Adjustable thresholds for tracklet-to-tracklet correlation of optical observations

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ABSTRACT

An up-to-date catalogue is crucial for Space Situational Awareness. To keep the orbits of known objects accurate and find new objects regular observations have to be performed. Surveys of the geostationary region produce large numbers of short observation arcs or tracklets. They can be correlated to catalogued objects via a tracklet-to-orbit correlation process. If the observed object is not catalogued, an initial orbit determination can be attempted. Due to the length of the tracklet, this might not lead to a satisfying result. Therefore, one tracklet should be correlated with at least one other tracklet to gather enough information for a precise initial orbit determination. As part of this work, a boundary value method for tracklet correlation was implemented. The goal was to find factors influencing the performance of the method. In the implemented method, angular positions from two observations are used to calculate an orbit. To evaluate the fit of an observation with the hypothesized orbit, their angular velocities are compared. The Mahalanobis distance between the angular velocities of the real observation and a simulated observation from the calculated orbit is used as a measurement of closeness between the orbit and the tracklet. When dealing with optical observations, the orbit between two observations is not uniquely determined. To find one unique orbit, the range to the object can be fixed for both observations. A downhill simplex algorithm was chosen to optimize the range and find the best-fitting orbit for two observations. When provided with several tracklets a correlation matrix is created containing the lowest possible Mahalanobis distance for every possible combination of two tracklets. The program was tested with simulated observations of 300 objects, covering geostationary and geotransfer orbits. For the simulation, perturbations due to geopotential, drag, solar radiation pressure, and third bodies were considered. The length of the tracklets varied between 30 and 120 seconds. The time between two observations was between 3 hours and 5 days. For the simulated survey campaign, 5 different sensor locations across the globe were used. The approach was shown to work well for observations less than a day apart. More than 90 percent of the tracklet pairs were identified. It worked decently well for observations that were multiple days apart. In this case, 60 percent of correlations were found. From the simulations, major influences on the quality of the correlation were identified. The main factors found to be affecting the fit between correlated tracklets are the time between observations, the orbit of the observed satellite, and the position of the observation on the satellite's orbit. It was found that using this information can help in deciding which tracklets belong to the same object. Different methods were applied to the correlation matrix to find tracklet pairs belonging to the same object. The simplest way to decide whether two objects are correlated is by applying a fixed threshold on the Mahalanobis distance between them. This typically leads to a significant number of false positives and false negatives in case the data is from different sensors and objects. It was shown that it can be beneficial only to consider the best possible correlation for each tracklet. While this might lead to missing other correct correlations, one correlation is enough to calculate a preliminary orbit. The number of true positives can be improved by adjusting the threshold to the hypothesized orbit of the tracklet pair. A low threshold can be used for GEO objects, while a higher one is applied to GTO objects. This can improve the number of true correlations without affecting false positives.

1. INTRODUCTION

Having an accurate, up-to-date, and complete catalogue is the key to Space Situational Awareness [3]. Several challenges present themselves when building and maintaining a catalogue. Regular measurements from known objects are necessary to perform orbit determinations and keep the orbits accurate [13]. Telescopes and Radars generate measurements in tracking mode or in survey mode. While it is well-known which object was observed in the first case, it is more difficult in the survey mode [3]. When measurements are obtained without knowing which object was observed some tagging is necessary to avoid losing the information [10].

Different ways of treating untagged measurements are available. In most cases, the object will already have been catalogued. If the orbit is accurate enough a track-to-orbit correlation can be performed. In that case, the observation can be compared to known orbits to find the best match [10]. This process is rather efficient and accurate. It is possible that no catalogued object matches the observation well enough. This can happen when the observed object was recently launched or created, or when the catalogued orbit is so outdated that no successful correlation was possible. If the track contains enough measurements over a sufficient amount of time it is possible to perform an initial orbit determination [6]. In many cases, short optical tracks, spanning a few seconds to minutes, do not contain enough information to reliably calculate an accurate orbit. To use the information from these tracklets a track-to-track correlation has to be performed.

While a single tracklet might not contain the information needed to compute an orbit, it is possible to collect multiple tracklets of the same object and perform an orbit determination with those. The challenge then becomes, finding out which tracklets belong to the same object. Satellites that were recently launched might be moving close to the rocket stage that launched it or other objects from the same launch. When caused by a fragmentation, many new objects will be found in similar orbits. For these cases, it is important to avoid false positives in the correlation. Large surveys will produce a significant number of uncorrelated tracklets. To ensure successful correlations, a process is necessary that differentiates between correlated and uncorrelated tracklet pairs accurately. A pair-by-pair comparison of n tracklets will require a number of comparisons N:

$$N = \frac{n \cdot (n-1)}{2} \in \mathscr{O}(n^2) \tag{1}$$

showing the importance of efficiency when correlating a large number of tracklets. The process will work by comparing all possible combinations of tracklets and assigning a value to each of them. This value is then evaluated to decide whether they are correlated or not correlated.

Depending on the properties of the tracklets to be correlated different methods are available [10]. For Radar tracklets [8] describes a method. Range and range-rate information from radar measurements can be used to simplify the calculation of the orbit between tracklets. In case the tracklet is very short and effectively only contains one measurement 3 tracklets might be necessary to perform a correlation. A method for these cases is presented in [9]. This work is focused on optical tracklets that are too short to perform an IOD but long enough to estimate the angular velocity of the track. From the tracklets, attributables A are formed containing the angular position's azimuth β and elevation el and their first derivatives $\dot{\beta}$ and elevation $\dot{el}$ at the epoch t:

$$A_n = \{t_n, \beta_n, el_n, \dot{\beta}_n, \dot{el}_n\}. \tag{2}$$

Multiple types of correlation are available for this type of data. When comparing two attributables they split up available information into information used to calculate an orbit and information used to evaluate the fit of the orbit to the tracklets. So in general the steps to perform a correlation will be the following:

1. Pre-filter tracklet pairs,
2. Find best-fitting orbit for each tracklet pair,
3. Save a measure of fitness in the correlation matrix,
4. Evaluate the correlation matrix for correlated pairs,
5. Save correlated pairs and preliminary orbits.

