Evaluation Of Lunar Brightness Observing Models For SSA Scheduling

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ABSTRACT

Spot checks of the sky background brightness over a lunar cycle are compared against existing established models from the literature to validate the effectiveness of those models. Results reported across multiple standard band passes around typical astronomical "bright times" are presented. Suggestions for a process, which could be used to optimize a ground-based SSA system's scheduling, are considered. These include estimates of the magnitude of improvements, where such optimizations can be expected to provide improvements depending on a system's passband and where it may not be worthwhile to consider such methods of operations.

1. INTRODUCTION

The reflected sunlight from the moon can produce and increase in apparent sky-glow as the Lunar Phase changes. Consequently, some light is reflected into the Earth's atmosphere as viewed from a ground location. This outcome has a profound effect on the ability to perform the Ground-Based Space Domain Awareness (SDA) mission; the detection of faint resident space objects (RSOs) must compete with this increased background signal which may lead to its detection signal being overwhelmed. Compounding the issue are three additional aspects of Moonlight scatter, each of which in turn are tightly coupled to time and location varying aspects to varying degrees. These aspects are:

1. Lunar reflection intensity is dependent on the lunar surface scattering profile direction to the ground site as well as distance, which changes with perspective due to the Moon's motion in its orbit as well as the specific ground viewing location and look angles relative to the (constantly changing) lunar position.
2. The overall degree of Moonlight scattering in the atmosphere varies with the local environmental conditions at the site from which the moonlight traversing the atmosphere is viewed. This is a complicated function of the combination of temperature, humidity, airmass at viewing angle, local suspended aerosols and the random paths that the scattered light follows through to the detector.
3. Effective Lunar Albedo is a function of passband at both the detector, the different scattering profile the moon is viewed from, and the local atmospheric extinction profiles and airmass.

All of these aspects make it nearly impossible to analytically predict the effects and magnitudes of local moonlight. However, because of the noted impact against the SDA mission needs, an accurate prediction may be useful. This paper will investigate a few different methods against local results and draw conclusions based on a suggested relative comparison against the most accurate of these existing models. The conclusions will suggest, that for most systems, increased effort for better predictions is not justified against the relative measures of better bandpass filtering and/or system specific coupling to site conditions. The outline is as follows:

In Section 2, a brief survey of the various traditional analytical methods are provided along with insight to the high level of uncertainty they provide. The review will suggest that an empirical fit to observations is strongly favored.

Following the insight into the analytical models' limitations, Section 3 will present a review of two such existing empirical fits from the literature. These are compared against locally obtained spot checks across various standard astrometric bands and an estimate of the level of error in each method against a random small sample is later presented. The results include additional band passes not present in the empirical models, but which are extrapolated with respect to nearby band passes in them, with the unvalidated assumption that the scattering profiles will be similar. This provides a means to correlate, with offset, both the different local conditions as well as expanded bandpass samples back to the empirical functions of the existing models, granted the assumption of similarity in the prior section is valid.

Finally, in Sections 4 and 5, these extrapolations of expected relative differences in site and bandpass, we then consider magnitude of impact from the Moonlight against the SDA mission and present some related recommendations based on those results.

2. ANALYTICAL MODELING OF SCATTERED MOONLIGHT

As moonlight is scattered and reflected sunlight, it can be expected to interact with the Earth's atmosphere through similar mechanisms, similar to sunlight during the daytime. This has been used to form the basis for analytical models to predict the sky brightening as a function of lunar phase. The shorter wavelengths can be expected to be dominated by the well understood Raleigh scattering and with the longer wavelengths being dominated by Mie scattering. For brevity, a simple model covering the shorter optical ranges will be considered along with a handful of the many sources which would introduce errors into such an approach. Such demonstrated variability for only a subset of the many potentially impactful factors, will provide a strong argument for either empirically based models or direct measurement. Instead, The Beer-Lambert Law states that the scattered extinction will be of the form:

$$E = e^{K*AM} \quad (1)$$

Where:
"AM" is the number of apparent optical airmasses the light will traverse, and K is a constant representative of the combination of the local aspects impacting the optical depth such as the chemical composition and density of the atmosphere and how they manifest through scattering mechanism.

