Long-term N-body Stability in Cislunar Space

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ABSTRACT

For orbits between the Earth and Moon, elements of n-body dynamics arise, creating chaotic behavior over various timescales. Because of this chaos, TLEs and the restricted three body problem fail to predict the long-term behavior of cislunar orbits. Solving cislunar orbits over long timescales requires accurate gravity and radiation pressure models which are computationally expensive. However, not all cislunar orbits are equally chaotic, so low-cost computational modeling may suffice depending on the orbit and timescales involved. Leveraging Lawrence Livermore National Laboratory (LLNL) high-performance computing resources and LLNL-developed space situational awareness python packages, we create a library of high-fidelity cislunar orbits, integrated out to twenty-year timescales. Using this library, we can then recompute select cislunar orbits with varying force models and integrators to quantify the accuracy tradeoffs for various models. We present the results of our cislunar orbit simulations and analysis on the divergence of lower fidelity models from a high fidelity model at different regions of orbit initialization phase-space. Simulated orbits include Earth-centered and moon-centered orbits, all of which are initialized via Keplerian orbital elements spanning the full 6d parameter space.

1. INTRODUCTION: MODELING CISLUNAR SPACE

Cislunar space is generally defined as the spherical volume between GEO and the orbit of the Moon. For this paper that will be the assumed definition. Within cislunar space are many regions, often defined by unique periodicities of the orbits that can be found there. A breakdown of cislunar space into regions where either the Earth, Moon, Sun or a combination of all three dominate [2]. Understanding roughly where each gravitational body dominates is important, though to maintain accuracy of an orbit over time one will need to model all gravitational bodies. The break up of parameter space into families is a starting point, however chaos will move any orbit in cislunar space from one 'family' to another given enough time. Simulating the same orbit several times with various models we can quantify the errors due to a choice of cislunar model.

The use of cislunar orbits will continue to grow for several reasons, including commerical, national security, and scientific endavours. Successful developments in reusable and cheaper heavy lift vehciles open the door to proliferation cislunar space.

As orbits extend further from Earth, perturbations due to other factors, such as the Moon, Sun, planets, and thermal pressures, become more significant. These perturbations lead to chaotic behavior of the orbits, and no orbit will unaidedly repeat in a perfectly periodic manner. Chaos is unpredictable, and while there are established measures of chaos, they are not as clearly meaningful for cislunar space. In orbital mechanics, chaos indicators such as the Lyapunov Exponent or the Mean Growth of Nearby Orbits (MEGNO) are often used to signal the growth of chaos, which will result in a divergence at some multiple characteristic timescales in the future. We find that for cislunar space, these chaos indicators rapidly grow for essentially all orbits between the Earth and Moon. The indicators do not necessarily indicate 'instability' but rather the degree at which the cislunar orbit does not repeat periodically. For certain purposes, such as selecting orbits that can be sustained in space, it may not be crucial for us to determine whether an orbit will oscillate within cislunar space. Instead, our primary concern might be to ascertain whether a given cislunar orbit will intersect with the Earth, Moon, or any other celestial body.

2. SOFTWARE PACKAGES

There are countless software packages that may be used to integrate orbits in cislunar space. Packages commonly used in the SDA community are REBOUND, MATLAB, STK, FreeFlyer and GMAT. For this paper we will be comparing to GMAT, STK and REBOUND. We provide below a brief summary of the packages.

2.1 General Mission Analysis Tool - ”GMAT”

The General Mission Analysis Tool (GMAT) is an open-source software developed collaboratively by NASA and private industry [4]. Its primary function involves technical computing, making it a valuable tool for space mission analysis. GMAT has been utilized in various missions including LCROSS, Lunar Reconnaissance Orbiter, OSIRIS-REx, Magnetospheric Multiscale Mission, and Transiting Exoplanet Survey Satellite (TESS). GMAT provides a platform for space mission analysis and research across multiple fields including aerospace engineering, astronomy, mathematical modeling, and physics. The GUI provided by GMAT to initialize your missions is beneficial for many users to get started. GMAT can also be interacted with through scripts, though this is not optimal or even built for HPC computing systems. This is the main reason we have chosen to develop a software package for orbit simulation with HPC in mind.

2.2 REBOUND

REBOUND is a versatile and user-friendly software package designed for simulating the dynamics of planetary systems and celestial bodies. It employs various symplectic integrators, including high-order options like SABA and WH Kernel methods, ensuring accurate and stable long-term simulations. Notably, its hybrid symplectic integrator, MERCURIUS, handles planetary dynamics with close encounters effectively. The addition of the IAS15 integrator provides high accuracy with adaptive time-stepping. REBOUND supports collisional/granular dynamics and offers multiple collision detection routines. Written in C conforming to ISO standard C99, it serves as a thread-safe shared library and offers an easily installable Python module. The software’s strengths include real-time 3D OpenGL visualization, parallelized integrators for fast simulations, and minimal external library dependencies. With extensive examples, straightforward execution, and open-source accessibility on GitHub, REBOUND stands as a powerful tool for simulating celestial mechanics with ease and precision [7].