The goal of this work is to find, implement, and evaluate a correlation method. It has to be able to handle a very large number of tracklets. The objects of interest are those in and around the geostationary region. It should be able to handle tracklets originating from a global network of optical sensors. To achieve this a set of pre-filters is defined to save computational effort on unnecessary comparisons. For the correlation, a boundary value method is chosen to calculate orbits between tracklets. This is combined with a downhill simplex optimization to find the optimal fit between the observations. Finally, different approaches are investigated to find tracklet pairs in the resulting correlation matrix. Here it can be shown that instead of using a fixed threshold to decide about correlation, it can pay off to adjust the threshold. Changing thresholds depending on certain characteristics of the used tracklets can lead to a reduction of both false positives and false negatives. In the following, an overview of the detailed problem description and test cases is given in Section 2. A description of existing methods and the chosen approach can be found in Section 3. The results of applying it to the given problem are presented in Section 4 and a conclusion can be found in Section 5.

2. PROBLEM DESCRIPTION

When choosing and implementing an approach the problem that it is applied to should be taken into consideration. A method that performs well for a certain type of problem might not work well for another. The main factors for picking an approach are the orbital regime of interest, the used sensors, the length and quality of tracklets, and the number of tracklets to correlate. The orbit influences the perturbations that should be considered, the sensors and tracklet properties determine which information is available for the correlation. The number of tracklets sets requirements for the efficiency and robustness of the method.

This work is aimed at providing a method for correlations in the geostationary region. It includes not only objects in GEO itself but also objects that come close to GEO. Orbits that might affect GEO are GTOs, graveyard orbits, and orbits of debris originating from the GEO belt. Objects in this region will mostly be observed by optical sensors. Here the tracklets are assumed to be provided by a network of telescopes placed around the Earth. The tracklets themselves are between 30 and 120 seconds in length. While this is too short to perform IOD, it is sufficient to calculate accurate angular velocities. The method described here is aimed at correlating a very large number of tracklets, observed within the span of a few days. The goal is to find as many, previously unknown, objects as possible while keeping a low number of false correlations.

2.1 Test cases

Since no sufficient number of real tracklets were available at the time of writing, a process was set up to simulate accurate tracklets. To cover the observed region, it is split into three categories. The GEO category represents the actual geostationary satellites and debris released by them recently. Their semi-major axis is varied very little and eccentricity as well as inclination are close to zero. A second category is placed near GEO but with a larger variety in semi-major axis as well as eccentricity and inclination. These represent objects in graveyard orbits as well as uncontrolled objects that have drifted from GEO due to perturbations. Finally, objects in the GTO category have a perigee close to the Earth and an apogee around the altitude of the GEO belt. They have a larger variety in eccentricity and inclination. For each category 100 orbits are created by randomly selecting parameters within the boundaries described in Table 1.

	GEO	**GTO**	**Close to GEO**
SMA / km	42114 - 42214	23450.0 - 25650.0	41664.0 - 42664.0
e / -	$0.0 - 5 \cdot 10^{-4}$	0.69 - 0.73	0.0 - 0.023
i / °	0.0 - 1.0	0.0 - 27.0	0.0 - 15.0

Table 1: Simulated orbit categories.

The simulated orbits are propagated for 14 days with the numerical propagator NEPTUNE [1]. Atmospheric drag, geopotential, and third-body perturbations are considered for the propagation. To represent a sensor network, 5 stations are placed around the Earth as shown in Figure 1. All passes of the simulated objects over the stations are calculated via the OKAPI:Orbits API [2]. Noise and bias parameters are included in the stations to create realistic, simulated tracks.

[1] Open source at https://github.com/Space-Systems/neptune
[2] Documetation can be found at https://okapiorbits.space/documentation/

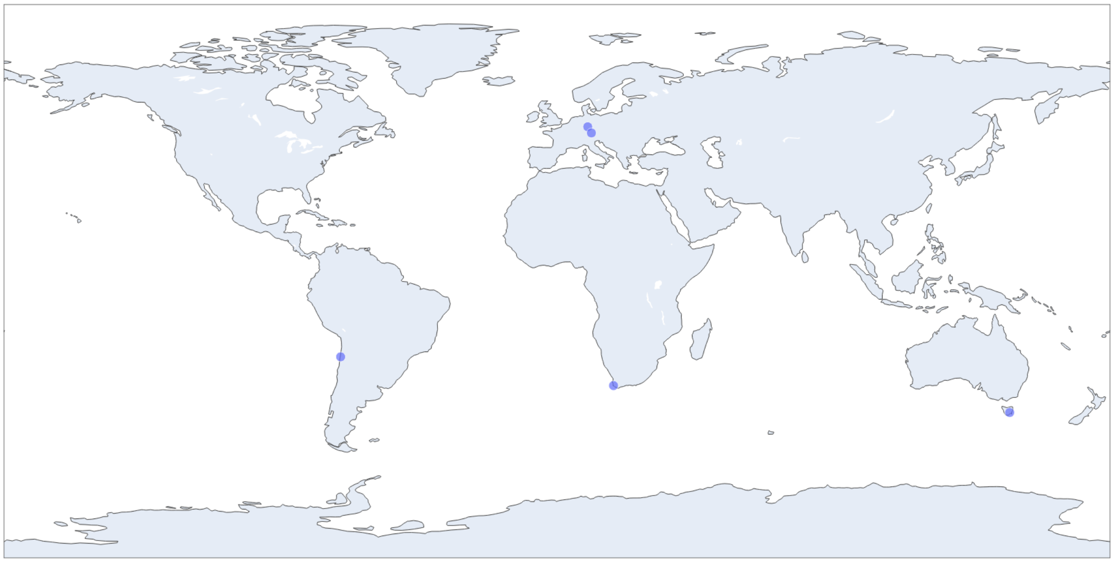

Fig. 1: Distribution of simulated sensor stations for simulated surveys.