It should be noted that the scalar equation in (1) can only be true for a specific frequency, away from which the optical depth would differ. In addition, the "apparent optical airmass" term is a function of viewing angle and geometry in that each frequency of light will undergo differing amounts of refraction. Because most atmospheric models feature a parallel plan approximation to the shape of the atmosphere with corrective terms to account for apparent curvature at lower angles, the air masses change more as the viewing angle nears the horizon and differ as the limits in their approximation become more evident. Since the Moon is constantly in relative motion with the Earth and includes a comparatively low inclination, the zenith look angles are not static and will span all possible elevation angles if the angular separation were held constant through the evening. An example is presented in Fig.1 where the airmass geometry from [1] is used for two hypothetical sites for which a normalized extinction coefficient in terms of k as "magnitudes per airmass" have been plotted.

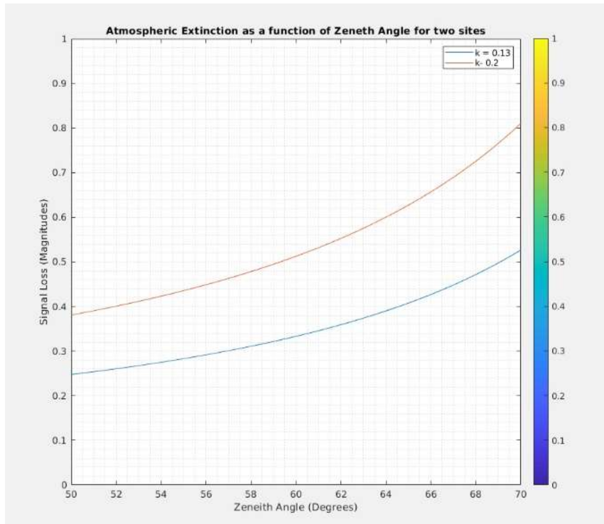

Fig. 1. Atmospheric Extinction as a Function of Zenith Angle

We can notice that the ambiguity in the degree of scattering, compounded by the ambiguity in optical airmasses, can drive many large errors. It is troublesome that most estimates for the extinction coefficient are modeled on an assumed Solar spectrum. However, the moon's geological components can be expected to modulate and change this spectrum. For example, at the lowest order, the less reflective features on the moon can be expected to absorb some of the incident light and then reemit it in accordance with the well-known black-body radiation Law. Thereby, changing the spectral irradiance of the reflected light in accordance with Kircoff's Laws. Unfortunately, the exact magnitude of such changes will vary widely as a function of both the viewing direction to the Lunar Surface, as well as the scattering profile at that angle, and finally as a finer-scale function of the specific terrain and material illuminated at that changing angle. One confirmation of this is that the Bond Albedo is different than the Geometric Visual Albedo, confirming the directional dependence. Because of the complication in changing viewing angles, due to the Lunar eccentricity and inclination compounded by an irregular orbital period which only repeats on the order of decades, any attempt to analytically predict scattered moonlight to very high precision would be computationally exhaustive even if a very detailed model of the lunar terrain and composition could be obtained.

In addition, the scattering profiles and mechanisms (e.g. Mie scattering at longer wavelengths) between both the Lunar surface and viewing angle differences, as well as due to the local atmospheric conditionals and compositions (e.g. the presence of air pollution or locally imbalanced aerosols and so on), introduce additional complexity when considering extensions of such models outside of the simple optical realm. Perhaps one of the most suggestive examples, of the difficulty in analytically modelling just the atmospheric portion of those drivers, is demonstrated by the large number of input parameters used by the standard MODTRAN routines. Finally, the atmosphere is unlikely to be similar in all directions. For example, in the direction of a distant storm, the atmospheric extinction may be greater for the same viewing elevation than in the direction away from a storm. The simple model considered above, can be seen to rapidly increase in complexity for any reasonable accurate site accounting. As a result of these challenges, such analytical models typically make broad simplistic assumptions leading to shortcomings in their accuracy as a result.