2.3 Systems Tool Kit (STK)

”Systems Tool Kit” (STK), developed by Analytical Graphics, Inc. (AGI), is often used for orbit analysis, though the package is a comprehensive multi-physics software application designed to perform intricate analyses of diverse platforms within the realms of ground, sea, air, and space [1]. At its core, STK features a robust geometry engine capable of dynamically determining the positions and orientations of objects, even under intricate constraining conditions. STK’s functionalities are supported by various components and modules. Its user interface offers graphical displays, customizable toolbars, and interactive 3D windows for intuitive analysis. The Integration module, known as ”Connect,” enables script-based interaction and integration with other applications. STK organizes analyses into scenarios, allowing users to create and manipulate objects like satellites, aircraft, targets, and communication systems. The software also encompasses reporting and graphic capabilities for detailed analysis. STK’s modular structure allows users to enhance its capabilities through the addition of specialized modules. These combined functionalities enable engineers and scientists to conduct a wide range of complex analyses in the fields of aerospace, defense, and beyond, making STK a versatile tool for multidisciplinary applications.

2.4 Space Situational Awareness Python - ”SSAPy”

Space Situational Awareness Python (SSAPy) is a package developed by a group of astronomers at Lawrence Livermore National Laboratory. The software is usable via Python scripting, although the core libraries of SSAPy are built upon the C and C++ languages. Plans are underway to open-source the entirety of SSAPy, and new utilities continue to be added. The capabilities of SSAPy are numerous, and a comprehensive paper documenting them is in progress for release.

On the surface level, SSAPy offers a customizable and programmable way to integrate orbits. As a user, you have several choices of built-in integrators, and the fidelity of the model can be built using a selection of Earth models, Lunar models, the Sun, planets, radiation forces, and atmospheric drag forces. All gravitational bodies have a point source gravitational model, while the Earth and Moon also include models of gravitational harmonics. For the Earth’s harmonics, WGS84, EGM84, EGM96, and EGM 2008 models are included within SSAPy [3, 6]. For the Moon, a single harmonic model is implemented: the Lunar Gravity Field - GRGM1200A [5]. The output frequency of the evaluated orbit is easily controllable and independent from other time steps, such as the integration time step. This is beneficial if you wish to maintain high fidelity over a very long integrated orbit.

In addition to being an orbit propagator, SSAPy has many other capabilities, such as estimating most astrophysical quantities associated with an orbit, dozens of transformations between coordinate frames, plotting utilities, file management, data linking, and parameter sampling. The platform facilitates uncertainty quantification, short arc probabilistic orbit determination, conjunction probability estimation, and Monte Carlo data fusion for robust decision-making. Its versatility extends to orbital state representations, including Cartesian, osculating Keplerian, and osculating Equinoctial forms. Further, SSAPy accommodates ground-based and space-based observers, enhancing its scope for real-world applications. The software's analysis tools encompass orbit determination, Bayesian posterior state inference via Monte Carlo techniques, and seamless parsing of standard data formats. This comprehensive suite underscores SSAPy's role as a vital tool in advancing space situational awareness and orbital analysis.

The major highlight of SSAPy is that the code is built to take advantage of array broadcasting and parallelization. The C and C++ code bases that SSAPy is built upon are fast and efficient. For similar fidelity models of cislunar orbits, SSAPy can integrate nearly an order of magnitude faster than GMAT or Systems Tool Kit. Unlike other software, SSAPy can also be easily scaled up on HPC systems, enabling the computation of millions of orbits within a 24-hour time period.

3. INTEGRATORS

The numerical integrator from SciPy, a Runge-Kutta (RK) 7/8 in SSAPy, and the symplectic integrator in REBOUND were tested against each other for numerical drift over long periods of time. Numerical drift slowly accumulates due to non-symplectic integrators.

A symplectic integrator employs a specialized numerical integration technique designed for Hamiltonian systems. These integrators fall within the subset of geometric integrators, known for their adherence to canonical transformations. Symplectic integrators ensure the constancy of energy and momentum in the integrated system over time. Due to this property, symplectic integrators find use in various domains, including nonlinear dynamics, molecular dynamics, discrete element methods, accelerator physics, plasma physics, quantum physics, and celestial mechanics.

Orbital systems will accumulate numerical drift over time. Since RK integrators are used in STK, GMAT, and SSAPy, we conducted twenty-year orbital integrations with initial conditions that expose any inaccuracies arising from numerical integration errors.

Figures 1, 2, and 3 show a GEO orbit integrated for 30 days, 1 year, and 20 years, respectively. We tested a 4th-order and an 8th-order Runge-Kutta numerical integrator, the standard SciPy integrator, and the SGP4 model. The numerical drift is defined as the current 3D distance the satellite is from the ideal perfect GEO orbit. A negative distance indicates a position within GEO, while a positive distance indicates a position outside of GEO. In all cases, the RK and SGP4 integrations result in minimal deviations from the circular orbit over each time span of up to 20 years. However, the SciPy integrator experiences a drift of 60 meters after just 30 days and 18 kilometers over 20 years. Over 30 days, the SGP4 shows essentially zero error, but after 20 years, the SGP4 finds itself up to 18 kilometers off the circular orbit as well. The RK methods maintain very low errors of less than 0.1 mm, even after 20 years. The error of the SciPy integrator does not seem to be bounded, while the error from RK is growing logarithmically or even approaching an asymptote.

3.1 Orbital drifts due to integrators and additional perturbative accelerations

Figure 4 shows the deviation from a perfect circular orbit at 1 GEO due to not only numerical drift from the integrator but largely due to perturbations of the Earth's harmonics, the gravitational force from the Moon, Sun, radiation pressures and atmospheric drag. The perturbations due to Earth's harmonics and the Moon seems to keep bounded within a 40 km distance from the perfectly circular GEO orbit.

