On the Increased Risk of Kessler Syndrome by Anti-Satellite Tests

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ABSTRACT

The militarization of outer space and development of counterspace technologies is a growing concern, especially as commercial and governmental entities are sending thousands of satellites into orbit. Direct-ascent anti-satellite (ASAT) tests and weapons put the sustainability of space at greater risks because the debris they generate can render space inoperable. The additional debris can kick-start run-away collisions and lead to an exponential growth of debris known as the Kessler Syndrome. This work uses our recent statistical framework to explore the effects of debris injection events on the probability of the Kessler Syndrome occurring. In particular, we use a set of toy models under our statistical framework to explore how the timing and amount of debris generation by ASAT tests can influence future debris growth. We define a risk parameter P_{Exp} as the probability of run-away debris growth, and show how ASAT tests can dramatically increase its value, especially in a proliferated satellite and debris-filled environment.

1. INTRODUCTION

The space environment is changing rapidly as governments and commercial entities launch increasing numbers of payloads into Earth orbit, primarily low Earth orbit (LEO). Data from `space-track.org` [1], a monitoring site run by the 18th Space Defense Squadron (18th SDS), shows that the rate of payloads launched into orbit has risen from roughly 300 per year in 2018 to roughly 1500 per year currently. Commercial mega-constellations, such as Space-X's Starlink, already have thousands of satellites in orbit and, according to current plans, will ultimately consist of many more. With the growing number of satellites, the development of counterspace technologies exacerbate the risk of long-term sustainability of space.

The potential problem of run-away collisions and exponential growth of orbital debris was first studied by Kessler and Cour-Palais in 1978 and is commonly referred to as the "Kessler syndrome" [14]. If the number of collisions grows out of control, the large number of debris produced could render parts or all of the Earth orbital environment inoperable. This is becoming a widely discussed concern as the commercial sector launches thousands of satellites into orbits. However, the discussion of the Kessler syndrome often focuses on the risk of accidental collisions. In contrast, debris production via the intentional destruction of satellites is less well known by the public. Most recently in 2021, in a direct-ascent anti-satellite test, a Russian PL-19 Nudol missile destroyed a defunct military Soviet satellite (Cosmos-1408).[1] According to the U.S. Space Command, the Russian ASAT test generated a cloud of more than 1500 pieces of trackable debris. It is estimated that hundreds of thousands of smaller fragments were also produced [4]. Some larger fragments put the International Space Station (ISS) in jeopardy. It is unclear what effects the trackable and untrackable debris from this event will have in the long term. Therefore, it is important to provide modeling to quantify impacts of such events on the orbital environment to guide policy and practical decision-making in the near future.

There are two primary classes of models that address long-term orbital debris evolution. One class updates the positions and velocities of all current objects dynamically with high fidelity propagators. When objects come close to each other, the simulation samples whether they will collide given the probability of collisions. Examples of such models are NASA's LEGEND model [18] and the DAMAGE model [16] by European

[1] Note that we will use ASAT interchangeably with direct-ascent ASAT throughout this paper.

Space Agency. These models are generally computationally expensive, making it difficult to generate a large statistical sample of future outcomes, especially for large number of simulated objects. The other class of models are source-sink evolutionary models, sometimes called particle-in-the-box (PIB) models. They model how the number of objects change at the population level given a set of sources and sinks (e.g., launch, decay, collisions) operating at various average rates [15, 6, 11, 5, 19, 21]. These models are governed by a set of coupled ordinary differential equations and are relatively easier to solve than the propagation-based models. Thus, one can explore the range of outcomes relatively easily by varying the input parameters. However, the parameters in these models are usually observationally unconstrained; they lack the detailed physics included in the high-fidelity propagation models like LEGEND. Moreover, as currently applied, PIB models do not provide insight into the full range of possible outcomes given a set of fixed parameters, because they generally do not account for the randomness of modeled processes (e.g., collisions).

