Unlocking the Value of Space Debris: An Investigation on Multi-shell Source-Sink Physical-Economical Model and Space Debris Value Definition

Di Wu and Richard Linares
Massachusetts Institute of Technology

ABSTRACT

Space debris is ubiquitous, thus available to all, but is not believed to be of help in incentivizing the avoidance of the "tragedy of the commons" in space. This study evaluates the definition and formulation of space debris value on a multi-shell multi-species physical-economical model. Analysis is done for different species. The resulted single-debris value is examined shortly before applying it into finding its temporal evolution. The multi-species values' low volatility with respect to the different numbers of space debris removal indicates a reliable value formulation. Introducing agent-based artificial space debris market (SDM), the investigation on its effect on the natural evolution of the space-debris value shows a drag effect, resulting in a spontaneous decrease. This result shows that introducing SDM is promising for creating a self-sustaining space development.

1. INTRODUCTION

The increasing presence of space objects in circumterrestrial space raises concerns about safety and sustainability. However, this perspective is limited to the current open-access era, which is now undergoing transformation. As we navigate this transition, a pivotal question emerges: What will replace the open-access era, and what principles will guide this new phase?

In the pursuit of addressing the challenges associated with space debris, prior efforts have introduced physical-economic models. However, the distinctive advancements of this study unfold through a two-fold breakthrough. Primarily, the physical components of existing models transition from intricate structures to simplified single-shell source-sink models. This transformation responds to the inherent diversity in orbital-use values across distinct spherical or circular shells, acknowledging that certain orbits possess distinct conditions with unique space application potential. While the conceptual framework of a multi-shell source-sink model has been envisioned in earlier studies, its full realization remains unexplored. This research endeavors to bridge that gap by actualizing the integration of such a multi-shell approach.

Calibration, a pivotal phase in this endeavor, involves aligning the model with historical data to determine parameter values essential for incorporating the multi-shell source-sink model or even getting first-handed up-to-date information like in [3]. However, a more crucial aspect lies in calibrating the economic values attributed to objects within these diverse shells. This calibration process gives rise to a new multi-shell physical-economic model, providing a refined depiction of the comprehensive landscape. In essence, this investigation propels the evolution of space debris modeling by embracing the intricate interplay between physical and economic dimensions within distinct orbital shells.

Built upon this foundational model, an additional layer will be constructed in the form of an agent-based artificial space debris market. This endeavor delves into the intricate realm of space debris value evolution with the overarching goal of identifying avenues for a self-sustaining framework to underpin space development. In the context of space exploration, the economic dimension has witnessed significant advancements. While conventional analyses have centered on optimizing space utilization and allocation, innovative perspectives have emerged. Rao, for instance, proposed a transformative concept of introducing an "orbital-use fee," envisioning a quadrupling of the long-term value within the satellite industry [7]. This paradigm shift prompts a reimagining of the economic landscape within the space sector.

Furthermore, these conventional physical-economic models often exhibit a narrow focus on active satellites, overlooking the latent potential that space debris holds for augmenting the overarching space economy. With the rapid proliferation and expansion of the space industry, the quest for fresh revenue streams and value propositions has gained prominence. Space debris, once deemed a hindrance, is now ripe for a paradigm shift, evolving into a valuable resource that extends beyond its physical presence to encompass the underlying orbits they occupy. Indeed, in the spatial domain, orbital space itself holds intrinsic value, rendering space debris an indispensable asset. In this study, our aim is to scrutinize the definition of space debris value and devise a market model that inherently encourages the recognition, ownership, and strategic utilization of space debris.

Against this backdrop, the integration of an agent-based artificial space debris market into the existing model signifies a bold expedition into uncharted realms, seeking innovative avenues to cultivate self-sustaining growth within the realm of space development. This audacious endeavor transcends traditional methodologies, immersing itself in the dynamic evolution of space debris value.

The paper formulates like this: in Sec. 2, we briefly summarize MOCAT-SSEM and its significance; in Sec. 3, we delve in the detailed analysis on formulating space debris value as well as its temporal evolution; in Sec. 4, agent-based artificial space debris market is introduced for the first time and we investigate the effect of such innovation; last but not least, we conclude with our findings in Sec. 5 and Acknowledgements.

2. MOCAT-SSEM

Source-sink evolutionary models (SSEMs) are used to simulate complex dynamic systems through sets of coupled ordinary differential equations that represent relevant quantities. SSEMs offer a computationally efficient approach to model space debris population evolution. These models segment space into orbital volumes and aggregate various anthropogenic space objects into populations within each volume. The MIT Astrodynamics, Space Robotics, and Controls Laboratory (ARCLab) has developed SSEMs as part of the MIT Orbital Capacity Assessment Tool (MOCAT). These models progressively incorporate additional behaviors, such as object properties, space weather effects, coordination between operators, orbit-raising behaviors, and trackability constraints.