To simulate the results of a survey campaign, the tracks are broken down into random tracklets of 30s to 120s in length. The time between tracklets of the same object and the number of tracklets produced per object is varied. This way catalogues of around 500 tracklets are produced that can be used to test and evaluate an implemented correlation process. From the tracklets, attributables are formed. These contain angular positions and velocities at one epoch. They also contain the covariance of the angular velocities.

As an additional case, fragmentations are simulated. This is achieved by duplicating one orbit 18 times and changing the velocity of the initial state by different values between 10 and 1000 $\frac{m}{s}$. A small cloud of objects with similar orbits is created. Tracklets are simulated between 1 and 10 days after the simulated fragmentation. This case is especially difficult due to how close all the objects are to each other.

3. APPROACH

To perform the complete correlation process an approach for each step described in Section 1 has to be chosen. The first step in the process would be filtering out unlikely correlations. Here the basic correlation process will be described first in Section 3.1. The choice of pre-filters then depends on the chosen approach and is explained in Section 3.2. A method to optimize the orbit of each tracklet combination is explained in Section 3.3. In Section 3.4 the process to decide on correlated pairs is explained and in Section 3.5 the implementation is described.

3.1 Correlation methods

Correlation methods can be differentiated mainly by which parameters are used to calculate an orbit and which are used to evaluate the fit of the orbit to the tracklets [10]. The main options are initial value and boundary value methods which are described in the following sections.

3.1.1 Initial Value methods

Initial value methods use all the information of one tracklet to compute an orbit. By comparing it to the second tracklet the correlation can be evaluated. For optical measurements, an attributable formed from a tracklet, as described in Equation 2, will contain the line of sight and angular velocity of the observed object. Without additional information, this is not enough to uniquely define an orbit. Range ρ and range rate $\dot{\rho}$ are therefore free parameters. They can be chosen in a way that leads to the best possible fit for the second tracklet. For evaluating the fit between orbits, a comparison can be performed in observation space [6] or in the 6-dimensional hyperplane of orbital elements [5]. In either case, a propagation to a common epoch is necessary to perform the comparison.

Initial value methods have the advantage of being able to consider any number of perturbations in the correlation process. This makes them interesting with objects affected by more significant perturbations or when the time between tracklets is large. Orbits can also be precomputed and propagated for each individual tracklet. When new observations are added, a re-computation of existing tracklets might be saved. On the other hand, the propagation step of the calculation is computationally expensive. Especially, when complex perturbations are considered. As pointed out in [10], the ρ, $\dot{\rho}$ search space can contain multiple local minima corresponding to the number of revolutions between the observations. A powerful optimization method has to be applied to find the global minimum. Alternatively, the search space can be separated into sections which creates more complexity [11]. Finally, using angular velocities to calculate an orbit can be problematic. For short tracklets, the angular velocities might be less well-defined than using just the angles. This means that the calculated orbits might have biases which can lead to lower rates of success for this method.

3.1.2 Boundary value methods

Instead of using all the information from one tracklet and comparing it to the second, boundary value methods split the information of both tracklets and use angles at both epochs to calculate an orbit. Angular velocities between the calculated orbit and attributables are then compared to evaluate the fit between it and the orbit [10]. There are again 4 fixed parameters to the orbit and 2 free ones to optimize. In this case with the ranges to both observations ρ_1 and ρ_2 position vectors at both epochs can be calculated. The angular velocities are used as discriminators to evaluate the fit of the orbit to the measurements. So the parameters of the attributables are split up as follows:

- Fixed parameters: $\beta_1, \beta_2, el_1, el_2$.
- Free parameters: ρ_1, ρ_2
- Discriminators: $\dot{\beta}_1, \dot{\beta}_2, \dot{el}_1, \dot{el}_2$

With these two position vectors, the full orbit can be calculated at those times via the Lambert Transfer [2]. The Lamber Transfer works by fitting an orbit through two position vectors, perturbations are generally not considered. It introduces two additional free parameters since the direction of motion and the number of revolutions between the states are not uniquely determined. Each of them can be treated as an individual optimization, containing one optimum. When both ranges, the direction of motion, and the number of revolutions are set, the 6-dimensional state is obtained at both observed epochs.

The cartesian state at the observed epochs can be transformed back into the observation space. While the angular positions of both the observed and the calculated measurements will contain the same information, the angular velocities can be used to evaluate the fit between calculation and observation. Comparing the four angular velocities, 2 for each epoch, can be done via the Mahalanobis distance [4]

$$d_m = \sqrt{(\vec{x}_1 - \vec{x}_2)^T C^{-1} (\vec{x}_1 - \vec{x}_2)} \quad (3)$$

For a fixed number of revolutions and direction of motion, the function Mahalanobis distance over the ranges to each tracklet is convex containing only one optimum. An example of such a function geometry is shown in Figure 2. In some cases the optimum lies within a thin valley surrounded by steep inclines. If the number of revolutions or the direction of motion is too far from the correct value, no optimum is found in the limits of the search space.