To understand this range, one can use a stochastic source-sink evolutionary model [22, 20, 12, 3, 10, 2]. In our recent work [17], we develop a simple stochastic evolutionary model for the orbital environment. This model mainly demonstrates how stochastic evolutionary models can apply to the study of orbital debris production. Similar to many evolutionary models in literature, we model satellite and debris populations under launch, decay, explosion, and collision processes. Unlike these models, however, our model generates a large number of possible future evolutionary paths for debris growth. Taken together, these evolutionary paths represent the statistical distribution of outcomes that could be generated by a fixed set of model parameters (defined by the processes such as launches and collisions; see section 2). Thus, the trajectories can be used to quantify the probabilities of environmental conditions.

In this paper, we explore how the timing and amplitude of the debris injection events (e.g., ASAT tests) can have an effect on the space environment as it gets more populated by satellites. Specifically, we are interested in the effects of injection events on the risk parameter P_{Exp}, defined as the probability of run-away growth.

2. METHODOLOGY

In this study, we use a suite of Monte-Carlo simulations, which consists of all the physical processes we described in [17] where additional details can be found. Here, we briefly summarize the modeled processes and the stochastic sampling.

2.1 Statistical Framework

The central object in an evolutionary source-sink model is the population vector $\mathbf{N}(t)$ representing the numbers of objects of various species at a given time t. In an orbital evolutionary model, species can be indexed by object type (active satellite and debris), length scale, altitude, and other parameters. The model also consists of a rate matrix Ω that governs how the state changes with time. These rates can be state- and time-dependent.

Typical evolutionary models propagate the population vector $\mathbf{N}$ over a series of time steps, producing a single trajectory for each species. However, given the underlying rates, several different system trajectories are possible. The time-dependent probability distribution $\mathbf{P}_{\mathbf{N}}(t)$ encapsulates the range of possible population values at time t. This probability distribution's evolution follows the Master Equation [9]:

$$\frac{d\mathbf{P}_{\mathbf{N}}}{dt} = \Omega \mathbf{P}_N(t) \tag{1}$$

Propagating this equation provides information on the full range of system states with time. When the number of possible system states is so large that solving Eq. (1) exactly becomes impractical. Instead, we approximate $\mathbf{P}_{\mathbf{N}}$ by stochastically sampling system trajectories, as described in more detail in Sec. 2.3.

2.2 Orbital Environment Model

We implement a simple stochastic model of the orbital environment similar to other models in the literature [23, 15, 11, 6, 5, 19, 21]. Our model consists of two species of objects: active satellites and debris. The population vector elements can be written as N_{α,ℓ_i^α}, where $\alpha = (\mathrm{sat}, \mathrm{deb})$ labels the object species and ℓ_i^α $(i = 1, ..., n_\ell^\alpha)$ is the length. The model changes the state (number of satellites and debris) of the system via various sources and sinks processes, such as launch, decay, collisions, and explosions.

2.2.1 Launch

We define the launch rate Ω_L as the number of satellites added to space due to launches. Each launch populates a number of satellites at various length scales, chosen at random. In this study, we apply constant launch rates in our simulations ($\Omega_L = L$).

2.2.2 Decay

Orbital decay due to atmospheric drag removes satellites and debris from the system. We assume objects decay at a given rate in a steady state. So the orbital decay rate for a species varies linearly with the number of objects in the system:

$$\omega_{D,\alpha,\ell_i^\alpha}(t) = r_{D,\alpha} N_{\alpha,\ell_i^\alpha}(t) \tag{2}$$

where α indicates the object type (satellite or debris) and ℓ_i^α is the length scale, with i indexing the possible choices. Different decay rate factors govern $r_{D,\text{sat}}, r_{D,\text{deb}}$ for satellites and debris, respectively.

2.2.3 Collisions

Our model includes all possible collisions between object species: satellite-satellite, satellite-debris, and debris-debris. The total rate of collisions can be expressed as a quadratic function of the total number of objects on orbit:

$$\Omega_C(t) = r_C N_{\text{tot}}^2(t) \tag{3}$$

where N_{tot} is the total number of all satellites and debris for all length scales. We treat r_C as a free parameter in our work, although its order of magnitude at $10^{-10}\,\text{yr}^{-1}$ is expected for LEO [17]. The relative collision rate between objects of type α and length scale ℓ_i^α and objects of type β and length scale ℓ_j^β is given by:

$$\omega^C_{\alpha\ell_i^\alpha,\beta\ell_j^\beta} \propto (\ell_i^\alpha + \ell_j^\beta)^2 N_{\alpha\ell_i^\alpha}(N_{\beta\ell_j^\beta} - \delta_{\alpha,\beta}\delta_{i,j}) \tag{4}$$