This paper presents the integration of these features into a unified SSEM framework called MOCAT-SSEM. Built in an object-oriented MATLAB-based framework, MOCAT-SSEM aims to be user-friendly and expandable. The ongoing work involves verifying and validating ARCLab's SSEMs against MOCAT-MC, a higher-fidelity Monte Carlo model. Once sufficiently developed, MOCAT-SSEM and its models will be released as open-source tools for the community. It will be utilized in future ARCLab projects related to space sustainability decision-support and adaptive space governance.

MOCAT-SSEM restructures previous SSEM work into an object-based paradigm, serving as a foundation for future space debris SSEMs. This approach automates equation development based on input objects and parameters, along with facilitating figure production, optimization, and capacity metric computation. This shift enables more complex sources, sinks, and flows within space debris SSEMs and enhances usability for new users.

The schematic diagram in Fig. 1 elucidates the formal interactions inherent within the various species. The dynamic of the active satellites population predominantly hinges on the influx of new launches, dictated by the annual launch rate (λ). In parallel, the adherence to Post-Mission Disposal (PMD) protocols entails the retirement of active satellites from the simulation following a designated operational lifespan (Δt). Notably, the transition from active satellites to derelicts is characterized by the probability of successful PMD implementation. Furthermore, collisions represent a pivotal element within the intricate inter-species dynamics and are extensively expounded upon in detail in D'Ambrosio et al. [4].

We refer interested readers to Lifson et al. [5] for details.

3. PHYSICAL-ECONOMICAL MODEL

In this chapter, we focus no evaluating the values of different debris under a 1 year frame. We will ignore this 1 year frame in the following content. To assess the value of space debris, we followed a structured approach encompassing three key stages:

- Modeling Baseline Costs: We constructed a comprehensive model to quantify the expenses incurred by satellite operators due to interactions with debris. This involved factors like maneuvers to avoid collisions (maneuver)

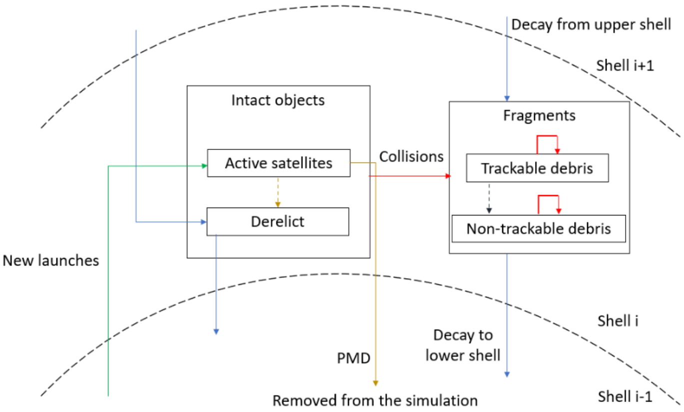


Fig. 1: Qualitative MOCAT-SSEM model.

and damages resulting from impacts (collision). This model established the baseline costs linked to an initial condition.

- MOCAT Perturbation Estimation: We introduced variations to the debris environment, such as the removal of a number of debris fragments from orbits. Employing the established model, we gauged the resultant alterations in costs for spacecraft operators relative to the baseline. Notably, any reduction in existing expenses was categorized as “value”, representing the benefit of owning a space debris.

We employ the value definition composed by maneuver part and collision part, e.g. Eq. 1. The maneuver part is calculated based on the number of maneuvers taken and the cost each maneuver; the collision part is the number of collisions and the cost each collision. We list the values we employ in Table 1.

$$V_d = V_m + V_c, \tag{1}$$

where V_d is the total value, V_m is the total value due to maneuvers, and V_c total value due to collision.

The values come from NASA report [3]. Despite of different existing numbers in [3] due to different operator classes, we assume all the cost would converge to commercial level. This means the ‘Commercial Bespoke’ value or even lower would be applicable for maneuver cost. Meanwhile, mass production-level satellite industry would drive satellites costs to some standard values as well. This would make satellite to be categorized into three different groups: small satellite, whose cost per collision is given by ‘CubeSat/SmallSat’ in [3]; medium satellite, cost per collision given by ‘Commercial Large Constellation’ in [3]; and large satellite, cost per collision given by ‘Commercial Small Constellation’ in [3].