4. SSAPY VS GMAT VS STK

Figure 5 shows a comparison run between SSAPy and GMAT. For consitency between models the start date, data output frequency (10 seconds), coordinate frame (GCRF), state vectors, and forces are all the same.

Figure 6 are two orbits initialized in the same manner in both STK and SSAPy. For consitency between models the start date, data output frequency (10 seconds), coordinate frame (GCRF), state vectors, and forces are all the same. The orbits are fairly Keplerian, with a semi-major axis of 2 GEO and eccentricity of 0.3. The drift between STK and SSAPy grow rather rapidly, reaching 6 km after only 90 days. The models depart from one another by entire orbits over the course of 5 years.

5. MAGNITUDE OF GRAVITATIONAL ACCELERATIONS ON CISLUNAR ORBITS

5.1 Solar and Planetary Accelerations

Figure 7 illustrates the calculated gravitational tidal accelerations experienced by a satellite in both a circular GEO and an eccentric 8 GEO orbit over a time period of 1.5 times the orbital period of the planets. Although these accelerations are small, they are comparable to radiative effects and can result in kilometer-level drifts or even significant orbital deviations over the course of years. For any mission where the orbit beyond 5 GEO where the Moon's affects are more dominant, small perturbations become increasingly important, as kilometer-level differences can result in drastically different orbital outcomes.

Figure 8 is another illustration of the the gravitational forces due to each planet on a orbit of a given semi-major axis from the Earth. This acceleration is measured when the orbit crosses the x-axis, which is parallell the vernal equinox.

5.2 Non Gravitational Accelerations

Figure 9 shows the magnitude of the accelerations due to Earth and solar radiation pressures on 1 GEO and 8 GEO orbits. Panels (a) and (b) are for a satellite with a cross-sectional area of 1 m^2 and (c) and (d) a cross-sectional area of 100m^2.

6. CISLUNAR MODELS

As observed at the conclusion of section 3, and unsurprising to any reader, additional forces will perturb an orbit significantly over an extended period. While station-keeping is crucial for all satellites, objects in intricate cislunar orbits may demand even more attention. In this section we present the difference between the same orbit integrated under various gravitational models. All models are integrated for 1 year and compared between one another. The models where the Moon is only considered as a point source still move the Moon on its actual eccentric orbit. This is produces more accurate orbits than those from the CR3BP, as in that model the Moon is fixed to a circular orbit.

The models listed by relative accuracy for cislunar orbits:

1. Earth as a point source
2. Circular Restricted Three-Body Problem (CR3BP)
3. SGP4 model
4. Earth and Moon as point sources
5. Earth + EGM84 harmonic model with Moon as a point source
6. Earth EGM96 + Moon as point
7. Earth EGM2008 + Moon as point
8. Earth + EGM84 and Moon + harmonic model
9. Earth + EGM84 and Moon + harmonic model + Sun as a point source
10. Earth + EGM84 and Moon + harmonic model + Sun and all planets as point sources
11. Earth + EGM84 and Moon + harmonic model + Sun with non-gravitational radiation pressure and atmospheric drag
12. Earth + EGM84 and Moon + harmonic model + Sun and all planets with non-gravitational radiation pressure and atmospheric drag

Figure 10 panel (a) and (c) show the drift due to considering the moon on a GEO and 8 GEO orbit. panel (b) and (d) show the drift when including the EGM 84 harmonic model to a point source Earth and Moon. By 5 GEO the pull from the Moon causes 300,000 km deviations between the models within 1 year. This highlights the failure of TLEs for cislunar space. Keplerian motion is non existant over even day time-scales beyond 5 GEO.

Figure 11 shows the drift between a 1 GEO and 8 GEO orbit due to differences in Earth's harmonic gravity model.

Figure 12 shows the drift of the cislunar orbits when adding gravity from all of the Solar System's planets. Perturbations from the planets becomes increasingly important the further from Earth the orbit travels. Within 5 GEO the drifts remain at a couple kilometers but at 8 GEO the orbits are pulled apart by thousands of kilometers.

Figure 13 shows the drift of the cislunar orbits when adding non gravitation forces to the models. The effects of radiation pressure are very similar to the effect of including gravity from all of the planets. The magnitude of the accelerations of each are similar and result in the same order of magnitude drifts between cislunar orbits.

Figure 13 shows how quickly cislunar orbits deviate when the Sun is included in the model. Perhaps unsurprisingly, the Sun is the dominate perturbation besides the Moon. Including the gravitational forces from Sun are important even over day to week time scales if you wish sub kilometer accuracy on your orbit. As with each of the other perturbations the Sun becomes more important the further from the Earth an orbit travels. The simple addition of the Sun will drive precession in any cislunar orbit, and will cause rapid deviation from the CR3BP model.

7. CONCLUSIONS

From the results in this paper we may conclude that Runge-Kutta integrators can be used for integrating cislunar orbits at least over 20 year time spans without any significant error. The need for a sympletic integrator is not required as any loss in fidelity is easily washed out by any other slight model inaccuracies.

SSAPy, STK and GMAT do not exhibit perfect agreement in terms of orbit integrations. Additional effort may be necessary to comprehensively comprehend the discrepancies between them. Errors between the models are on the order of kilometers over 90 days and hundreds of kilometers over 5 years. These differences are not surprising though a better understanding of why these differences occur is needed.