where the delta function covers the case of two objects of the same species colliding. We normalize the relative rates so that their sum equals the total rate in Eq. (3). Our model includes a lethal debris length scale, ℓ_{lethal}. Debris objects with sizes below the lethal length scale cannot cause catastrophic collisions, which we define as collisions that create additional debris. The model uses the NASA Standard Breakup Model (SBM) to model debris production from collisions. The SBM model for the number of debris above a given length scale ℓ generated by a catastrophic collision is:

$$n_{\text{deb}}(\ell' > \ell) = 0.1(M_1 + M_2)^{0.75}\ell^{-1.71} \tag{5}$$

where $M_{1,2}$ are the masses of the two objects involved in the collision. We calculate the mass for each object as the area divided by the area-to-mass ratio (AMR), which draws from the conditional distribution defined in the SBM [13].

2.2.4 Explosions

Although satellite explosions can be a source of debris and one can model their effects, we assume there are no explosions in the current set of simulations for simplicity. Details of our explosion model can be found in [17].

2.3 Stochastic Sampling

In practice, it is difficult to solve the Master Equation directly for our model. This is because the number of coupled differential equations scales with the number of objects in space, which can grow unbounded. Instead, we generate sample trajectories stochastically. Even the standard strategy of using the Gillespie algorithm for sampling can be difficult because the timestep is inversely proportional to the number of events (e.g., collisions). In the present problem, the timestep can also be extremely small given the run-away number of collisions. Therefore, we apply the tau-leaping approximation to the Gillespie algorithm to generate these samples [7, 8]. The tau-leaping approximation allows us to draw from a Poisson distribution for the number of events in some finite time Δt (timestep of the simulations). We then update the state based on the number of generated events. We repeat this for each trajectory to produce the range of sampled outcomes.

2.4 Risk of Exponential Growth

Our goal is to assess the risk of runaway debris growth (the Kessler syndrome). We define this time-dependent risk as the fraction of trajectories that have experienced runaway growth by a given time: $P_{\rm Exp}(t) \equiv n_{\rm Exp}(t)/n_{\rm total}$, where $n_{\rm total}$ is the total number of simulated trajectories. We consider a trajectory to have experienced runaway growth if it exceeds a high threshold[2] and is increasing quickly. For these trajectories, we define the start time of runaway exponential growth $t_{\rm Exp}$ as the point at which the logarithmic growth of debris experiences the steepest increase as the natural characteristic timescale,

$$t_{\rm Exp} = \mathrm{argmax}\left(\frac{d\ln N_{\rm deb}}{dt}\right) \tag{6}$$

Given this timescale $t_{\rm Exp}$, one can compare it to a given time t to decide whether a particular trajectory has gone exponential, which allows one to count $n_{\rm Exp}(t)$ and compute $P_{\rm Exp}(t)$.

2.5 Debris Injection Event

In this work, we extend the model in [17] by adding debris injection events. We model this by simply adding a number of debris objects to the state of the system at specified times. Mathematically, this can be expressed as:

$$\Delta N_{\rm inj}(t) = N_{\rm inj}\,\delta(t - t_{\rm inj}) \tag{7}$$

where $\delta(t - t_{\rm inj})$ is a Kronecker delta function at $t_{\rm inj}$. The number of debris and the timing since the beginning of the simulations (i.e., $\Delta t_{\rm inj} = t_{\rm inj} - 2022$) are free parameters that we wish to study. As discussed below, we use these events to model the effect of an ASAT test on the orbital environment.

3. RESULTS

The goal of this work is to explore the potential effects of debris injection on the risk of runaway debris growth $P_{\rm Exp}$ using a simple set of models. In particular, we examine how the timing and the amplitude of the debris injection by events like ASAT tests will increase $P_{\rm Exp}$. In what follows, we show the results of the two sets of simulations focusing respectively on the timing and amplitude of the debris injection events.