3. EMPERICAL MODELING OF SCATTERED MOONLIGHT

Because of the limitations of the analytical models, an alternate approach of directly measuring the conditions over one or more lunar phase in many directions and handpasses were used <ref> , ref. The latter of these references makes a relative comparison and reports, that errors of up to 20% were measured and corrected for, with the reported updated empirical model. However, we noticed a magnitude of difference, which at this level is suggestive of the same differences present in the analytical models due to site-specific extinction coefficients. These may combine, resulting in a combination of model-independent real-world difference in site conditions, such as weather, as well as the differences inherent to modeling fidelity. To gauge this, we spot-checked our dark site location near a full moon and present the sky brightness relative to our full unfiltered bandpass thought narrower filters and compare the reference V-Band values to those published in [2]. The process used to normalize to V-Band is described later in this section. Furthermore, to obtain a relative measurement to the reference, we based a reference location at the reference's published Lunar Phase and distance/angle from the moon and defined our measured results to be the basis for measurement- following their approach of reporting subsequent values as difference-in-magnitude vs. angular separation. We then computed the offset between the empirical model's expected values and our measured values. The reference compared to in the table, in turn within that paper, compares its own results to the prior values in [3] where they report a magnitude of error between the latter model to their improved empirical results with differences on the same order as those we observed here. In general, this suggests that the empirical models have reached the point at which additional reduction in error must be driven with corrects obtained via measurements at each specific site near the time when they are to be utilized.

Note: all measurements presented were obtained two days prior to a Full Supermoon Moon with the Sun greater than 18^{o} below horizon.

Table 1. Comparison- actual vs. empirical model

Measurement Number	Reference Angular Separation (Deg)	Expected Delta (Mag)	Measured Delta (Mag)
1	51.0	0.1	0.072
2	50.0	0.8	-0.2
3*	75.5	-0.5	-1.2
4*	49.75	0.1	1.83

Of interest are the outliers denoted with an asterisk. When solving against reference star catalogs it was noticed that Measurement sample 3 fell in the direction of the Galactic Plane, accounting for the discrepancy. Measurements 4 was an alarmingly large difference which became evident when the background gradient which we compute to compare to correct for sensor bias prior to comparison to a reference catalog was performed. In this case, as presented in Fig. 2, insufficient baffling at this look angle for our system clearly allows for the introduction of unwanted glare. This was a welcome discovery as it underscored that that even small measuring system differences may drive errors on the order of all of the natural sources mentioned prior—and this became the basis for one of our concluding recommendations presented in a later section.

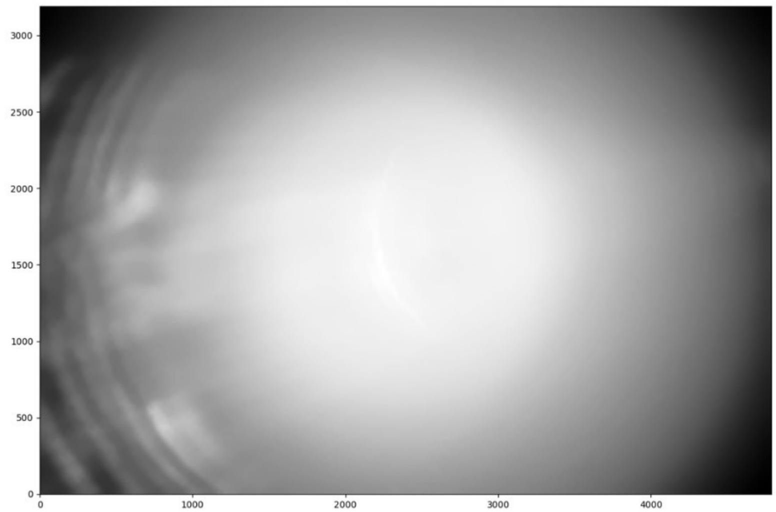

Fig. 2. Measurement 4 background gradient.
Notice the clear optical flaring from curved surfaces in the far left of Measurement 4's gradient responsible for the large difference in spot-check