The difference between Earth's harmonic models, EGM 84/96/2008 is the least important factor in computing high fidelity cislunar orbits. The inclusion of a harmonic gravity model is important for maintaining accurate integrated orbits, but the distinction of which harmonic model does not appear to be nearly as important. This is called out because using high order terms in these models and cause signicantly increase computation costs. If the model being used to integrat the orbit is not already including the Sun, Moon's harmonics, planets and non-gravitation forces, then the choice of a high order harmonic model is lost in the noise with the expense of slower computation.

The perturbations from the planets specifically Venus, Mars, Jupiter and Saturn are on the order of those due to radiation from the Sun and Earth. The inclusion of planets should be considered if several thousand kilometer inaccuracies are not reasonable after 1 year integrations. The computational cost of adding the planets is equivalent to simple radiation pressure models. If complex satellite models are being evaluated to calculate radiation pressures, then planets should be included for higher accuracy at a relatively low computational cost.

8. ACKNOWLEDGMENTS

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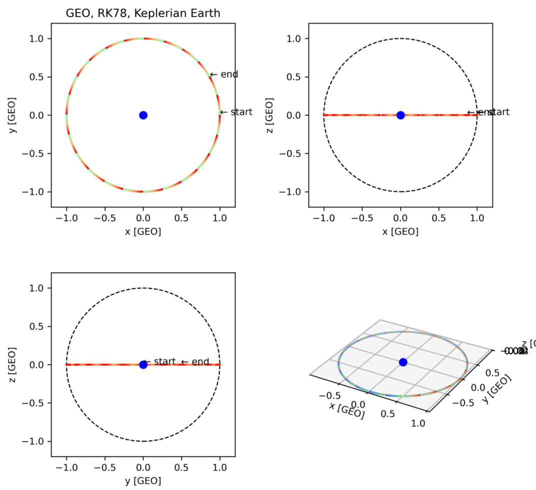
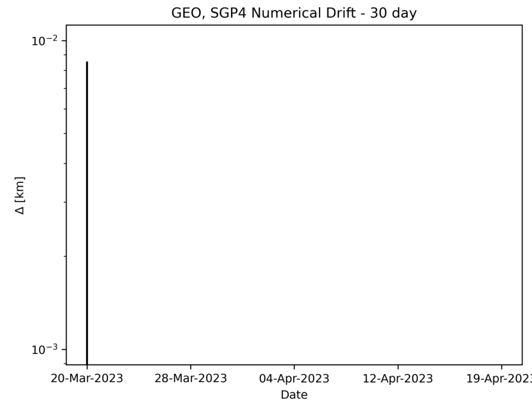
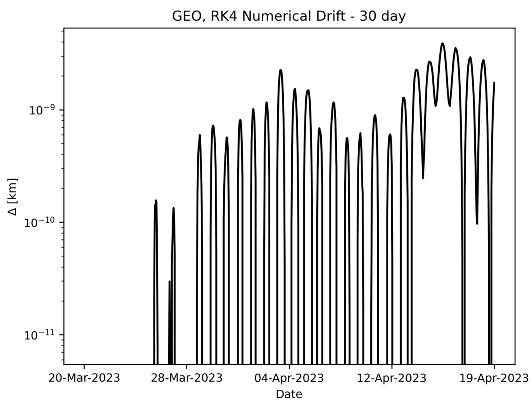


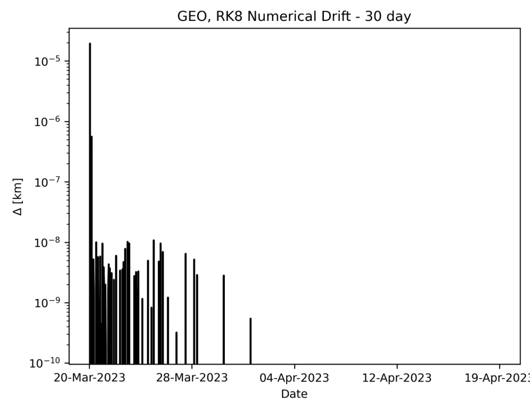
(a) The integrated orbit at 1 GEO.



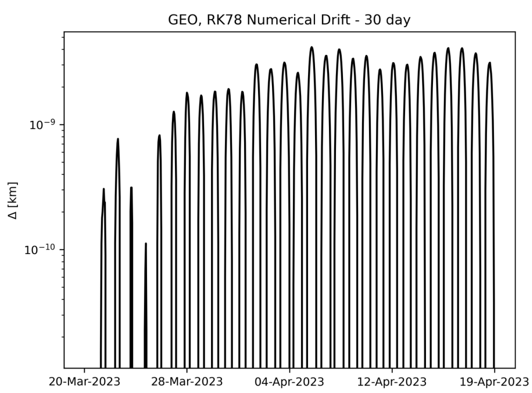
(b) Drift due to SGP4 integrator.



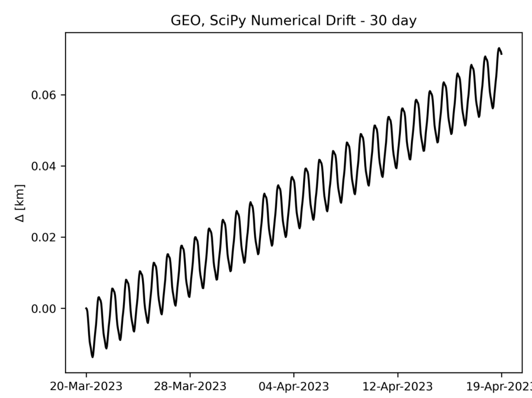
(c) Drift due to the RK4 numerical integrator.