It’s important to recall that in comparison to the baseline level of anticipated debris costs, the removed debris result in such a value that they are naturally credited to that it. The initial condition for the baseline is retrieved from TLE on January 23rd. The composition of the values, from maneuver and collision, are depicted in a log scale in Fig. 2. The value corresponding to the maneuvers only take a very small portion of the overall cost.

Single-debris value, in a general sense and disregarding the shell information, depends on the specie of the debris shown in Fig. 3, the overall accumulated value, and the number of debris considered. It comes from averaging the accumulated value by the number of debris considered, as in Eq. 2.

$$C_d = \frac{V_d}{N_d}, \tag{2}$$

where,

$$V_d = \sum_{shell=1}^{max\,shell\,No.} V_{d,shell}, \tag{3}$$

Cost class	Small S (e.g., unslotted 5kg satellite.)	Medium S (e.g., unslotted 250kg satellite, slotted 306kg satellite.)	Large S (e.g., slotted 1250kg satellite.)
Cost per maneuver	\$462	\$462	\$462
Cost per collision	\$300,000	\$1,000,000	\$3,000,000

Table 1: Summary of the operation costs used to calculate the value of the Baseline and MOCAT Perturbations. The values come from NASA report [3]. Despite of different existing numbers in [3] due to different operator classes, we assume all the cost would converge to commercial level. This means the ‘Commercial Bespoke’ value or even lower would be applicable for maneuver cost. Meanwhile, mass production-level satellite industry would drive satellites costs to some standard values as well. This would make satellite to be categorized into three different groups: small satellite, whose cost per collision is given by ‘CubeSat/SmallSat’ in [3]; medium satellite, cost per collision given by ‘Commercial Large Constellation’ in [3]; and large satellite, cost per collision given by ‘Commercial Small Constellation’ in [3].

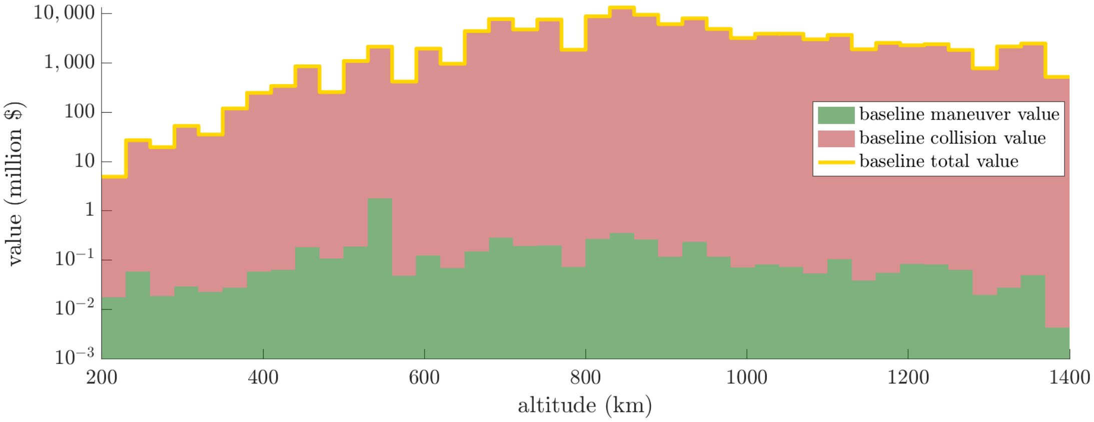
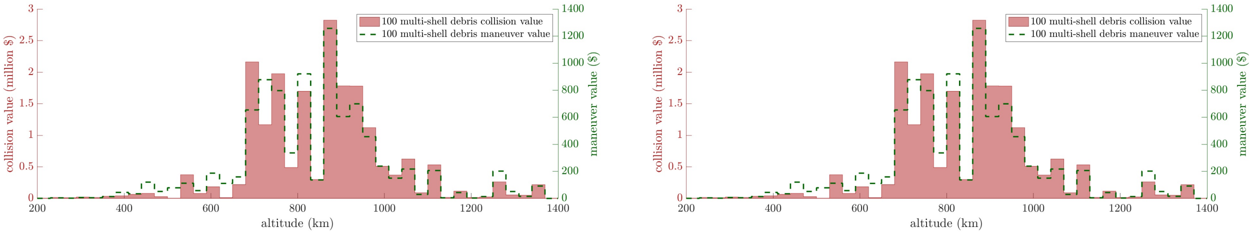
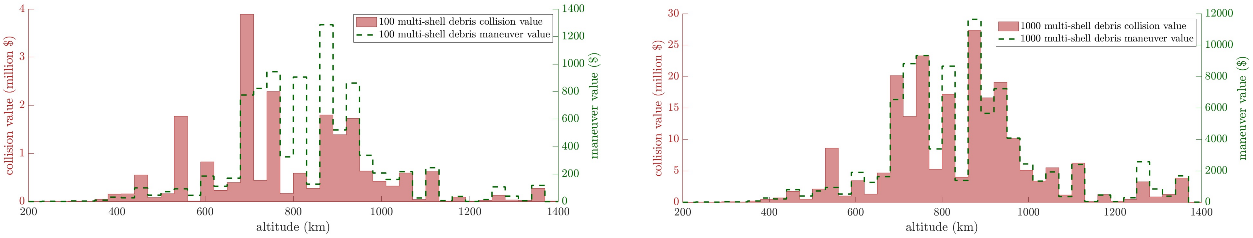