3.1 ASAT Tests in an Increasingly Crowded Space

As discussed in the introduction, space is getting more crowded as both commercial and governmental entities launch satellites at increasing rates. The potential risk of run-away collisions can be exacerbated with additional debris injected into the already crowded volume. To explore this potentially compounding effect, we run a series of models with sudden debris injections at certain points, varying the timing of these injection events. We define a baseline model with a launch rate of 300 or 1500 satellites per year, a satellite de-orbit timescale of 25 years (in concordance with the historical 25-year rule for satellite de-orbit), a debris decay rate with $D_{\rm deb} = 1/95\,\mathrm{yr}^{-1}$ (typical timescale for debris to decay from 800-1000km), and a nominal collision model with $r_C = 1.77 \times 10^{-10}\,\mathrm{yr}^{-1}$.[3] With this baseline model, we vary the time of the injection events ($\Delta t_{\rm inj}$) since the start of the simulation. In order to not confuse the outcome of the simulations, we only inject once at $\Delta t_{\rm inj}$ with $N_{\rm inj} = 1500$ pieces of lethal debris. The sizes of these debris objects follows a power-law: $N_{\rm deb} \propto \ell_{\rm deb}^{-1.6}$.

Fig. 1 shows, as an example, the simulated trajectories for one of the models where we inject 300 pieces of lethal debris 40 years after the start of the simulation. The orange lines on the left figure represent random sampling of possible debris trajectories due to the stochastic behavior of the simulation. Note that we turn

[2] The outcome is not sensitive to the choice of this threshold, provided it is sufficiently large.
[3] We derive the value of r_C to match the historical collision rates given by Equation 3.

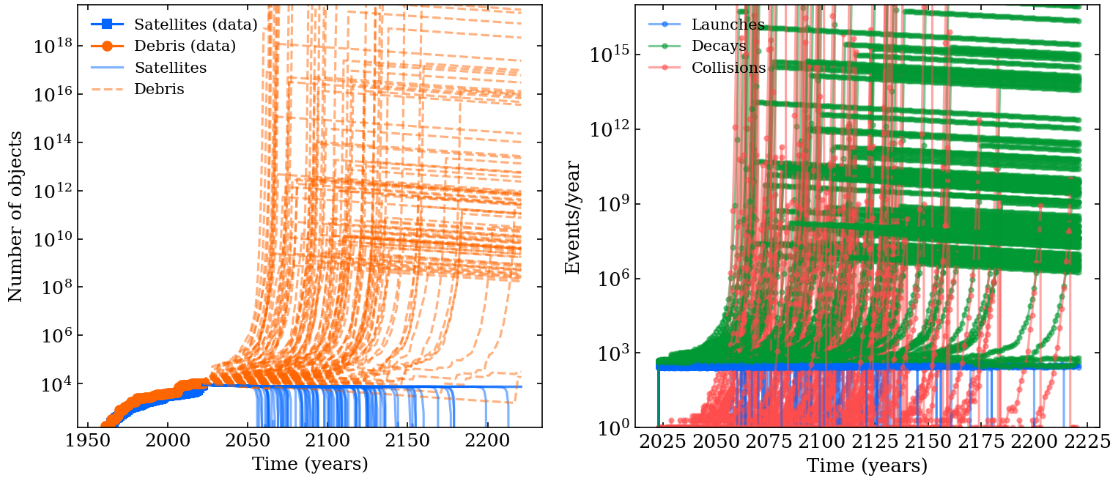

Fig. 1: This figure shows the set of simulated trajectories with a launch rate of 300 satellites per year. Left panel shows the evolution of debris and satellite over time. The blue and orange points represent historical data. Each line of the same color is a trial in the simulation, experiencing a variation in the sampling of the process rates. At some point during the exponential growth of debris, all satellites are destroyed (blue line drops to zero). This is also where we have turned off satellite launches. The orange lines correspondingly decay with the rate that we set. The panel on the right shows the corresponding rates for various types of events. Although it is not visible in this scale, we have also injected 1500 pieces of lethal debris 40 years after the start of the simulation (2022).

off launch when we reach a state where a given trajectory is experiencing exponential growth. This is where satellite (blue lines) are downturning to zeros because all satellites are destroyed. Subsequently, the debris (orange lines) decay according to the rate we set. The panel on the right of Fig. 1 shows the corresponding counts for various events that occurred throughout the simulation for all the simulated trajectories. For example, the red lines for collisions drive the debris growth on the left. As additional debris introduced to the environment due to increased collisions, the number of decaying events (green lines) also grow until the number of collisions slows down, corresponding to the same behavior in orange lines on the left panel that shows the decay in debris counts.

