Reducing Uncertainty in Satellite Conjunction Analysis

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ABSTRACT

This paper lays out a method of improving the estimates of neutral density within the thermosphere used in orbit propagation. An ensemble Kalman filter is used to combine an orbit propagator with precise orbit information and an atmospheric model. A Kalman filter is an algorithm used to find the optimum way of combining observational data with model output to produce an estimation of the state. The largest uncertainties in orbit propagation for satellites in low Earth orbit (LEO) arise from difficulties predicting neutral density. This is important for the Space Situation Awareness (SSA) community because of the increasing number of satellites in LEO and the growing amount of debris. More accurate satellite positions can help the number of possible collision warnings.

1. INTRODUCTION

The last decade has seen a dramatic increase in the number of satellites in low Earth orbit (LEO). On top of the number of satellites there is also an increasing amount of orbital debris to contend with. In January 2019 there were estimated to be 34 000 debris objects larger than 10 cm in size, 900 000 objects between 1 cm and 10 cm and 128 million objects from 1 mm to 10 cm. This expansion is producing challenges for satellite operators due to the increase in collision warnings. The best way to reduce the number of possible collision warnings is to reduce the size of the error in a satellite's position. For satellites in LEO the largest uncertainties in position arise from difficulties predicting atmospheric density. Neutral densities within the thermosphere can be challenging to model and predict as they show both temporal and spatial variations. The temporal variations can be abrupt over the time scales of minutes to hours, as well as diurnal and on scales of solar rotation and solar cycle. Spatial variations include those in latitude, longitude and altitude.

Orbit propagators are used to predict the tracks of satellites. Errors between the propagated and recorded position mainly arise from difficulties measuring and predicting density within the thermosphere. As these differences arise from errors in density predictions the approach can be reversed and density estimated from errors in orbit propagation. This study aims to use an ensemble Kalman filter to combine an orbit propagator with precise orbit information and an atmospheric model to produce improved estimates of density within the thermosphere leading to better predictions of satellite position.

Kalman filters present a data assimilation technique which is used widely in the fields of meteorology and oceanology. A Kalman filter is an algorithm used to obtain the optimal combination of observations with a model by minimising variances. It is ideal for use in systems which are continuously changing. Kalman filters are recursive; an estimate of the current state is updated to provide a predictive step. This predictive step then becomes the current state and the process is repeated. Here an estimate of the state is created using an atmospheric model. An ensemble Kalman filter is a Monte Carlo implementation of the Kalman filter. Here the input parameters of solar irradiance and magnetic activity will be perturbed to produce a range of ensemble members. For each member the density will be estimated and used to calculate the atmospheric drag felt by the satellite. This can then be used as input for the propagator. Propagated positions for each member can then be compared to recorded position to make an estimate of the density the satellite has passed through. By repeating this process for a range of satellites in different orbits a global view can be built up.

This picture of the thermosphere can then be used as input to a data assimilation model to improve the predictions of density made into the future.

Section 2 describes a preliminary study used to show the differences changing the atmosphere used in propagation can have. Section 3 looks at methods of observing the thermosphere. Section 4 describes the process of data assimilation and section 5 sets out the method used in this study.

2. EFFECT OF CHANGING THE ATMOSPHERE

A preliminary study has been made to see the effect of changing the atmosphere used for orbit propagation. Three different models have been tested: a simple exponential atmosphere, which models thermospheric density as an exponential decrease with altitude; NRLMSISE-00, which is an empirical model which calculates the neutral atmosphere from the surface to the lower exosphere [7] and DTM-2000, a semi-empirical model describing the temperature, density and composition of the Earth's atmosphere [3].

To perform the study the orbit propagator Orekit [4] has been run for a period of 24 hours over 4th September 2019. This is a day of low solar activity with a mean Disturbance Storm Time Index (Dst index) of -19. Orekit has been chosen for this study due to its robustness, large number of options and its current use in operational settings [1]. To start the propagation Orekit has been given the initial position and velocity of Swarm-A. This is then propagated forward for 24 hours using numerical propagation. Propagation includes gravity perturbations and the perturbations modelled by the chosen atmosphere. The position predicted has been compared to the satellites recorded position and the Euclidean difference between these two points has been plotted in Figure 1. For the first 15 hours the performance of the propagation run with the empirical models NRLMSISE-00 and DTM2000 show a greater agreement to the recorded position than the simpler exponential model or running without atmospheric perturbation. After this the agreement diverges and by 20 hours the difference is larger than when no atmospheric perturbation is applied to the propagation.