Fig. 2: Baseline value composition: maneuver value, collision value, and total value. The value from maneuver portion is expressed in dark-green; from collision is fire-brick; total, sum of maneuver and collision, is given in gold. 40 shells evenly spaced between 200km to 1400km above Earth radius. Inside baseline, there are maneuverable slotted satellites, maneuverable unslotted satellites, and unmaneuverable debris of multiple sizes.



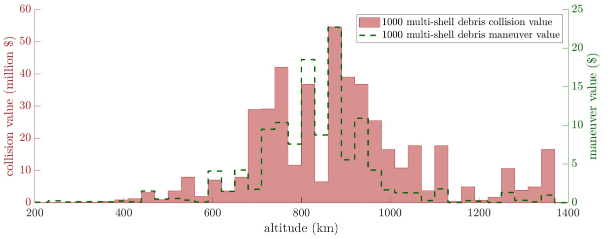
(a) 100 multi-shell medium(large) debris. Debris has a physical characters of ∼ 1.419kg and ∼ 1.5m diameter in a cannonball assumption.

(b) 100 multi-shell medium(small) debris. Debris has a physical characters of ∼ 15kg and ∼ 22cm diameter in a cannonball assumption.



(c) 100 multi-shell small(large) debris. Debris has a physical characters of ∼ 5kg and 30cm diameter in a cannonball assumption.

(d) 1,000 multi-shell small(small) debris. Debris has a physical characters of ∼ 1.419kg and 10cm diameter in a cannonball assumption.



(e) 1,000 multi-shell tiny debris. Debris has a physical characters of ∼ 0.001419kg and 1cm diameter in a cannonball assumption.

Fig. 3: Accumulated multi-shell debris values of multiple species are depicted in altitude. The debris value composes of collision value (solid space, left axis, and in fire-brick color) and maneuver value (dashed line, right axis, and in dark-green). Collision value is the total benefits of avoiding collisions that would happen if not removing the multi-shell tiny debris. Maneuver value is the total benefits of avoiding maneuvers that would happen. Collision values are the major part in the composition as maneuver costs, mostly labors, are relatively low comparing to the collision cost given in 1. Notably, the scale of collision value is at millions of dollars (million $) while the maneuver value at thousands of $, except for the case of tiny objects. Different numbers of debris (e.g., 1,000 and 100) are employed because of the different existing amount of debris and physical characters (listed in the sub-figure captions) of the debris, potentially directly linked with technological levels of difficulties.