Using the Lambert Transfer as described above makes considering perturbations more difficult compared to the initial value methods. Separating solutions by the number of revolutions, on the other hand, becomes much easier. Since direction of motion and number of revolutions can be fixed in the Lamber transfer the optimization can be split into a limited number of problems, each with just one optimum. This allows for a less complex optimization method to be used.

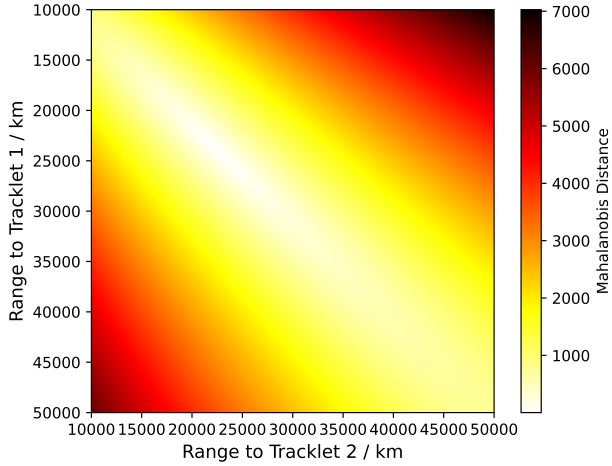

Fig. 2: Figure of the convex geometry based on the Mahalanobis distance of the boundary value method.

3.2 Pre-filters and constraints

The comparison of two tracklets takes a significant effort. To speed up the process, some comparisons can be skipped if it is clear that the tracklets do not belong to the same object. For the optimization of free parameters, the search space can also be limited with some heuristics.

Skipping correlations

The easiest way of increasing the speed of the process is to not perform correlations when a priori knowledge exists that hints at the objects not being correlated. One has to be careful though not to exclude correlated pairs while also sorting out as many uncorrelated pairs as possible. For tracklets from the same station, a simple check can be applied. When the time difference between the tracklets is less than 4 hours, tracklet pairs can be excluded if their angular distance is too large to be travelled in the time between observations. With the azimuth β and the elevation el, the angular distance between tracklets d_{ang} is

$$d_{ang} = \arccos(\sin(el_1) \cdot \sin(el_2) + \cos(el_1) \cdot \cos(el_2) \cdot \cos(\beta_1 - \beta_2)). \tag{4}$$

The combined angular velocity $\dot{d}_{ang}$ can be calculated via

$$\dot{d}_{ang} = \sqrt{\dot{el_1}^2 + \dot{el_2}^2 + \dot{\beta_1}^2 + \dot{\beta_2}^2}. \tag{5}$$

And the time difference Δt between measurements is

$$\Delta t = |t_2 - t_1| \tag{6}$$

. A limit can be set to consider tracklets to be uncorrelated if

$$d_{ang} >= \dot{d_{ang}} \cdot \Delta t \tag{7}$$

. This should not be applied if the tracklets are further apart in time. In those cases, the assumption of constant angular motion fails. When tracklets from multiple stations are processed, an object may be observed from different positions at the same time. While this is unlikely to happen randomly, survey campaigns might aim at observing the same area of the sky from different positions. Two tracklets that were observed from different stations less than x seconds apart can be pre-processed. The closest point between the two lines of sight can be calculated from there.

It can be assumed that the tracklets are of the same object if:

- The closest point between lines of sight is ¡ 1 km apart

- The distance to the earth is ¡ 100000 kilometres at that point
- The distance to the earth is ¿ 6700 kilometres at that point

If these points are fulfilled it is almost certain that the same object was observed from both stations. In this case, information from both measurements can be combined to find the position of the object at the time of observation. Although the tracklets are correlated, no orbit is calculated from them in this case. Instead, both tracklets are updated with information on the range to the object. When an updated tracklet is correlated only one range will have to be optimized, thus saving effort. For tracklet pairs that do not fulfil the criteria, it can be safely assumed that the tracklets are not correlated.

Constraining the search space

To speed up the optimization, the search space can be reduced. This can also help to avoid divergence. With the chosen approach, multiple optimizations are performed for each tracklet pair. For each optimization, the number of revolutions between the tracklets is set. The number of revolutions n_{rev} and thereby optimizations can be constrained by setting upper and lower limits for the orbital period T_U:

$$n_{rev,max} = int(\frac{T_{obs,2} - T_{obs,1}}{T_{U,min}}) + 1 \tag{8}$$

$$n_{rev,min} = int(\frac{T_{obs,2} - T_{obs,1}}{T_{U,max}}) - 1 \tag{9}$$

Objects affecting the GEO region will have orbital periods ranging from around 10 hours for objects in Geo Transfer Orbits to more than 24 hours for graveyard orbits.

For each individual optimization, the starting point can be set by taking the most likely orbit and distance to the sensor into account. Depending on the optimization method one or multiple starting values for the ranges to each tracklet have to be chosen. In general, optimizations will perform better when the initial guess is close to the optimum.

More complex initial constraints based on the angular velocities and directions of motion can be applied. These rely on an accurate estimate of these velocities. Especially for very short tracklets, these might not be given. So more complex constraints might not be applicable in the general case. If no other information is present, starting ranges between 35000 and 45000 kilometres are assumed.

3.3 Optimization

A great variety of optimization methods offer themselves to be applied to the given problem. Taking the properties of the loss function into account, a suitable method can be chosen. As described in Section 3.1.2 the function geometry of boundary value methods contains only one minimum. This is beneficial since it avoids the possibility of converging to a local but not global minimum. While stochastic methods like genetic algorithms or simulated annealing might be able to deal with these cases [1], they are not necessary here. Deterministic methods find optima fast and reliable for convex cases like this.