(d) Drift due to the RK8 numerical integrator.

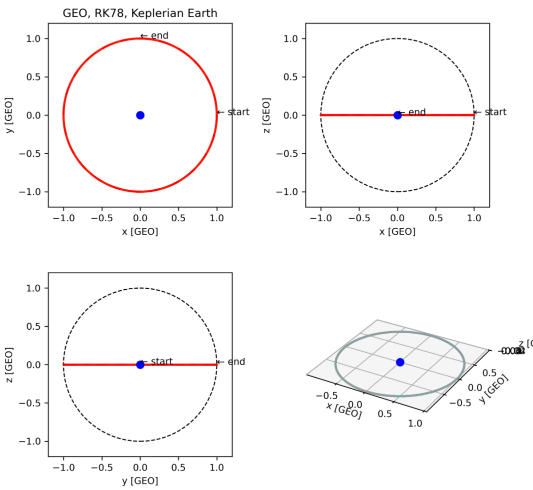


(e) Drift due to the RK78 numerical integrator.

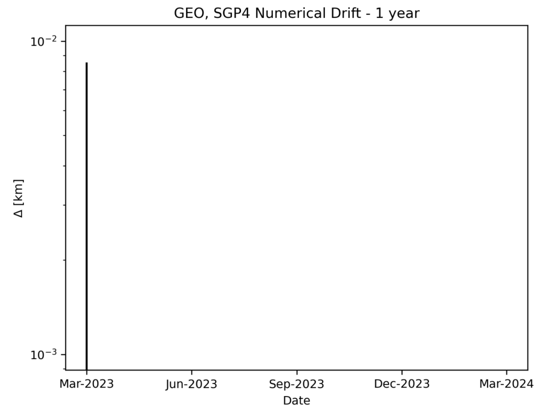


(f) Drift due to the SciPy numerical integrator.

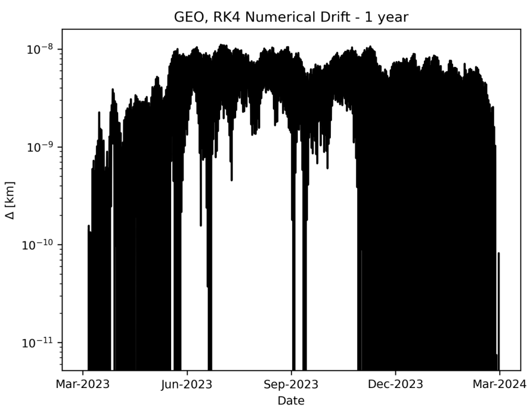
Fig. 1: The numerical drift from a perfect circular orbit at GEO over the course of a 30 day integration.



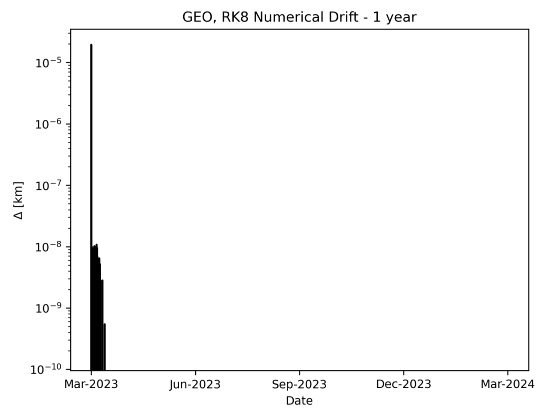
(a) The integrated orbit at 1 GEO.



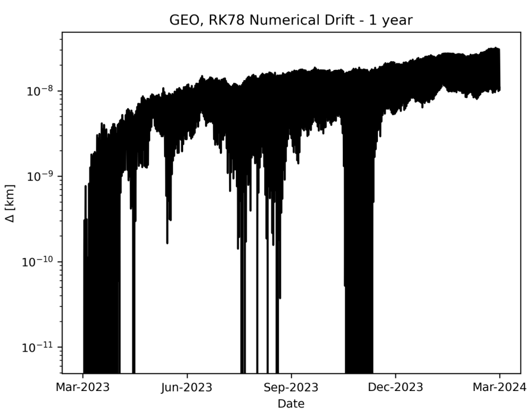
(b) Drift due to SGP4 integrator.



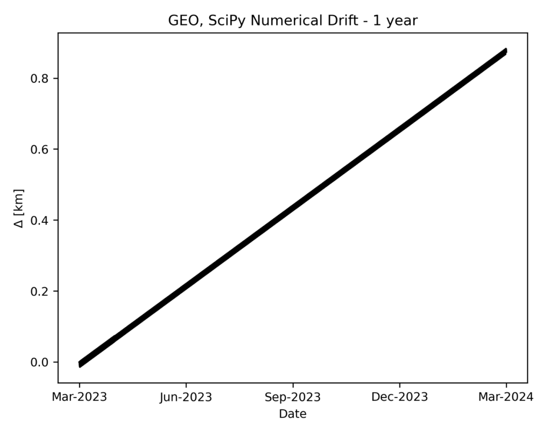
(c) Drift due to the RK4 numerical integrator.



(d) Drift due to the RK8 numerical integrator.

