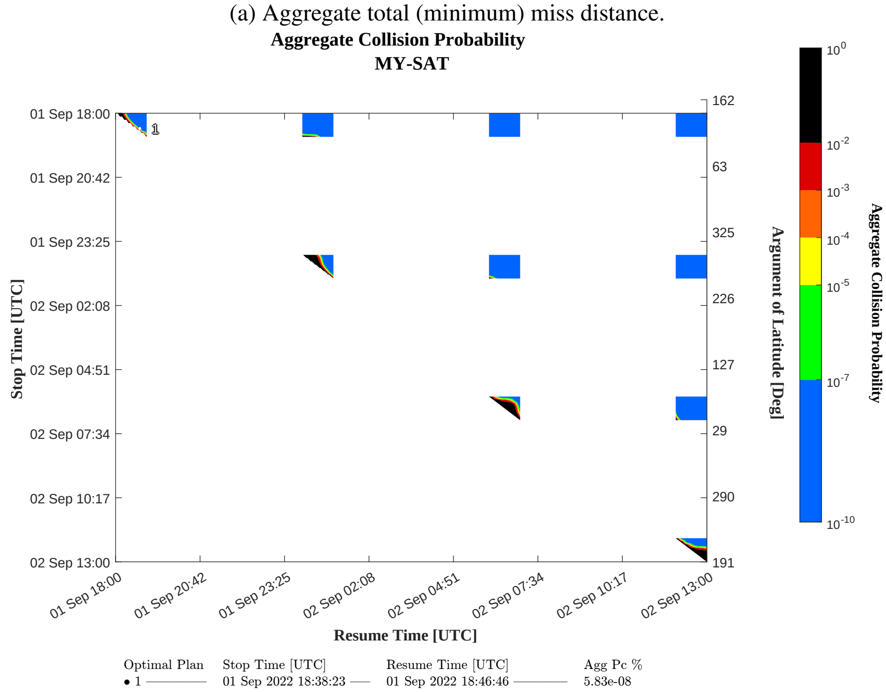



(b) Aggregate probability of collision, $P_c$

Fig. 15: Resulting shifted maneuver trade spaces with contours plotted.




(a) Aggregate total (minimum) miss distance.




(b) Aggregate probability of collision, $P_c$

Fig. 16: Resulting stop time *vs.* resume time maneuver trade spaces with contours plotted.

Looking at Figure 16b it can be seen that the aggregate $P_c$ is reduced to $5.83 \times 10^{-8}$ with a stopping maneuver lasting from 18:38:23 to 18:46:46 on September 1, 2022, a total change of 8 minutes and 23 seconds from the nominal ascent maneuver plan. Similarly it can be seen that the aggregate miss distance is increased to 100 m in Figure 16a. Table 1 and Table 2 list the changes to the maneuver plan start and stop times. Note that there is an additional row in Table 2 as the stopping maneuver falls within the first maneuver, splitting it into two maneuvers.

Table 1: Nominal ascent maneuver plan for maneuvers falling in MTS.

| Maneuver Number | Maneuver Start Time | Maneuver Stop Time |
|---|---|---|
| 1 | 01 Sep 2022 18:00:00 | 01 Sep 2022 19:00:00 |
| 2 | 02 Sep 2022 00:00:00 | 02 Sep 2022 01:00:00 |
| 3 | 02 Sep 2022 06:00:00 | 02 Sep 2022 07:00:00 |
| 4 | 02 Sep 2022 12:00:00 | 02 Sep 2022 13:00:00 |

Table 2: Trimmed ascent maneuver plan for maneuvers falling in MTS.

| Maneuver Number | Maneuver Start Time | Maneuver Stop Time |
|---|---|---|
| 1 | 01 Sep 2022 18:00:00 | 01 Sep 2022 18:38:23 |
| 2 | 01 Sep 2022 18:46:46 | 01 Sep 2022 19:00:00 |
| 3 | 02 Sep 2022 00:00:00 | 02 Sep 2022 01:00:00 |
| 4 | 02 Sep 2022 06:00:00 | 02 Sep 2022 07:00:00 |
| 5 | 02 Sep 2022 12:00:00 | 02 Sep 2022 13:00:00 |