The main purpose of Fig. 1 is to show there exists variations in the simulated trajectories, in particular the spread of exponential growth timescales. It is perhaps more illuminating to consider a figure that summarizes this variation using the risk parameter P_{Exp}. Fig. 2 shows P_{Exp} over time, and compares simulations with different injection timing (e.g., Δt_{inj} since 2022). These are four simulations with the same baseline parameters and Δt_{inj} varying from 5, 25, 40, to 100 years. As shown in the left panel of the Fig. 2, with a launch rate of 1500 satellites per year, and nominal satellite and debris decay rates, the risks of Kessler syndrome climbs quickly toward unity well within a hundred years given an additional ASAT test after 2022 . For the simulation where debris injection is delayed until $\Delta t_{\mathrm{inj}} = 100$ (i.e., 2122), the risk grows slowly compared to the others initially. However, as soon as the injection happens, the risk jumps to 100% within a year or two. However, one might find the risks intolerable even without injection; this figure suggests that a launch rate of 1500 satellites per year, given the current modeling assumptions, is simply too aggressive.

If we lower the launch rate to 300 satellites per year, as shown in right panel of Fig. 2, the overall risks are smaller and the rate of increase also slows down. Furthermore, the timing of the debris injection changes when and how quickly P_{Exp} grows. In particular, one can see that the risks in simulations with injections at $\Delta t_{\mathrm{inj}} = 5$ and $\Delta t_{\mathrm{inj}} = 25$ are growing faster than others where injections have not happened yet. Clearly, debris injected in the early times of the simulations can initiate additional collisions that would otherwise not happen. Their effect is both increasing the overall risk and the rate at which it climbs given the collision cascade. To put it differently, the risk curve for $\Delta t_{\mathrm{inj}} = 100$ grows the slowest initially compared to the others; but it jumps nearly instantaneously at the time of debris injection (100 years after 2022) given the more

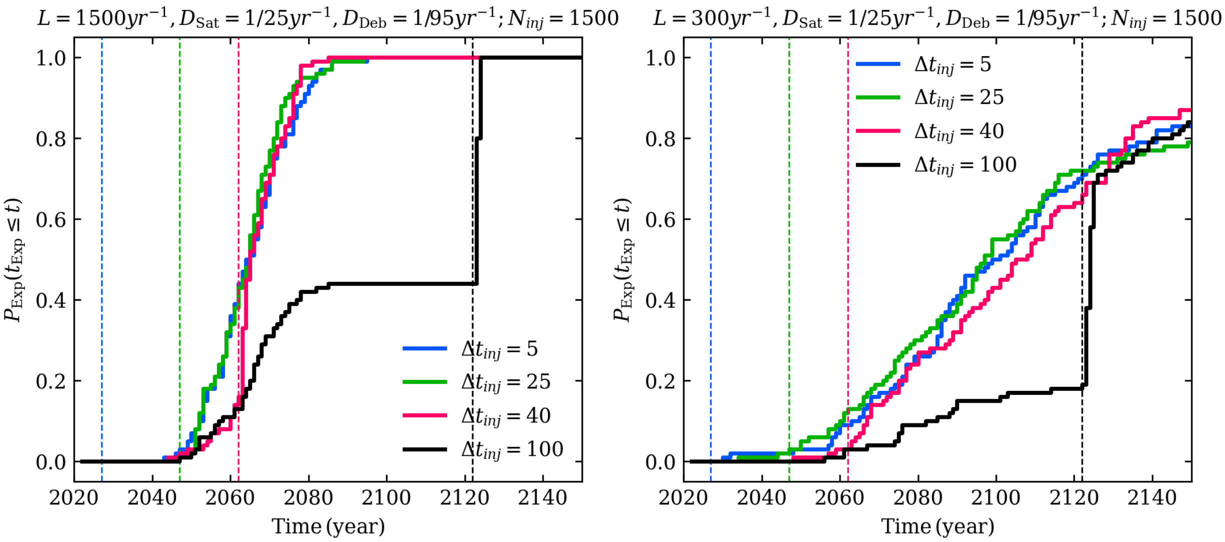

Fig. 2: Simulations with varying debris injection time Δt_{inj} for a set of baseline parameters. The left and right panels have share the same parameters except the launch rate (1500/yr for the left panel and 300/yr for the right panel). The vertical dashed lines correspond to the timing of the injection of the same color in the solid lines. For a launch rate of 1500/yr, P_{Exp} climbs toward unity in all simulations given the aggressive launch rate. The black lines show the slower increase of the probability until a debris injection happens, which causes the probability to shoot up to the level of the other scenarios.