Figure 1 plots the Euclidean distance between propagation and recorded positions. To extract the direction of this difference it has been converted into the satellite centred radial, cross-track, in-track (RCI) frame. This frame is described in Figure 2. This transformation has been performed for each of the four propagations shown in Figure 1. The results of this are shown in Figure 3. In all cases the largest difference is seen in the in-track direction meaning the predicted position is too far ahead or behind. This shows the errors arise from difficulty predicting the craft's acceleration. Acceleration is influenced by the drag felt by the craft. For the two empirical models NRLMSISE-00 and DTM2000 at approximately 10 hours the satellite's predicted position moves from being behind the recorded to in front. From around this time the position starts to deviate steeply. Beyond 15 hours this divergence causes the estimation to become less accurate than that performed with a simple exponential atmosphere.

This preliminary study shows how that the atmospheric model used in orbit propagation influences the accuracy of the prediction. This study was performed at a time of low solar activity. Repeating this study at a time of higher activity is suspected to show larger differences.

3. OBSERVATIONS OF THE THERMOSPHERE

Observations of the thermosphere are challenging due to its location. Starting at 85 km it is too high for measurements to be made on board aircraft or balloons. To reach heights of the thermosphere rockets need to be used. These have been used to sound the thermosphere and measure its physical and chemical properties, however rockets are expensive to launch and provide only very localised observations. Above 200 km altitude the satellites provide another tool. As these crafts orbit within the thermosphere they provide a way to directly assess its state. The further into the future a satellite's position is predicted, the larger the errors between the propagated and actual positions become. For satellites in LEO this error mainly arises from errors in prediction of the drag felt by the satellite as it travels through the thermosphere, this force is proportional to the density. This error can then be used to gain information on the character of the error in predicted density. If the satellite moves faster than predicted the density was too small and vice versa. With the ever increasing number of satellites orbiting within the thermosphere, this provides a useful data source with good spatial and temporal coverage. Figure 4 shows the errors in propagation as they have developed over a 24 hour period. Over time the errors become larger so the track becomes darker on the plot. After 24 hours