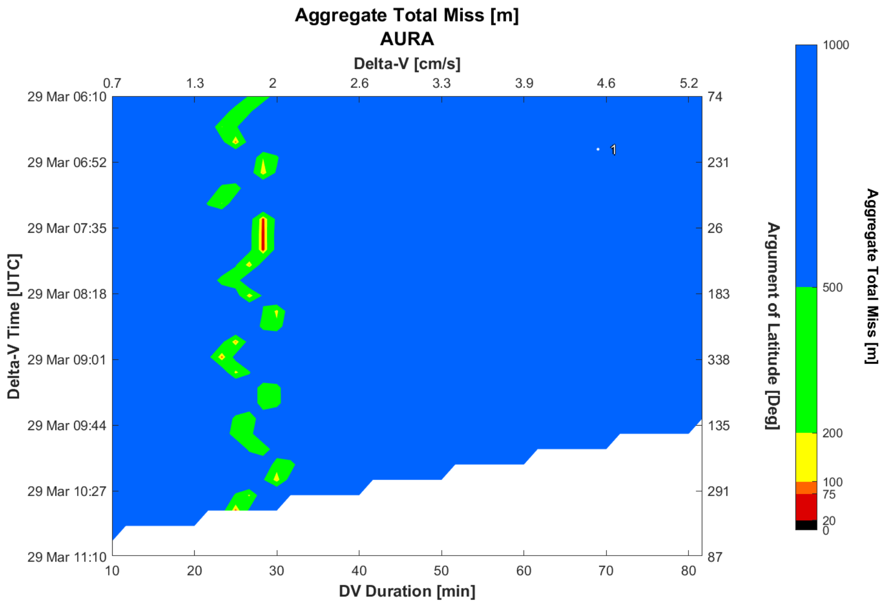
### 5.2 Dual Maneuver Risk Mitigation Test Case

The dual maneuver MTS approach was tested by generating a nominal ephemeris with an HIE and limitations on the maximum and minimum achievable maneuver duration. The HIE had a TCA occurring around 2019-04-01T19:10:49 UTC with $P_c \approx 10^{-4}$. The MTS was defined to search for first maneuver times occurring between maneuvers 2019-03-29T06:10:00Z and 2019-03-29T11:10:00Z with a maneuver timestep size of 400 seconds. The unconstrained single maneuver MTS was defined to evaluate maneuver sizes ranging from 600 s to 49 000 s with a 50 s step size. The individual maneuver duration limits were set to have a minimum duration of 600 s and a maximum duration of 3600 s with maneuvers occurring one revolution apart. The thrust used was 0.031 N. The variable responsible for setting the first maneuver value, $C$ from Equation (16), was set to be 0.90. Note that this test case corresponds to the MTS shown in Figure 8 and as such does not require the $\beta$ and $\vec{v}$ variable definitions.

In Figure 17b it can be seen that the aggregate $P_c$ was reduced to $8.05 \times 10^{-8}$ with two maneuvers that both meet desired magnitude constraints and does not contain any more total maneuver duration than is required. The final maneuver plan is also listed in Table 3.