Gradient methods, like Quasi-Newton, have been applied successfully to the problem of tracklet-to-tracklet correlation [10]. The disadvantage here is that gradients are not directly available as a result of the Lambert Transfer. They can be computed numerically through finite differencing. This may cause some additional difficulties. Alternatively, methods that do not use gradients can also be applied. An example of such a method would be Nelder-Mead [12], which works reliably and reasonably fast for the given problem. Figure 3 shows an example of the first steps the optimization takes for the correlation. From an initial simplex of three points, it works by replacing the worst of the three points through contraction, expansion or shrinking. It does not rely on gradients. Even in thin valleys, it performs well by adjusting the shape of the simplex.

3.4 Decision-making

The optimization of an orbit for each tracklet pair results in a correlation matrix containing the lowest possible Mahalanobis distance between every combination of tracklets. An example is shown in Figure 4. Brighter colours represent a better fit. Tracklets were sorted, so correlated pairs appear on the main diagonal. To decide which tracklets belong

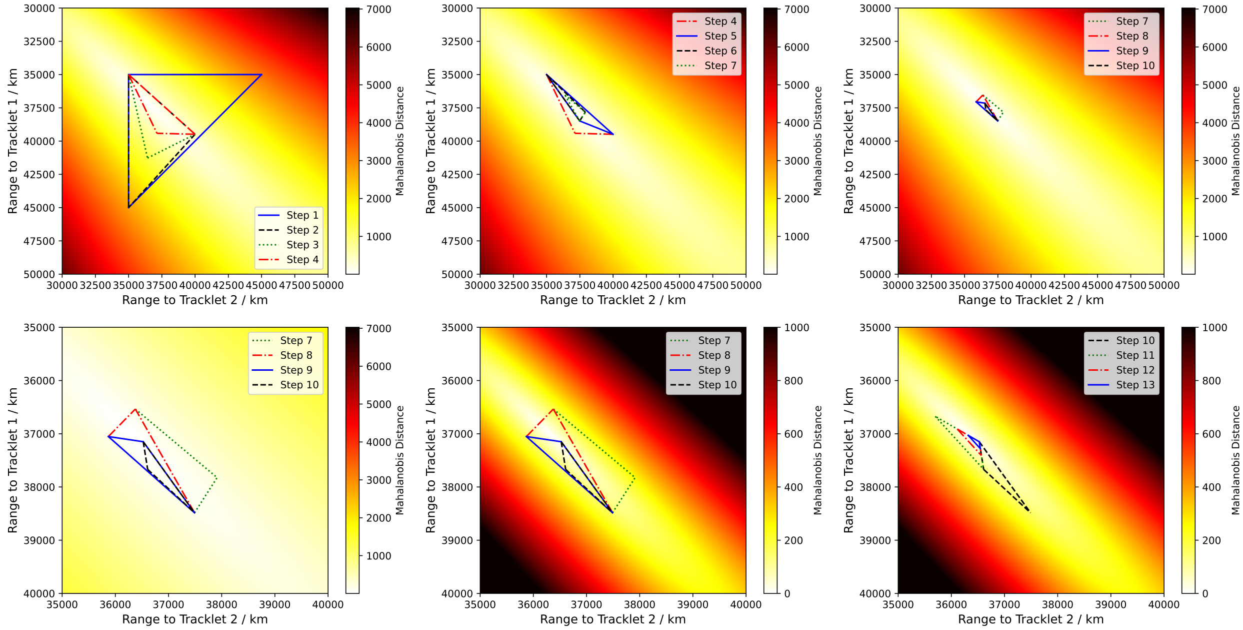

Fig. 3: First steps of the optimization process, finding the optimal ranges to each optimization based on the Mahalanobis between measured and calculated angular velocities.