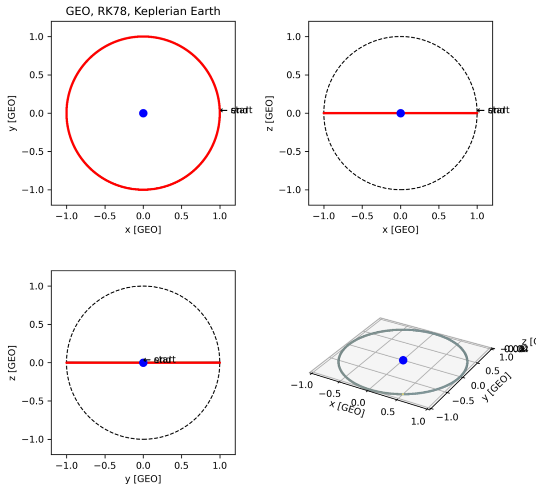


(e) Drift due to the RK78 numerical integrator.

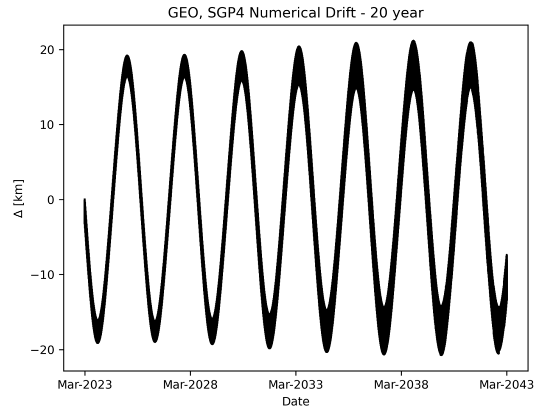


(f) Drift due to the SciPy numerical integrator.

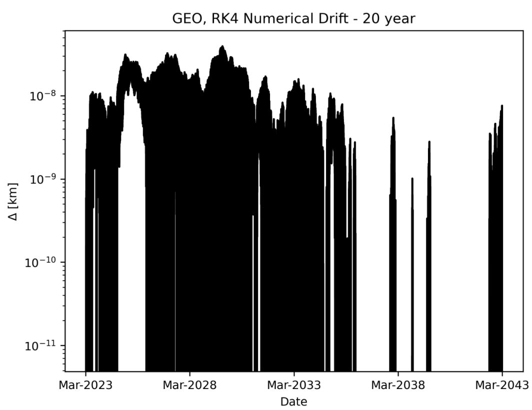
Fig. 2: The numerical drift from a perfect circular orbit at GEO over the course of a 1 year integration.



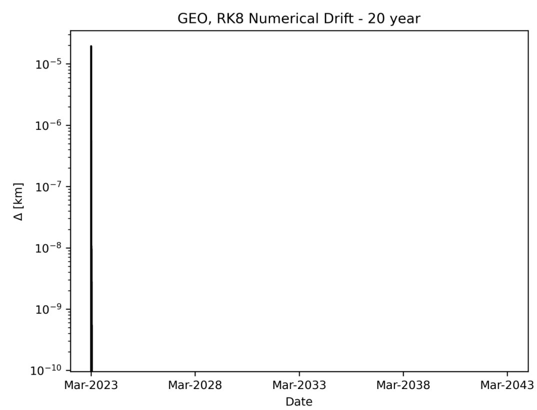
(a) The integrated orbit at 1 GEO.



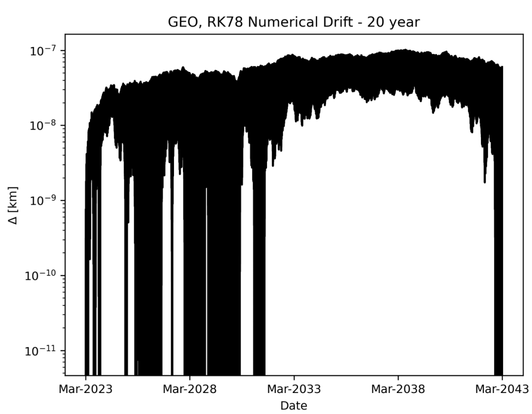
(b) Drift due to SGP4 integrator.



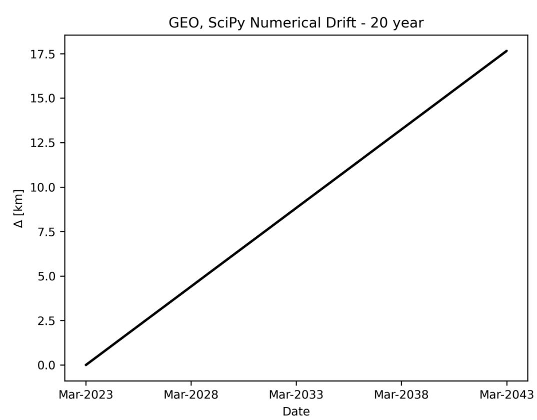
(c) Drift due to the RK4 numerical integrator.



(d) Drift due to the RK8 numerical integrator.

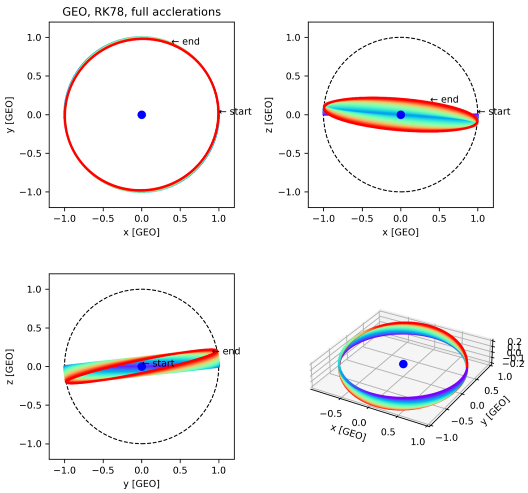


(e) Drift due to the RK78 numerical integrator.