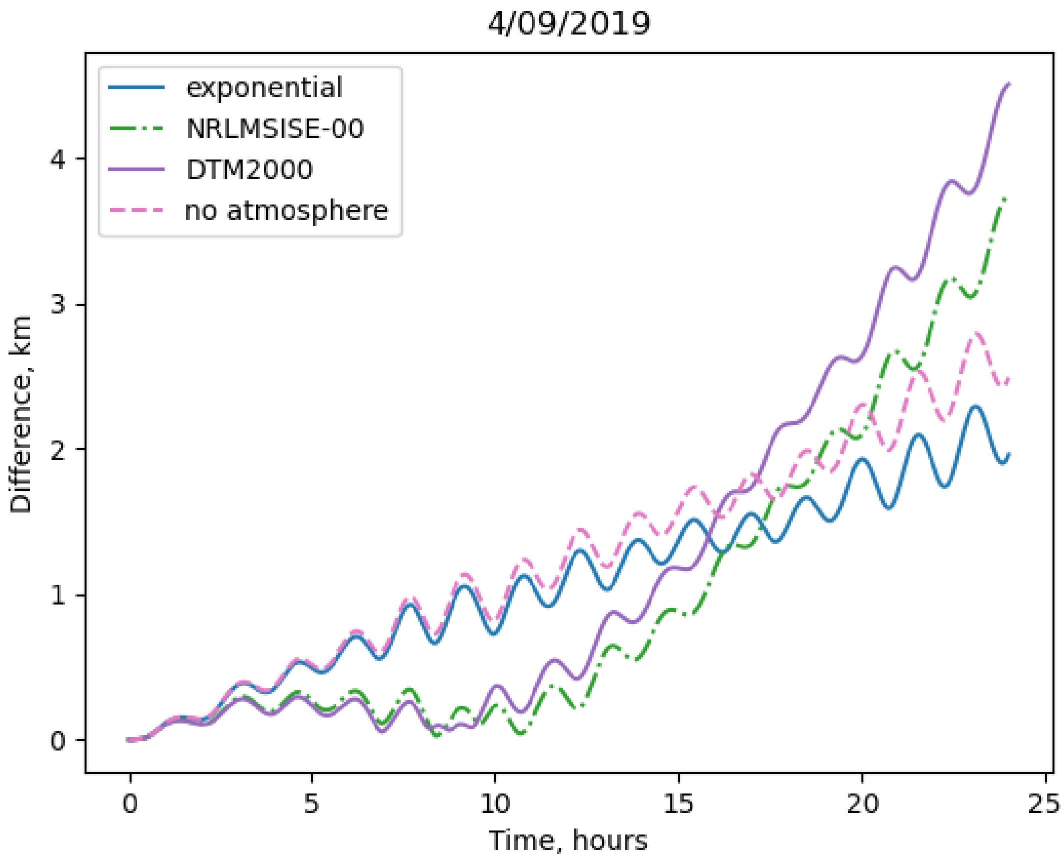

Fig. 1: The orbit of the Swarm-A satellite has been propagated for 24 hours over the 4th September 2019 using Orekit for three atmospheric models (exponential, NRLMSISE-00 [7] and DTM2000 [3]) in this study and without an atmospheric perturbation for comparison.

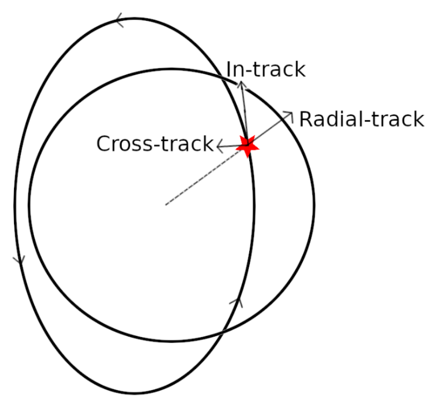

Fig. 2: The radial, in-track, cross-track frame is a satellite centred frame. Where the axes are orientated with respect to the satellites motion.

of propagation these errors have typically reached the size of 2-3 km [2]. These errors can be higher at times of high solar activity.

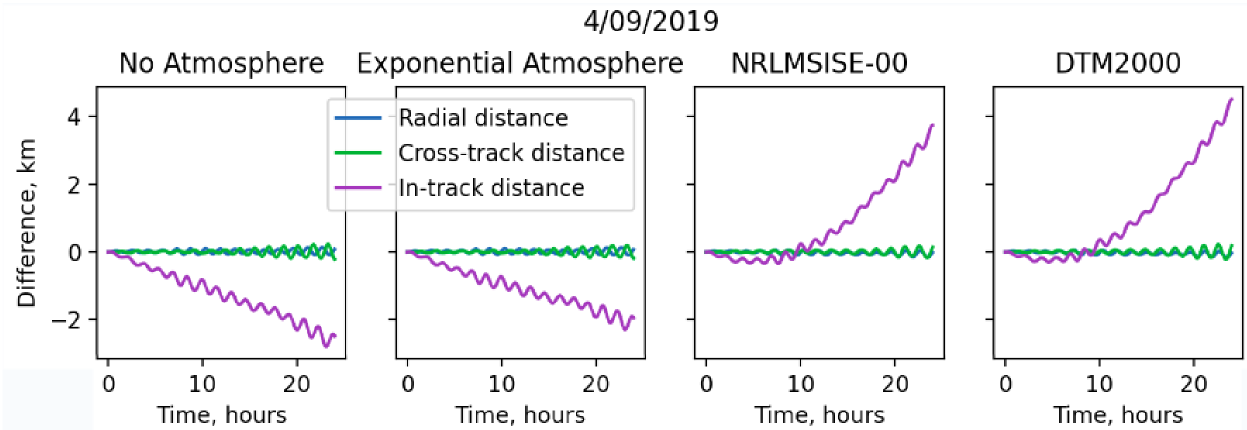

Fig. 3: The propagation seen in Figure 1 has been split into radial, in-track and cross track components. The largest difference is seen in the in-track direction.