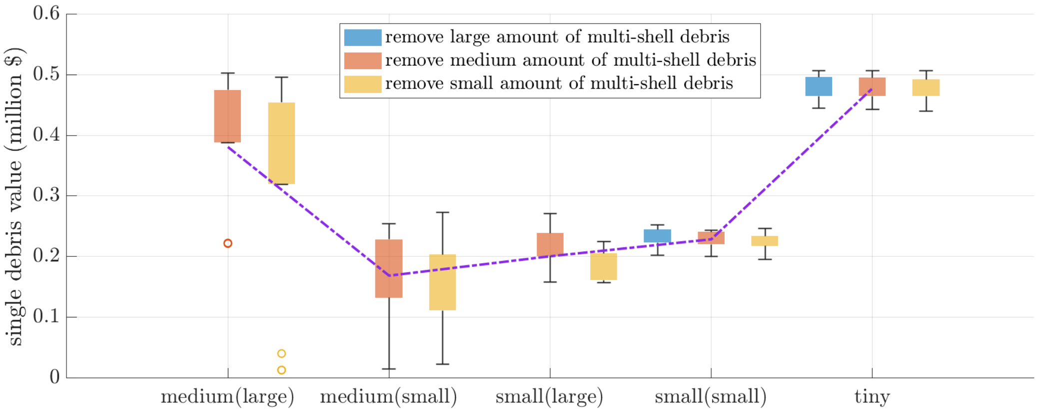


Fig. 4: Single-debris values for medium(large), medium(small), small(large), small(small), and tiny debris. The averaged values for each species are plotted in a purple dash-dotted line. Monte-Carlo simulation is employed in each case of removing different amounts of debris for every specie. Large amount of debris removal in each specie is marked in blue box; medium in red; and small in yellow. The lower and upper quartiles are shown as the bottom and top edges of each box, respectively. The whiskers, or lines that extend below and above the box, have endpoints that correspond to the smallest and largest values. For the medium(large) debris specie, the outliers are marked in circles. The averaging to get the purple dash-dotted line is conducted within species by averaging over all cases from different removal amounts.

and N_d is the total number of debris removed in the consideration; d stands for different species of debris, e.g., medium(large) debris (in Fig. 3a), small(small) debris (in Fig. 3c), tiny debris (in Fig. 3e), etc..

We notice that, from math, it is unrealistic to average single-debris value calculated from dividing debris values by the number of debris in each shell, given that we know the number of debris in each shell. Although it is well-known that the sum of the averages is not equal to the average of the sums, we restate it here for reference. For single-debris value in a single shell, the computation could be repeated by setting the removed debris all come from a single shell. We leave this work for future exploration for now as the focus of this paper is to explore the definition of space debris value and a general work frame for its calculation and application.

The calculated single-debris values for different species are shown in Fig. 4. For any specie of single debris, two or three levels of debris removal situation are examined. The level of debris removal is given by the number of debris removed. These numbers, depending on the existing quantity of a debris specie and the physical characters, thus the removing technology difficulty, are selected to be, respectively large amount, medium amount, and small amount: for medium(large), medium(small), and small(large) debris, 50, 100, and 500; small(small), and tiny debris, 500, 1,000, and 10,000.

The single-debris values of medium(large), medium(small), small(large), small(small), and tiny debris, from different removal amounts, are shown in Fig. 4. Monte-Carlo simulation is employed in each case of different debris' removal amounts for every specie to generate those box charts. Averaging all the cases for each specie, we get the averaged single-debris value. The purple dash-dotted line connects those values in Fig. 4.

To generalize the single-debris value, we extend the values creating the purple line. Debris come in different shapes and sizes, thus a physical character different from the five species. The five exemplary species create a range between about 0.168 million \$ to 0.381 million \$. We extend these values by creating a uniform random distribution bounded by these two values. In this way, for any general single debris, we sample through this uniform distribution to extract its single-debris value.

Based on this single-debris value, we take this metric for extrapolating values over longer temporal horizons. To streamline our calculations, we make the simplifying assumption that this debris removal action remains constant throughout all subsequent years. This assumption permits us to extend the value incurred by a removal not solely to the year of its occurrence but also across all subsequent years for which it remains relevant. Incorporating these