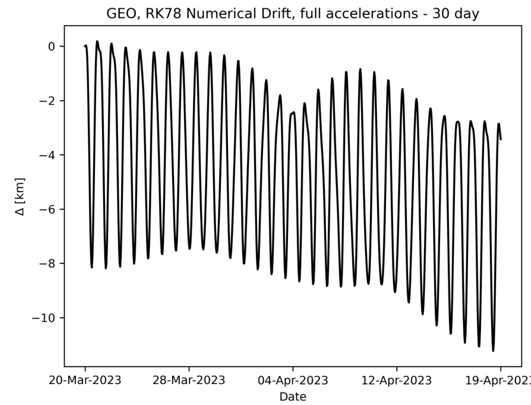


(f) Drift due to the SciPy numerical integrator.

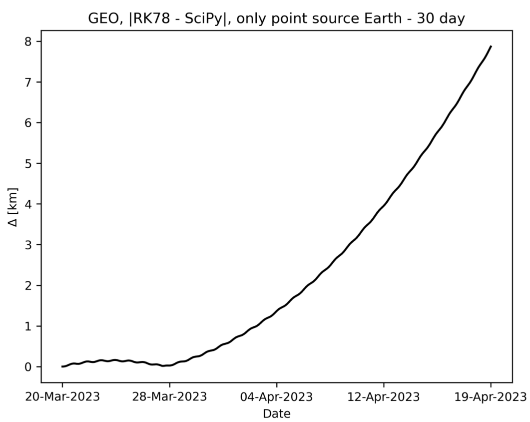
Fig. 3: The numerical drift from a perfect circular orbit at GEO over the course of a 20 year integration.



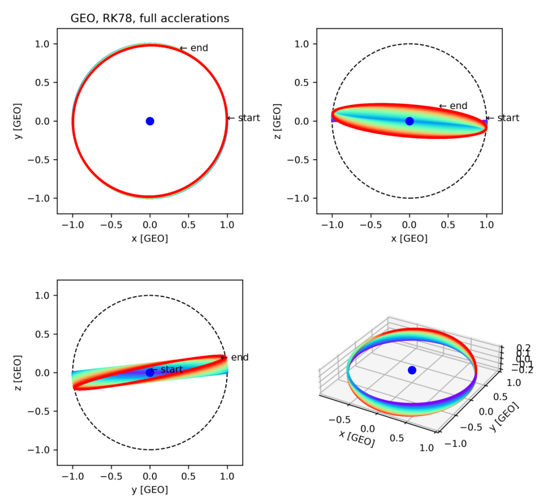
(a) The 30 day integrated orbit at 1 GEO with all accelerations turned on.



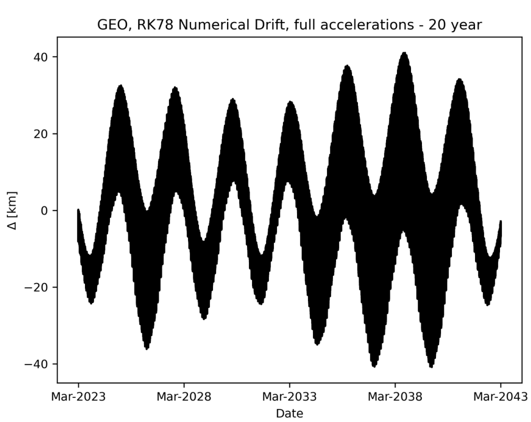
(b) Drift due to gravitational perturbations using the RK78 integrator over 30 days.



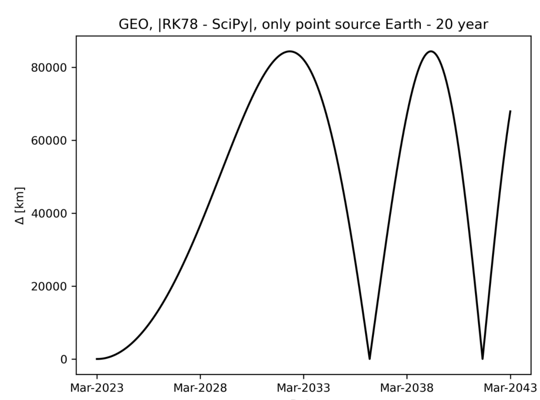
(c) Difference betweem drift using the RK78 numerical integrator and SciPy.



(d) The 20 year integrated orbit at 1 GEO with all accelerations turned on.

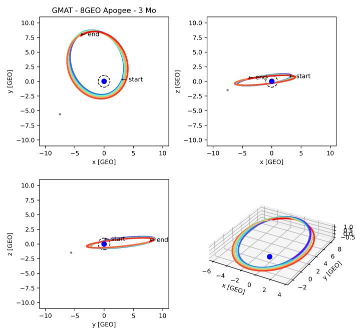


(e) Drift due to gravitational perturbations using the RK78 numerical integrator over 20 years.

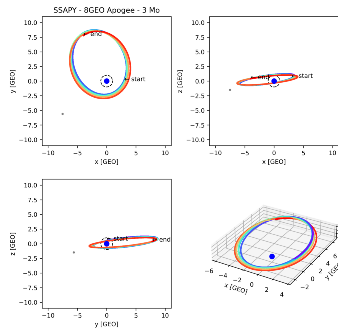


(f) Difference betweem drift using the RK78 numerical integrator and SciPy.