Additional Sloan Digital Sky Survey (SDSS) filters were used to obtain relative measurements to the V-Band values here, and those results are presented in Table 2. Because of the issues mentioned above, measurements 3 and 4 are omitted from the additional SDSS band-pass comparisons.

Table 2. Relative differences in moonlight skyglow between SDSS Bands

Measurement Number	**Filter**	**Difference from g' (Mag)**
Reference Position	Full System	+0.71
	r'	-0.92
	i'	-2.35
1	Full System	+0.79
	r'	-0.76
	i'	-1.91
2	Full System	+0.61
	r'	-1.38
	i'	-2.81

In the remainder of this section a brief description of the methods and setup used to perform our spot checks are provided. The measurement system utilized images from a commercial Celestron 8 Schmidt-Cassegrain telescope with a set SDSS u', g', r', and i' filters inserted into the imaging chain between an a Sony IMX-477-based Electro-Optical (EO) detector. Although we expected maximum Rayleigh Scattering in the shorter u' bandpass, we determined later that the system band pass has virtually no response in this band and that filter was disused for the remainder of the study. A tracking equatorial mount was used to precisely control pointing over a large sample of different azimuth and elevation angles for images obtained through each filter.

Fig. 3. Measurement System

The Moon's position from the test site was then referenced to calculate the relative position between the samples and the Moon. Finally, the series of images were blind-solved using Astrometry.net and center field (flat region) sources extracted using the established DAOFIND standard astronomical algorithm. The extracted sources were then correlated with the GAIA Data Release 3 catalog and the results filtered for known color index sources for conversion to Johnson V-Band as published in the SDSS literature [4]. These results were filtered to only include for Solar-Like color index sources and a resulting collection of peak pixel values for the filtered extracted sources were obtained for each image in each direction. Then, an exponential trendline was fitted to this collection, which provides any arbitrary pixel value to magnitude mapping. Finally, a representative "blank area" of the image was selected and an average background pixel value was applied to the trendline equation in order to estimate the sky background magnitude in that pixel. This was normalized for the telescope's known plate scale to obtain magnitudes per arcsecond squared. The test site selected was a rural dark sky location with minimum light pollution, where imaging occurred after astronomical twilight to relegate background signal to natural skyglow and scattered moonlight alone. In addition, relatively long exposures of 1second each were used to ensure the physical sky signal overwhelmed the lower contributions from sensor noise.