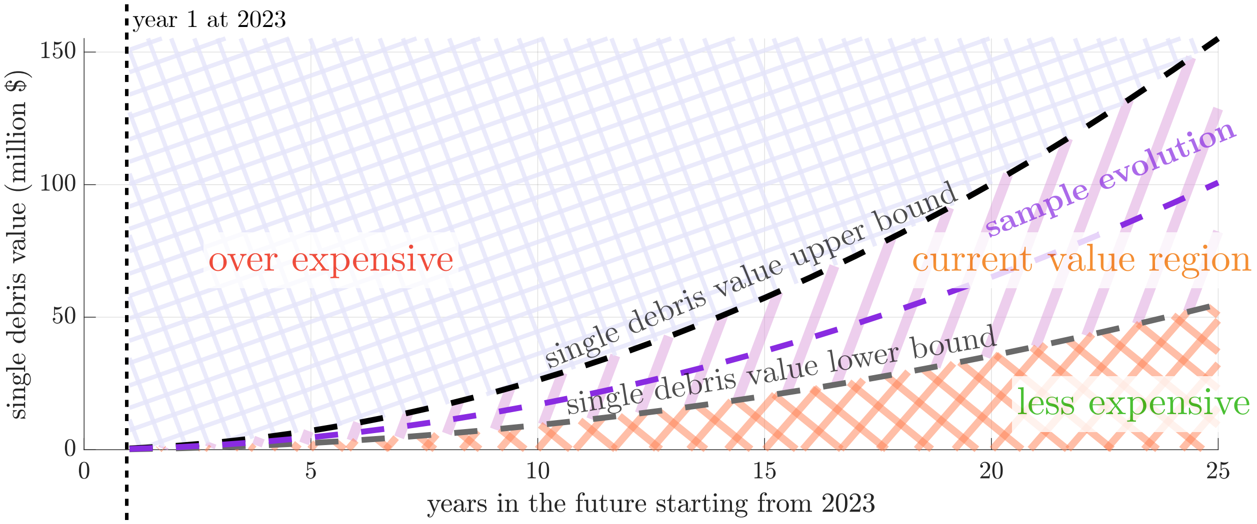


Fig. 5: Temporal single-debris value evolution. This evolution is bounded by the growth expanded from the two boundary conditions in initial year 2023. We call this region (purple-line hatched region) the "current value region" as it is created by examining the current single-debris value. Below it, the region (orange-cross hatched region) is simply "less expensive" region as the region, firstly, are cheaper than the "current value region"; secondly, could be possible when further development drops the maneuver or/and collision costs. A initial-sample-based value evolution drawn. This sample evolution follows the same polynomial growth indicated by Eq. 5.

considerations, the accrued benefit over a defined timeframe T encompasses both the mitigated risk and the cumulative costs saved. The single-debris value for future horizons is delineated by Eq. 5. This temporal value mirrors the risk relationship and contributes to the comprehensive evaluation of benefits realized over the specified time span.

$$
\begin{aligned}
C_d(T) &= \sum_{t=1}^{T} C_d(T-t+1) && (4)\\
&= \frac{1}{2} C_d(T^2+T) && (5)
\end{aligned}
$$

The boundary conditions that we get in Fig. 4 would bound the temporal evolution depicted by Eq. 5. This is shown in Fig. 5. Moreover, the boundary provided by this polynomial growth serve similarly as the boundary conditions in the initial year. A initial-sample-based value evolution, shown in the figure, is extracted by employing the same equation Eq. 5 on the uniform randomly sampled initial condition. This would represent an ideal natural evolution of a single-debris value.

4. AGENT-BASED ARTIFICIAL SPACE DEBRIS MARKET

Space debris market (SDM) is a market where space debris will be traded. Three kinds of agents involved in the market are: debris miners, chartists, and random traders. Debris miners have a major goal of accumulating space debris through updating their debris mining infrastructure. Chartists represent speculators. They usually issue buy orders when the price is increasing and sell orders when the price is decreasing. Random traders trade randomly and are constrained only by their financial resources as in work. They issue buy or sell orders with the same probability and represent people who are in the market for business or investing, but are not speculators. A similar setup is introduced in [1, 2] in formulating an agent-based artificial market for modeling bitcoin mining/trading and analyzing bitcoin price.

At each time step, a new agent enters the market at the designated time. These agent can fall into one of three categories: debris miner, random trader, or chartist. Initially, the market is comprised of agents who possess an initial amount of fiat currency and space debris. These existing agents encompass individuals who were already involved in the market, including those engaged in debris mining and trading before the simulation commenced.

As the simulation progresses, new agents entering at a later time other than the initial time bring with them only a quantity of fiat currency, signifying their investment in the market. These individuals represent those interested in entering the market and allocating their funds.

The distribution of wealth among agents adheres to a Zipf law [6], and the set of agents joining the market is pre-generated prior to the simulation's start. These agents are initially established according to a Pareto distribution of fiat cash, with random selections made to determine those entering the market at specific time steps. Likewise, the initial wealth distribution of agents in terms of cryptocurrency also follows a Zipf law.

In addition, we employ a general exponential model to model the number of agents in SDM every year shown in Eq. 6:

$$N(T) = a * e^{b*(608+T*365)}, \tag{6}$$

where $a = 2$, $b = 0.007$. The agent number needs to be an integer so we round down the calculated number to the closest integer. This exponential model gives us an initial 3 agents and 460 agents after 25 years. This numbers could be more carefully examined to be calibrated with real world data, or tested with different initialization. Yet for method validation purpose, they provide a good starting point to test the involvement of SDM.

Specifically, we set the debris owned by the initial agents equal to 20. Then, we leveraged the power-law's ranking property [8] to construct Zipf's distribution. The total number of agents and the total number of debris owned by them follows a Pareto law with exponent $\alpha = 1$. We set the cash of those initial agents equal to five times the debris values, e.g. debris numbers multiply by the value of single debris. Similarly, the cash distribution follows a Pareto law with exponent $\alpha = 0.6$. The richest agent after the initial time is about 5 times the cash owned by the richest initial agent. The Pareto law is expressed as Eq. 7.

$$c_i = \frac{c_1}{i^{\alpha}}, \tag{7}$$

where c_1 is the richest cash, i is ranking number of cash index.