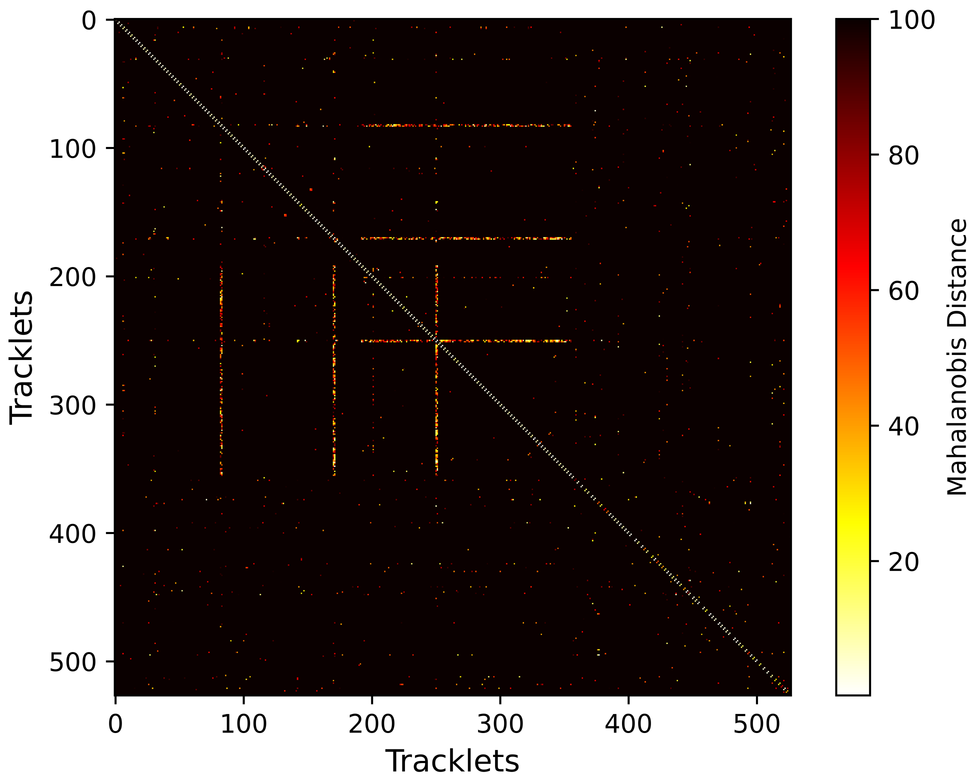

Fig. 4: Correlation matrix showing the best possible fit between each possible combination of tracklets.

to the same object, a decision criterion has to be set. The goal of this process is to find as many correct correlations (true positives) as possible while keeping the number of falsely correlated objects (false positives) low.

For picking a suitable threshold the distribution of Mahalanobis distances of correlated pairs can be taken into consideration. Figure 5 can give an idea of how many correct correlations a threshold will result in. It should be noted that this distribution depends on the sensors, observed objects, distribution of measurements and other parameters. So the decision-making strategy has to be adapted to the data that it is applied to.

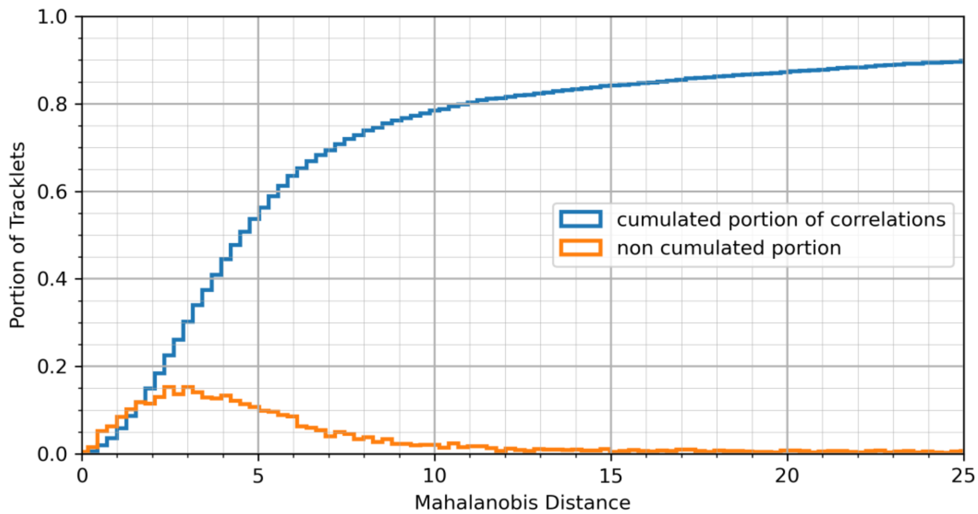

Fig. 5: Distribution of Mahalanobis distances for correlated tracklet pairs.

The Mahalanobis distance of correlated tracklet pairs depends on multiple parameters. Some of these are known when a decision on correlated pairs is made. Knowing the expected behaviour of the correlation process can help decide which tracklets belong to the same object. The boundary value method that is being applied to calculate orbits does not consider perturbations. This means that it has to be expected that the accuracy of the orbit will decrease with time between observations. Orbits that are more significantly affected by perturbations are also less well-modelled. A less accurate orbit will lead to a worse fit between calculated and real angular velocities. Therefore, it is expected that tracklet pairs of objects affected by more significant perturbations will have higher Mahalanobis distances on average.

From the simulated objects, those in GTO will be less well described by two-body motion. This is shown in Figure 6. The histogram shows that objects in GTO produce tracklet pairs that have a higher Mahalanobis distance while objects in GEO tend to have a lower Mahalanobis distance. It can also be seen in Figure 7 that the average Mahalanobis distance grows with the time between observations. These factors can be made available for the decision-making. The orbit that is calculated from the tracklets via Lambert Transfer will not be entirely accurate [10]. For classifying the orbits into categories the calculated orbital parameters are sufficient.

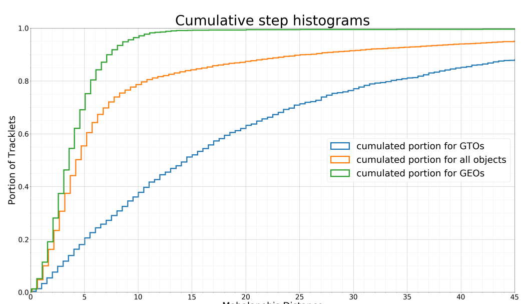

Fig. 6: Distributions of Mahalanobis distances for correlated tracklet pairs separated by orbit.

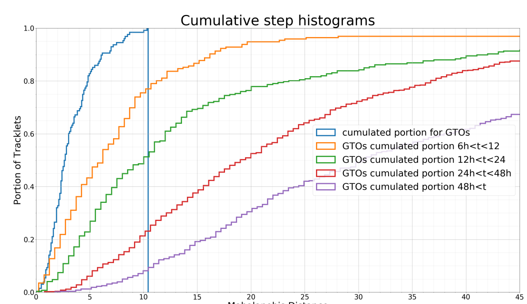

Fig. 7: Distributions of Mahalanobis distances for correlated tracklet pairs separated by time.