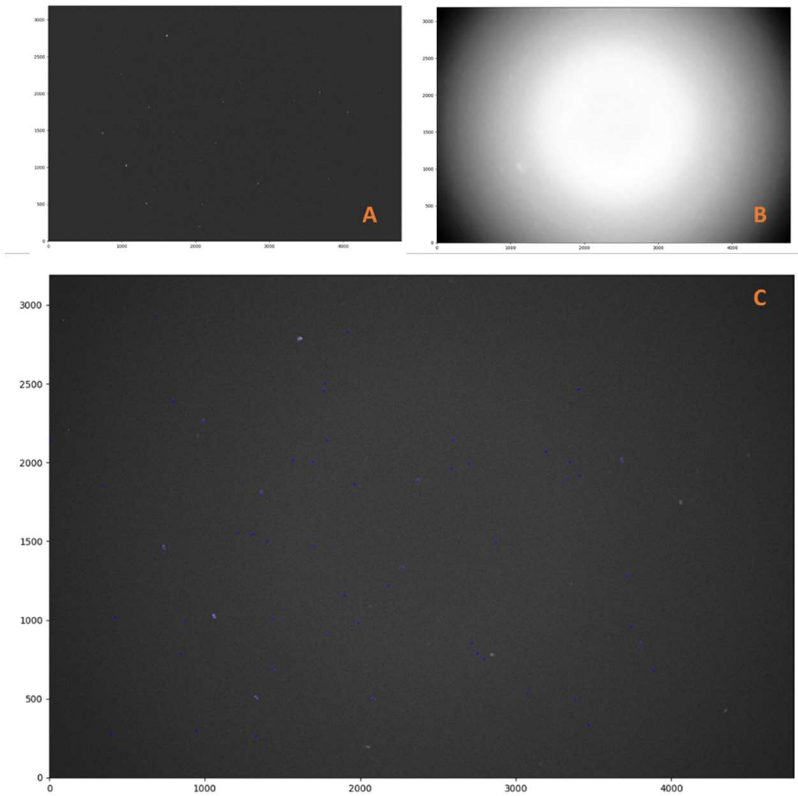

Fig. 4. Where A is a raw image, B is the background to be removed, and C (enlarged) shows the extracted sources for correlation to GAIA.

4. MOONLIGHT SCATTERING IMPACTS ON SSA SCHEDULING

This section evaluates if the attention provided to the differences, in both the prediction and measurement of the change in sky background due to scattered moonlight in the study, are warranted for an SDA system. If the standard academic objective for better understanding were not sufficient in itself, the substantial practical impacts to the SDA mission add additional weight. First, it should be noted that on average half of the time the moon is Gibbous. Discounting the smaller effect that the directionality of the scattering means a non-linear increase in skyglow which provides a little less skyglow until past Gibbous phase, meaning at least half the time approximately an SDA system must contend with bright sky backgrounds regardless of site location and weather. From this "frequency of impact" perspective, even small impacts are often enough to make their consideration practical. As shown below, the magnitude of impact in terms of search rate and sensitivity are substantial at any frequency.

Consider two "sensitive, normalized" systems, which are defined to mean two identical systems capable of providing detections near the natural skyglow noise floor operating under identical conditions -- save for the difference in moonlight caused by sky background. Because the systems are assumed to be identical, and taken to be operating at times with reasonably bright moonlit conditions, a simplified charged coupled device (CCD) equation for which the sensor impacts have been removed may be used for relative comparison:

$$SNR = \frac{\dot{S}\sqrt{t}}{\sqrt{\dot{S} + \dot{B}}} \quad (2)$$

Where:
$\dot{S}$ is the Electro-Optical (EO) converted signal count in photoelectrons, pe^-, at the focal plane, and
$\dot{B}$ is the background sky signal count in similar sensor focal plane pe^-.

For simplicity, the sensor noise terms have been assumed to be much smaller than the sky background counts due to the bright moonlight conditions, and the pixel plate scales are assumed well-tuned to the optical point-spread function. It is assumed that a SNR of 10 is needed for a detection and that the system, under natural dark conditions of 21.5mpas, could provide that trigger in a given arbitrary normal basis of time (allowing the sqrt(t) to be evaluated as “1” unit of time.). From equation (2), a plot was created with represents an example of how much additional time is needed for a 20% error in estimation of sky brightness as the skyglow is varied from dark conditions to bright conditions. The values used were for a system which resulted in 10pe- sky background under dark sky conditions. For such system, a nearly 17% hit to search rate (or increased latency/well) would be needed to overcome a poor estimation of sky brightness.