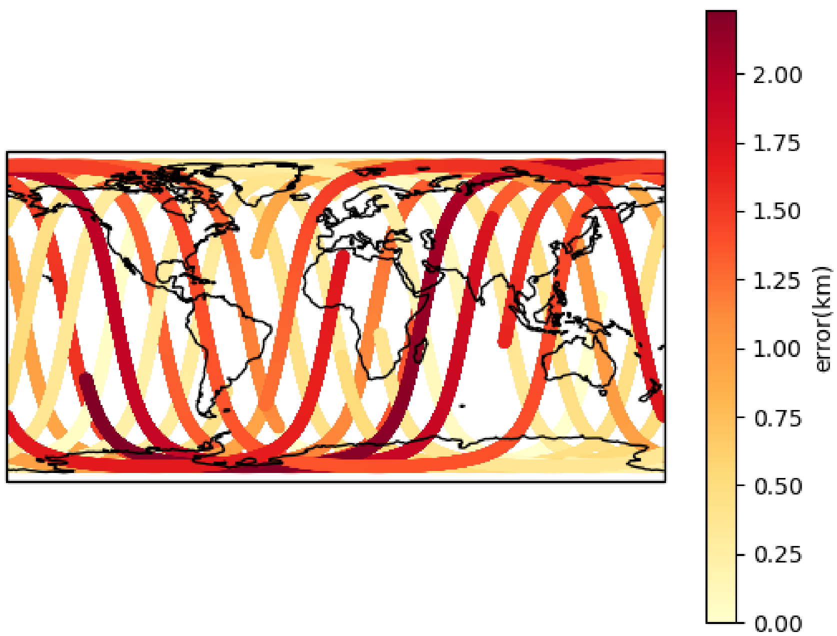

Fig. 4: Global heat maps showing the error between propagated and recorded satellite positions. As the position is propagated further into the future the error increases.

4. DATA ASSIMILATION

Data assimilation is the name given to the process of combining observations and modelled forecasts. The aim is to produce the most accurate representation of the system. A Kalman filter is an algorithm which obtains the optimal combination of observations with a model by minimising variances [5]. It is ideal for systems which are continuously changing. The Kalman filter is built upon Bayes' theorem. This theorem provides the fundamental framework for all data assimilation methods. Bayes states that "the probability of x given y is equal to the probability of x, multiplied by the likelihood of y given x, divided by the probability of y" [6].

$$f(x|y) = \frac{f(x)f(y|x)}{f(y)}. \tag{1}$$

This can be re-written for a state variable, x, and its prior probability distribution function (pdf), f(x), and a vector of observations, d, with a likelihood, f(d|x). Here Bayes' theorem will define the posterior pdf, f(x|d).

$$f(x|d) = \frac{f(x)f(d|x)}{f(d)}. \tag{2}$$

Bayes' theorem assumes variables have Gaussian errors, are linear and non-biased.

In the case of a Kalman filter an estimate of the system is produced using an iterative two step process.

1. an estimate of the state is made which can then be propagated forward to time t_0. This is known as the priori estimate or background state vector ($\mathbf{x}_b$) and has an associated co-variance matrix (**B**).

2. the background state vector is updated to include any observational data recorded at t_0 (y_0 with co-variance matrix **R**). This produces an updated state vector $\mathbf{x}_a$.

$$x_a = x_b + K(y_0 - Hx_b), \tag{3}$$

$$A = (I - KH)B, \tag{4}$$

$$K = BH^T(HBH^T + R)^{-1}. \tag{5}$$

where **K** is the Kalman gain matrix and **I** is an identity matrix, H is the observation operator used to map variables from model space to observation space, **A** is the co-variance matrix associated with x_a. This can then be propagated forwards again to become the background state vector for the next iteration.

$$x_b = Mx_a, \tag{6}$$

where **M** represents the model used to update the state to the next time step. For an ensemble Kalman filter the single background state vector is replaced by a set of k background ensemble members where the mean is assumed to be the best estimate of the state before observations.