The usual procedure for deciding on correlated objects involves applying a threshold that is used for every pair. When this is applied to tracklets of different origins from multiple days, a high threshold would be necessary to capture the correlations from all objects. This is not practical as it would lead to many false correlations. Objects in GEO are so close together that they cause false correlations when too high thresholds are applied. Using a low threshold instead would lead to few successful correlations of GTO objects.

To improve upon this, the threshold can be adjusted to the properties of the tracklets and the orbit during the decision-making process. The orbital parameters of each correlation can be stored along with the Mahalanobis. This way a low threshold can be applied to the densely populated GEO region, while a higher threshold is used for GTO objects. In cases where the calculated orbit intersects the Earth or has an eccentricity > 1, correlations can also be sorted out. Additionally, using only the closest correlation avoids false correlations in case multiple objects are close to each other.

3.5 Implementation

A boundary value method using the Nelder-Mead method to optimize ranges to the tracklets and Lambert Transfer to calculate orbits is implemented in Fortran. The program takes a list of tracklets and forms attributables containing one pair of angles and angular velocities. Then it iterates over all possible combinations of tracklet pairs. The first iteration is done to filter out unlikely pairs, as described in Section 3.2. In the second step, another iteration is performed over all the tracklet pairs that were not sorted out previously. This process is parallelized since individual correlations do not affect each other.

For every correlation, three pairs of starting ranges are chosen. From each of them, an orbit is created, and the angular velocities are evaluated. This is used as the initial step of the optimization. In the steps of the optimization, new orbits are calculated and evaluated until the process converges. The converged results are stored with their Mahalanobis distance, orbital parameters, and time between tracklets for each combination. Decisions on which pairs are correlated are made by a separate program to allow one to easily try different strategies on a correlation matrix once it is computed.

4. RESULTS

To evaluate the performance of the correlation method, it is applied to the test cases described in Section 2.1. The timeframe of the simulated survey is varied between 3 hours and 3 days. The different approaches are applied to the resulting correlation matrix. Comparing the results to the correct correlation pairs yields the portion of true and false positives for different thresholds. As an additional test, the correlation is applied to a smaller subgroup of tracklets of very similar orbits. This is to represent a fragmentation or recent release of objects.

4.1 Fixed threshold

When a threshold is applied to the correlation matrix, it has to be carefully selected to achieve a good result. The quality of a result is measured by how many correlated pairs can be successfully identified. A trade-off has to be made since an increased number of found correlations also leads to a higher number of falsely identified correlations. Figures 8 and 10 show the portion of found combinations and the portion of false correlations depending on the threshold. The true positives are given as a fraction of all correlated pairs, while the false positives are given as a portion of the found correlations.

A priority can be set to either focus on finding as many correlations as possible or to keep the number of false correlations as low as possible. Even though false positives are not entirely avoidable, it is clear that raising the threshold to very high levels would not be advisable since the number of false positives would grow much faster than the number of true positives. For a fixed threshold between 5 and 10, the correlation process yields good results when the correlated tracklets are observed during the same night. If the correlated tracklets are more than a day apart, the results get significantly worse. The same threshold will result in both fewer true positives and more false positives when applied to tracklets with more time between them. While the results are still decent, fewer correlations will be found and more post-processing is necessary to filter out false correlations.

4.2 Adjustable threshold

As pointed out in Section 3.4, tracklets from different orbital regimes behave differently, when correlated. While tracklets from GEO often lead to false positives due to the closeness to many other objects, tracklets from GTO can lead to false negatives because their orbits are less well described by two-body motion. Therefore, it makes sense to apply different thresholds depending on the orbit of the observed object. To apply this, all objects that have a perigee < 10000 kilometres use a threshold that is 2.5 times larger than the general threshold. Orbits with a perigee < 6300 kilometres or above 60000 kilometres are also sorted out as they will not be affecting the GEO region. In addition to that, only the best correlation is used for each tracklet. The resulting improvements are shown in Figures 8 and 9 for tracklets from the same night, and in Figure 10 and 11 for tracklets more than a day apart.

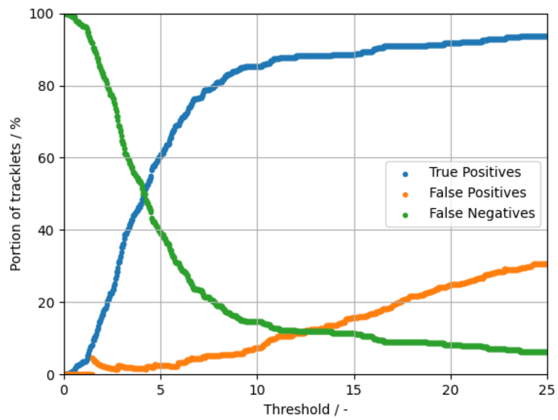

Fig. 8: Successful correlations over threshold (Fixed threshold, same night).

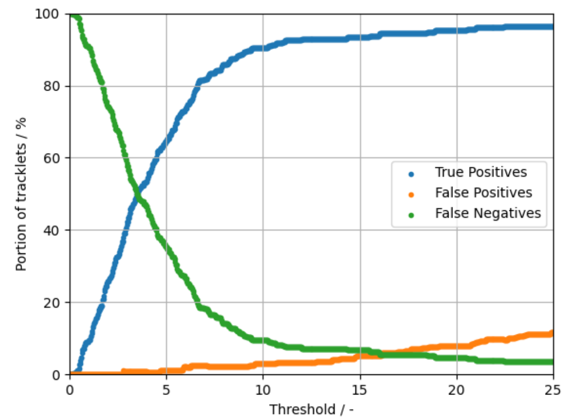

Fig. 9: Successful correlations over threshold (Adjusted threshold, same night).