crowded space. Those debris injected at a later time, in a more crowded volume, have a larger effect.

If we assume the 1500 satellites launched per year, reality from the last few years continues into the future, one can consider how the de-orbit rules can help suppress the long-term growth in the presence of ASAT tests. Recently, the Federal Communications Commission in the United States has adopted a shorter de-orbit rule, changing from the nominal 25 to 5 years.[4] Fig. 3 shows the model results with this change. The amplitude and growth rates of the risks are very similar to those in right panel of Fig. 2, which had a lower launch rate of 300 satellites and longer de-orbit timescale (25 years). Therefore, the 5-year-rule seems to be one way that adjusts the new normal (high launch rate) back to historical growth in terms of the risk of Kessler Syndrome.

3.2 Amplitude of the Debris Injection by ASAT Tests

Another parameter one could vary within our model framework is the amplitude of the debris injection N_{inj}, i.e., the number of lethal debris injected into the environment.

Fig. 4 shows the growth of the risk parameter with varying N_{inj} for launch rates of 1500 and 300 satellites per year, respectively. For both sets of simulations, we have set the injection time $\Delta t_{\mathrm{inj}} = 5$ years after the start of the simulations. We are interested in how that initial injection amount can influence the future growth of the risk parameter. As shown in Fig. 4, the additional debris injection only has a moderate effect on the risk curves given the launch rate is already so high (i.e., relative difference in P_{Exp} is small due to different N_{inj}). They all quickly reach unity around 2080 given the current modeling assumptions. While this may be alarming, we should emphasize the caveats that current model has many simplifying assumptions, such as modeling the entire LEO as a single volume where all objects can collide with each other. This assumption in particular can overestimate how quickly the risk can grow to unity if at all. Nevertheless, these simple models can provide insights on how the debris injection can shape the future. For example, under the assumption that launch rate stays at $L = 300$ satellite per year, right panel of Fig. 4 shows P_{Exp} grows much slower than those with $L = 1500$/yr. Moreover, the changes in the risks due to different N_{inj} is

[4] https://www.fcc.gov/document/fcc-adopts-new-5-year-rule-deorbiting-satellites

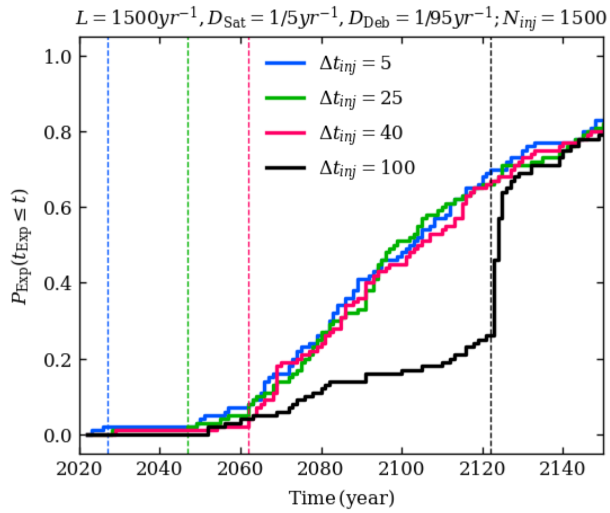

Fig. 3: Simulations with varying debris injection time Δt_{inj} with 1500 satellites launched per year and a reduced de-orbit timescale of 5 years. The number of injected lethal debris remains the same at 1500 pieces. The magnitude and growth rates are similar to those with a lower launch rate of 300 satellites per year but with a longer de-orbit timescale of 25 years.

more dramatic because the injected debris are relatively more important when the launch rate is smaller and consequently a less crowded environment.