For every time step, after new agents appear in the market, we need to find to which agent group this new agent belongs. We use a general exponential model, Eq. 8, to model the probability of a new agent belonging to debris miner.

$$p_M(T) = a * e^{b*T}, \tag{8}$$

where $a = 0.9425$ and $b = -0.002654$.

We use a 7/3 random trader to chartist ratio. So for an agent not a debris miner, the probability of this agent being a random trader is 70%, and chartist 30%. Thus the probability expressions of random trader $p_R(T)$ and chartist $p_C(T)$ are shown in Eq. 9 and Eq. 10.

$$p_R = 0.7(1 - p_M) \tag{9}$$

$$p_C = 0.3(1 - p_M) \tag{10}$$

Moreover, the complexity goes into whether random traders are active, what kind of orders, and how much they are issuing; so are they for the debris miner and chartist. The detailed setup is referred from [2]. the chartists within the context described can be categorized as "rational" traders. These chartists embody speculators who strategically engage in the Bitcoin market with the intention of generating profits. Their approach is grounded in speculation, whereby they anticipate that ongoing price trends will persist. If prices exhibit an upward trajectory, chartists expect this ascent to continue, and if prices demonstrate a downward trend, they anticipate a further descent. As part of their strategy, chartists often execute buy orders when prices are on the rise and sell orders when prices are declining. This speculative approach is a defining characteristic of chartists within this framework.

At every time step, the order book [2] holds the list of all the orders received and still to be executed. The system maintains an order book with sorted buy and sell orders. New orders are added, and matches trigger transactions. Orders are executed fully or partially, and this process repeats until no more matches are found. The order book ensures execution before processing new orders. This is called price clearing mechanism.

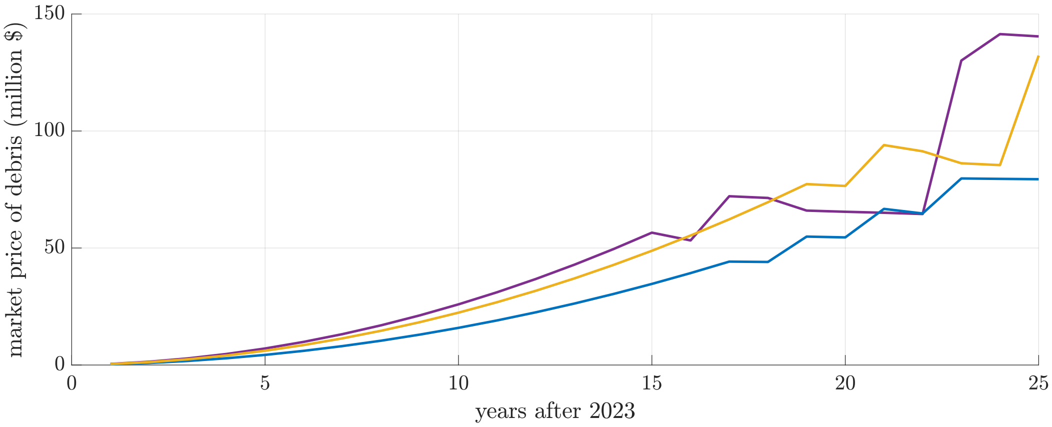


Fig. 6: Involving SDM on three cases of space debris' value evolution. At the beginning, the value evolution follows an ideal natural evolution of a single-debris value. After more agents are involved in the market, the trading effect emerges and drags the market price down in general. A less valuable debris means it would be less expensive to keep a sustainable space environment.

All of above are generating agents and preparing for trading actions. After which, we employ the price clearing mechanism to find the evolution of a single debris value evolution throughout the time. Three cases of space debris' are reported. According to the evolution showing in Fig. 6, the space debris values are dropping.

The Monte-Carlo running leads to Fig. 7. The evoluions basically cover the whole region noted as 'current value region'. Moreover, the space debris did not goes above the upper bound. Astonishingly, we observe the effect of dragging down the value of space debris through introducing SDM. Yet, there are cases that go below the lower bound. This natural evolution shows that through trading process, the value of debris could decrease naturally. This is appealing as this kind of operation would be self-sustainable and naturally brings the cost down without losing the flavors. with the house dog could help with the greenhouse report.

5. CONCLUSIONS AND OUTLOOK

Our model endeavors to broaden the valuation perspective of the space industry, transcending the realm of space debris, by harnessing a sophisticated multi-shell source-sink framework. By defining space debris values based on data from NASA report and integration with MOCAT-SSEM, our model fosters a culture of responsibility and sustainability within the space sector, catalyzing awareness and accountability for the management of space debris among industry stakeholders. This paradigmatic shift holds the potential to catalyze the emergence of novel business paradigms and revenue streams, incentivizing ethical conduct in space endeavors.