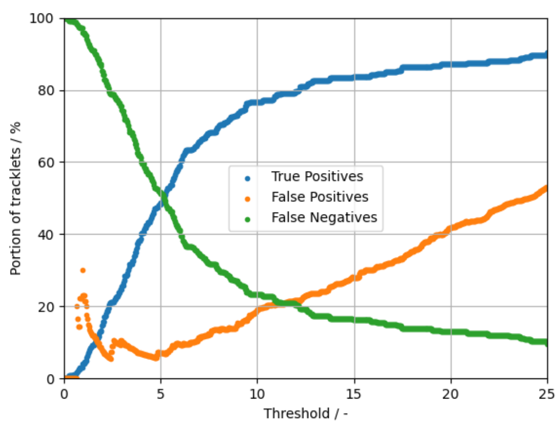

Fig. 10: Successful correlations over threshold (Fixed threshold, multiple nights).

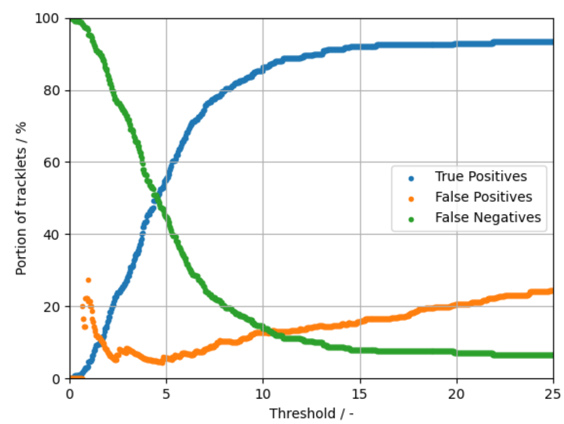

Fig. 11: Successful correlations over threshold (Adjusted threshold, multiple nights).

The plots show a clear improvement for all cases. False positives are at a lower level and more true positives are found with the same threshold. It works for both observations that are a few hours apart and for observations from different nights. The process of adjusting the threshold for decision-making to the properties of the tracklet pair performs better than the use of a fixed threshold.

4.3 Fragmentations

An especially challenging task is to correctly correlate objects that are in orbits very close to each other. This might happen due to fragmentations, the release of multiple satellites from a rocket stage, or during rendezvous and docking operations. For these cases the probability of false positive correlations is high. Figures 12 and 13 show the correlation matrix for tracklets from a group of objects, originating from the same position but with small differences in velocity. The matrix for the correlation on the day of the fragmentation shows many closely correlated pairs. These can potentially lead to false correlations. After 5 days the correlation matrix shows only the correct pairs as being closely correlated. This is because the objects have drifted sufficiently far apart to not be correlated.

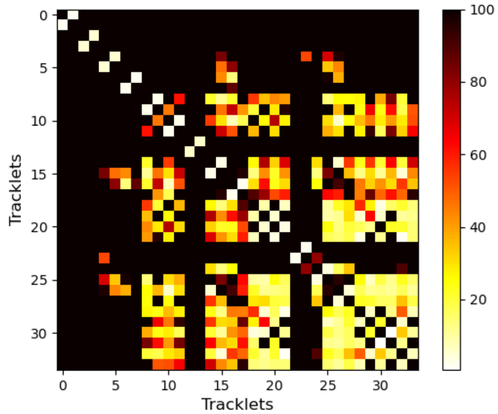

Fig. 12: Correlation matrix for objects resulting from a fragmentation, observed at day of fragmentation.

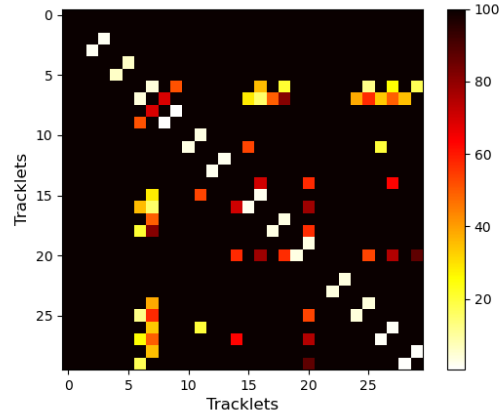

Fig. 13: Correlation matrix for objects resulting from a fragmentation, observed 5 days after fragmentation.

5. CONCLUSION

The implemented boundary value method for tracklet correlation of optical tracklets from the GEO region performs well for tracklets separated by less than a day. Tracklets from near circular orbits, close to GEO are reliably correlated, even when observations are further apart. Objects in GTO on the other hand produce tracklets that are more difficult to properly correlate with the method. This behaviour can be improved upon if thresholds are adjusted to the orbital region. Applying a lower threshold to the densely populated GEO region, while using a higher one for GTOs leads to a higher number of true positives and a lower number of false positives for all cases.

When trying to correlate tracklets from very close orbits, for example from a recent fragmentation, some success is possible in GEO or when the objects have drifted apart sufficiently. Fragmentations in GTO lead to a more difficult correlation process. This is only successful if days have passed since the fragmentation.

As a next step, more categories for orbits, tracklet length, the time between tracklets, and sensor systems could be created, to apply the most suitable thresholds for each tracklet pair. When used for real observations, this requires an analysis of the system behaviour to find appropriate thresholds for different cases. An improvement to the correlation process for real observations could be achieved by taking perturbations into account for the Lambert Transfer [7]. Since this would increase computational effort, this more complex approach might only be applied to a pre-selected part of the tracklet pairs. Calculating the expected magnitude of the error in the calculated orbit would also allow a more exact adjustment of the threshold.

6. ACKNOWLEDGEMENTS

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