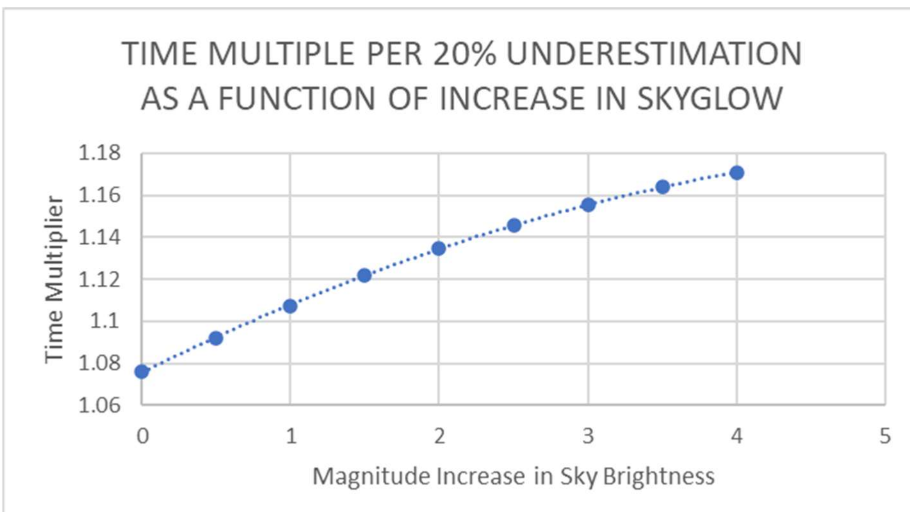

Fig. 4. Time Multiple Per 20% Underestimation as a Function of Increase in Sky Brightness

In general, such system features a noise floor near the sensor’s limit (after scientific correction of the image, i.e. bias and dark current subtraction and flat-fielding) for which a reasonably low signal may also be detected. In such case, any increase in noise may have a larger impact. When either term grows much larger, given the CCD equation features a square root of either term, results in the changes having less and less of an overall impact. In such cases, the increase in complexity to better estimate the moonlight scattering has the potential of overshadowing the lower returns it may offer.

5. RECOMENDATIONS

The impact of moonlight on a system is a variable function of how responsive to lower lightening conditions that system is to begin with. The impacts of poor lunar modeling can have a measurable impact on an SDA system's overall system throughput. It is an impact that could likely be overcome by larger lunar exclusion buffers up to a limit, where the scheduling time delays and/or misses due to loss of excluded search area, balance against the system and site-specific impacts to integration time due to increased background noise. The following recommendations are made:

1. The specific system parameters (how sensitive, how large, how close to the noise floor and how dark) can guide in a determination of the cost-benefit to performing in-situ skyglow measurements for better moonlight "bright time" exclusions zone scheduling instead of using a simple empirical model which will introduce error. <u>For most systems, the benefit will likely be less than 10% for the typical errors and such additional complexity is not warranted.</u>
2. If the sky background levels or the minimum discernible signal levels photoelectron counts are already considerably large (after scientific reduction of sensor parameters) under dark sky conditions, additional effort in better predictions for moonlight scattering beyond what the analytical and empirical models can already provide are likely not a large impact in terms of that systems effectiveness.
3. Selection of proper filtered passband can greatly help in reduction of moonlight impacts- more so than additional time and look angle modeling.
4. If in-situ measurements of the scattered moon light is determined to be useful, care should be taken to ensure the system performing the measurements have the same baffling and optical scattering characteristics of the system it is providing the inputs for.

6. CONCLUSION

Mechanisms for the cause of uncertainty in prediction of scattered moonlight were presented. The magnitudes of that variability were compared relative to each other, and spot checked against live-sky measurements. Extrapolation for SDA mission elements were made and the order of magnitude of the practical impacts for uncertainty were presented. The results suggest that for most practical systems, the level of uncertainty for most methods are acceptable overall with some corner cases wherein better predictions and/or in-situ measurements could be of some utility. Resulting recommendations to determine site-specific conditions, which would suggest if better fidelity predictions were beneficial were provided.

7. REFERENCES

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