4. DISCUSSION AND CONCLUSION

The two sets of simulation experiments, varying injection timing and amplitude, are both testing the same sensitivity to orbital capacity and the state of the space environment. In other words, when the system is close to being "full", a debris injection may push it over its limit and kick start the run-way collisions and debris growth. Given that the model is stochastic, this may not happen every time; there may be scenarios where enough debris decays before collisions can happen to kick-start the run-away process. In general, this stochasticity originates from the random sampling of the processes that we include in the model (e.g., launch, decay, collisions). The probability P_{Exp} is one way to quantify the increased risks as shown. However, this is not the only choice as discussed in our previous work [17]. In the current modeling, we assume a single volume for all LEO objects; they can all interact with each other via the coupling (i.e., r_C) that we assume in the collision model. In reality, when modeling the LEO region with multiple altitude shells, not all objects can interact with one another via collisions, especially with the assumption that all objects have circular orbits. So, we expect the run-away growth would not happen so dramatically in models incorporating altitude-dependent shells. In those scenarios, one can design a slightly different risk parameter, such as the fraction of simulated trajectories with debris greater than a tolerable threshold, i.e., $P(t, N_{\text{deb}} > N_{\text{deb,th}})$. We refer readers to [17] for the discussion of other limitations in our current simple toy models, other than to say that we will extend the model to include altitude shells and more realistic processes and associated parameters in future work.

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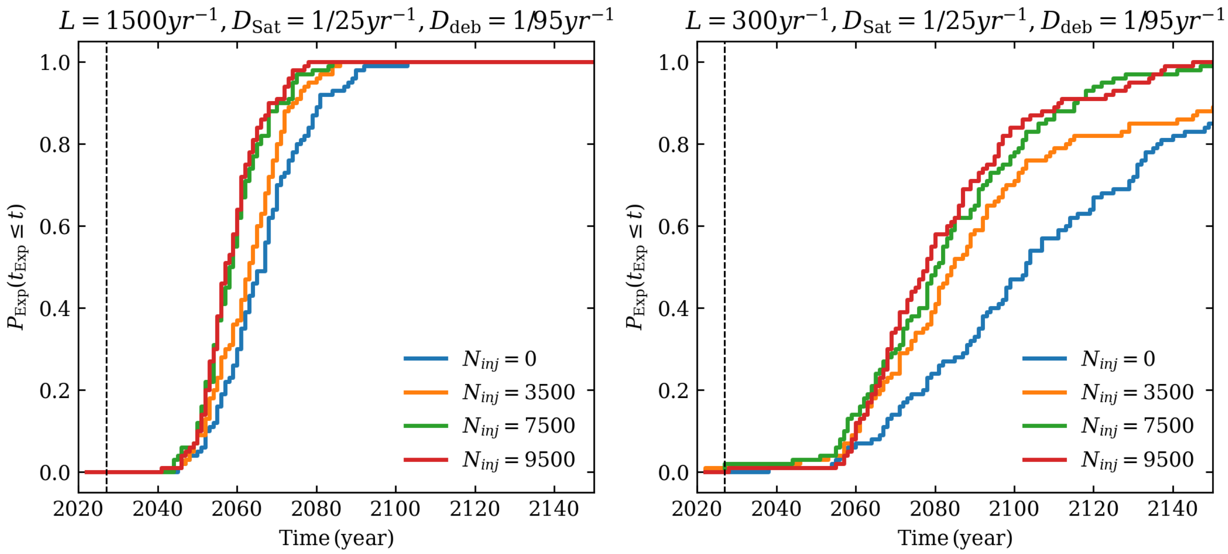

Fig. 4: Simulations with varying debris injection amplitude N_{inj} for launch rate of 1500 satellites per year (left panel) and 300 per year (right panel). The vertical black dashed line corresponds to the injection time 5 years after the beginning of the simulations. With a launch rate of 1500 satellites per year, the probability of run-away debris growth is increasing quickly for all simulations. Relatively to a baseline without any injected debris (blue line), large values of N_{inj} matter moderately for simulations for the given current set of modeling assumptions and parameters. In other words, the orange, green and red lines are roughly few to 20% higher than the blue line. In this case, the change in the risk is slightly more clear due to the relative higher fraction of injected debris to the total objects in space (i.e., lower launch rate).

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