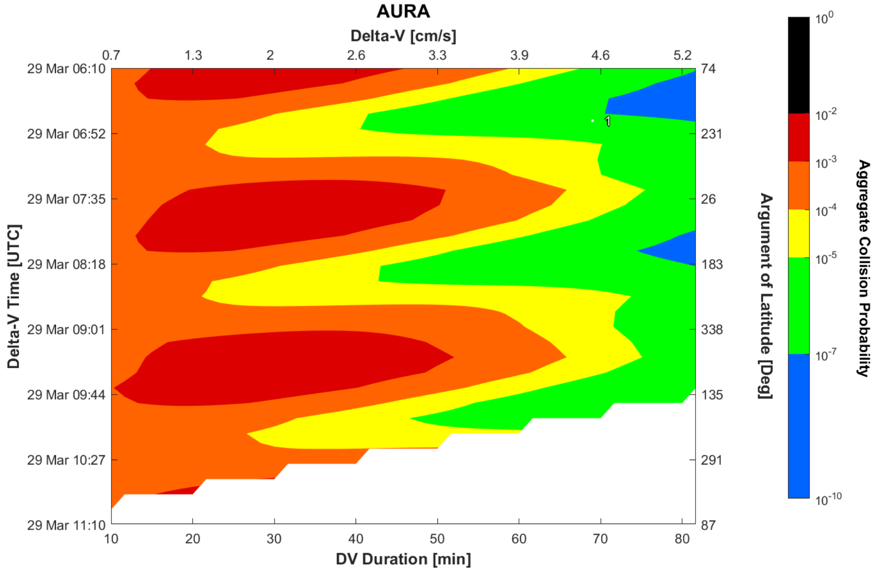
Table 3: Resulting dual maneuvers.

| Maneuver Number | Maneuver Start Time | Maneuver Duration |
|---|---|---|
| 1 | 29 Mar 2019 06:44:00 | 3540 s |
| 2 | 29 Mar 2019 08:23:00 | 600.06 s |




| Optimal Plan | Delta-V Time 1 [UTC] | Delta-V Time 2 [UTC] | DV Duration [min] | Agg Miss | DV Duration 1 [min] | DV Duration 2 [min] |
|---|---|---|---|---|---|---|
| • 1 | 29 Mar 2019 06:44 | 29 Mar 2019 08:23 | 69.001 | 1.50e+04 | 59.000 | 10.001 |

(a) Aggregate total (minimum) miss distance.




| Optimal Plan | Delta-V Time 1 [UTC] | Delta-V Time 2 [UTC] | DV Duration [min] | Agg Pc | DV Duration 1 [min] | DV Duration 2 [min |
|---|---|---|---|---|---|---|
| • 1 | 29 Mar 2019 06:44 | 29 Mar 2019 08:23 | 69.001 | 8.05e-08 | 59.000 | 10.001 |

(b) Aggregate probability of collision, $P_c$

Fig. 17: Resulting dual maneuver MTS with aggregate contours plotted. Note that “Dv Duration” represents the total duration across both maneuvers.

## 6. CONCLUSIONS

The power of a maneuver trade space approach to risk mitigation has been clear to the SSA community for some time now. Here we have been able to demonstrate that this same approach can be extended in order to handle more complex maneuver optimization while still maintaining a relatively straightforward methodology. By extending the MTS approach to more complex maneuvering situations, the new approaches to finding optimal maneuvers can be continuously added as they are designed. Overall an increase in the tools available to the SSA community when handling maneuvers makes space safer for all.

In order to further expand the MTS approach to finding optimal solutions to other scenarios, two things are required. First, a space that encapsulates the desired maneuver combinations to be considered and how points in said space map to a “real time” maneuver plan must be defined. Secondly, either a modified cost function or a metric that can work with the current cost function as defined by Equation (1) must be defined as well. In the future, an interesting extension of the dual maneuver trade space would be to support an arbitrary number of maneuvers. In a similar vein, it could be of interest to extend the dual maneuver MTS to use a unit-defining vector to define the maneuver times that are considered, similar to how the maneuver magnitude combinations are currently selected. On its own, fixing the maneuver magnitude sizes and looking only at an MTS which encapsulates different maneuver times could provide an interesting solution when looking for new ascent maneuver plans. In this configuration, rather than trimming the nominal maneuver plan, a shift in maneuver timing would be searched for.

## 7. REFERENCES

[1] S. Singer and J. Nelder. Nelder-Mead algorithm. *Scholarpedia*, 4(7):2928, 2009. revision #91557.