The background covariance matrix, **B**, can then be estimated from a matrix of background perturbations (X_b), the ensemble members with the mean removed from each member

$$X_b = x_{b(i)} - \bar{x}_b, \tag{7}$$

$$B = (k-1)^{-1}X_b(X_b)^T. \tag{8}$$

This technique is very useful for complex systems where it can be difficult to quantify the relationship between variables. It is traditionally used for high dimension systems as it can reduce computational complexity. After a prediction time step an analysis ensemble $x_{a(i)}$ is returned. The analysis covariance matrix can then be estimated in a similar way:

$$A = (k-1)^{-1}X_a(X_a)^T, \tag{9}$$

where

$$X_a = x_{a(i)} - \bar{x}_a. \tag{10}$$

5. METHOD

The method to be used is described below and set out in Figure 5:

- Set background models for ensemble members, altering the model input conditions (i.e. F10.7 and Kp) will produce a range of thermospheric densities

- Each background ensemble member can then be fed into the orbit propagator to produce an ensemble of orbit tracks.
- Here the Kalman filter is used to combine the modelled path with the recorded path of the satellite.
- This ideal state can then be used to move the propagation forward in time and the process can be repeated.

Using a constellation of satellites, which cover the globe, can give fuller coverage of the state of the thermosphere. This state can then be used to propagate the position of any satellite into the future.

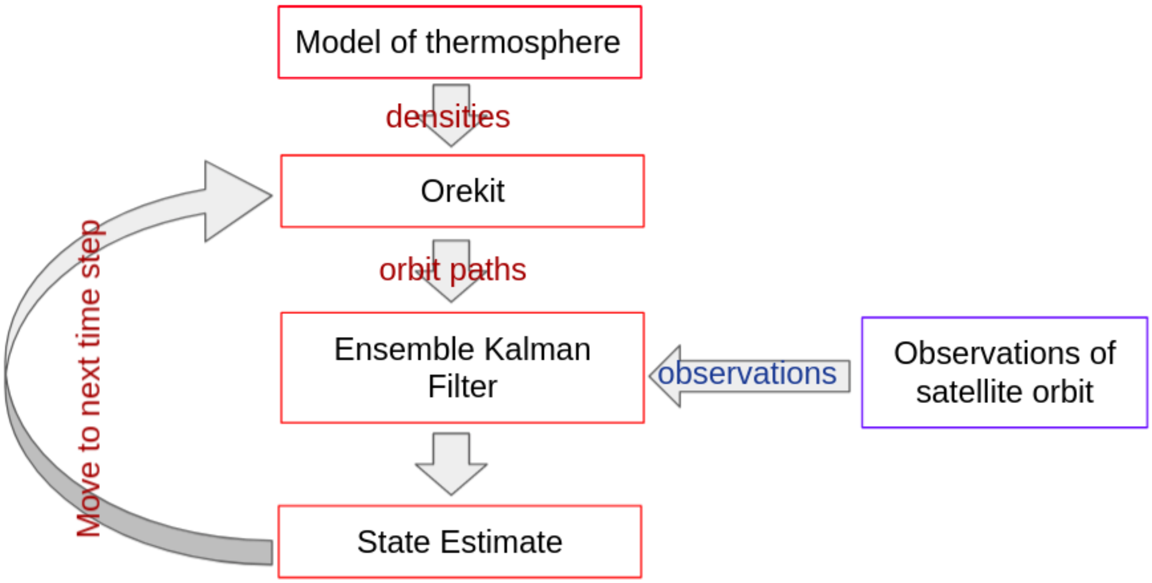

Fig. 5: Flow chart showing method of using an ensemble Kalman filter to combine a model of the thermosphere, orbit propagator and precise orbit location data from a satellite.

6. CONCLUSIONS

Predicting the path of satellites and other objects in LEO is important to avoid collisions and reduce collision avoidance maneuvers. For satellites in LEO the largest source of uncertainty arises from our ability to predict the density of the thermosphere. The differences between the predicted and recorded positions can provide a source of data and a way to directly observe the thermosphere. Here a method is set out to use an ensemble Kalman filter to combine observations of satellite positions with an existing atmospheric model to improve the estimated thermospheric densities used in orbit propagation.

7. REFERENCES

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