Moreover, the analysis of the composite debris value uncovers that collision values form a substantial portion, with maneuver costs being relatively lower in comparison to collision costs. Our results are presented across various debris quantities, acknowledging the diversity inherent in existing debris volumes and their distinct physical attributes, which are potentially intertwined with varying degrees of technological complexity. We have also seen a relatively stable single-debris value distribution between species, which further indicates the feasibility of single-debris value definition and calculation method.

Furthermore, our investigation delves into the application of the Space Debris Model (SDM) by introducing agent-based artificial space debris market. The analysis indicates a dragging effect on space debris value from introducing SDM, which makes space debris less expensive in the future. Intriguingly, through the analysis of Monte Carlo simulations, besides validating the "current value region", these simulations naturally extend into the "less expensive" region within a relatively short temporal horizon, notably within 15 years from 2023. This phenomenon underscores the prospects of cultivating a self-sustaining space environment, wherein the cycling of space debris becomes a catalyst for propelling the development of space infrastructures.

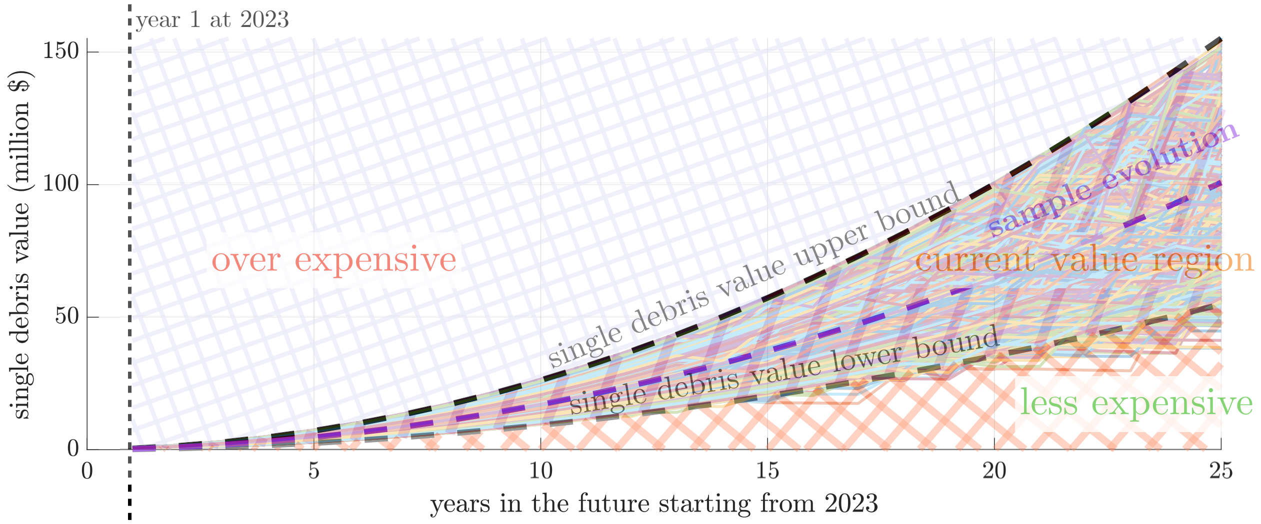


Fig. 7: Involving SDM on 1,000 Monte-Carlo cases of space debris' value evolution. Besides filling out the "current value region" as expected, the evolutions "overflow" into the "less expensive" region naturally as early as less than 15 years into the future from 2023. This makes the development of space environment self-sustainable when the cycling of space debris could fuel space infrastructures development.

There are many detailed works that could be done to make this work more complete. If we could calibrate with some existing knowledge on the space debris market side, we would have more confidence in the specifically calculated values. On the SDM side, despite the general finding that this innovation would create a self-sustainable market by itself. Both detailed scientific evaluation and economical strategy would be providing insights into the future of leveraging this emerging SDM. What's more, the SSEM model itself would need some tweaks and pulls to validate the indicators that are used to create the space-debris values.

In summary, our inquiry presents a holistic resolution to the intricate quandary of space debris evolution within circumterrestrial space. By assimilating insights gleaned from the SDM, we elucidate the potential trajectory of space environments toward self-sustainability. Through our scholarly endeavors, we aspire to contribute to the establishment of a more sustainable and responsible trajectory within the domain of space development.

ACKNOWLEDGEMENTS

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