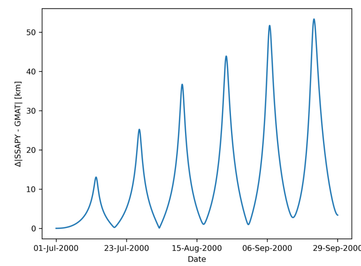
Fig. 4: The drift from a perfect circular orbit at GEO over the course of a 30 day and 20 year integration. This drift is not due to the only the integrator, rather the extra perturbations from Earth and Moon's non-symmetric gravitational forces.



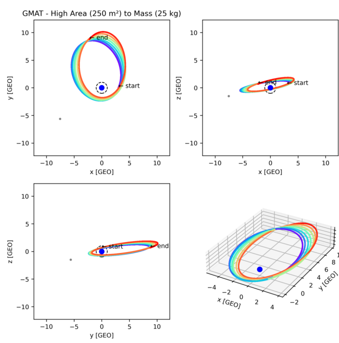
(a) Integrated by GMAT, an orbit of semi-major axis 8 GEO, 0.3 eccentricity and no initial inclination.



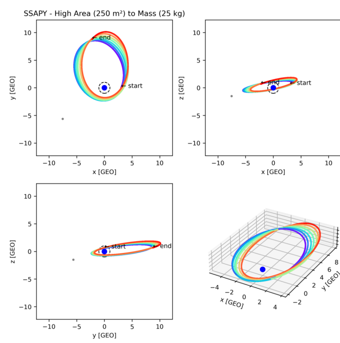
(b) Integrated by SSAPy, an orbit of semi-major axis 8 GEO, 0.3 eccentricity and no initial inclination.



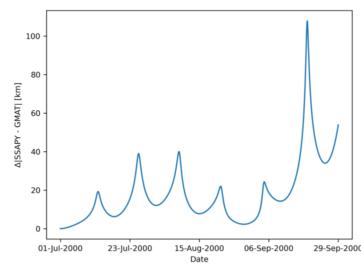
(c) Drift between GMAT and SSAPY over 90 days.



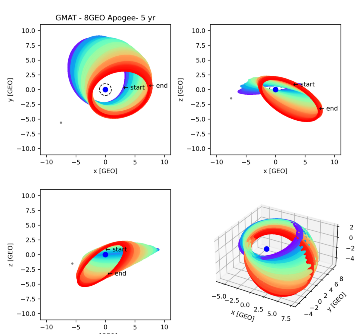
(d) Integrated by GMAT, same set up as subfigures (a)-(c) but now the area of the satellite is increased to 250 m^2 from 0.22 m^2.



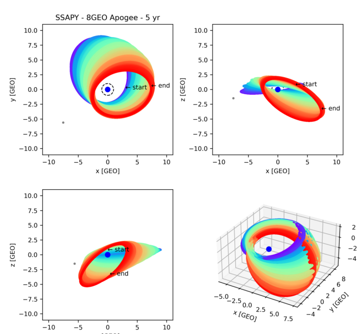
(e) Integrated by SSAPy, same set up as subfigures (a)-(c) but now the area of the satellite is increased to 250 m^2 from 0.22 m^2.



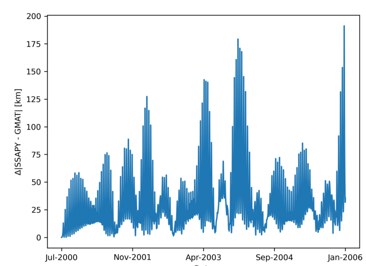
(f) Drift between GMAT and SSAPY over 90 days.



(g) A 5.5 year integration by GMAT.

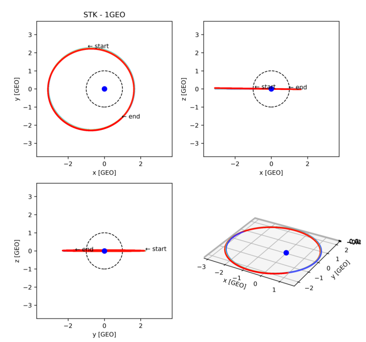


(h) A 5.5 year integration by SSAPy.

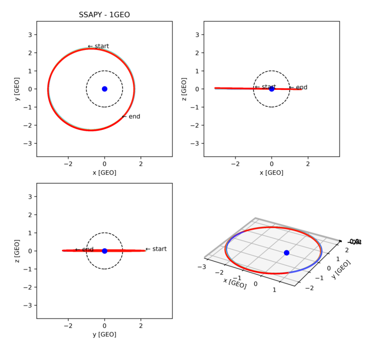


(i) Drift between GMAT and SSAPY over 5.5 years.

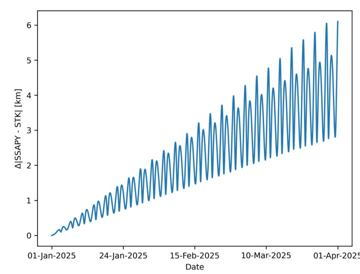
Fig. 5: The initial orbit is an orbit of semi-major axis 8 GEO, 0.3 eccentricity and no initial inclination. Perturbations from the Moon, Sun and radiation pressures cause the continuous drift of the orbit.



(a) Integrated by STK for 90 days.

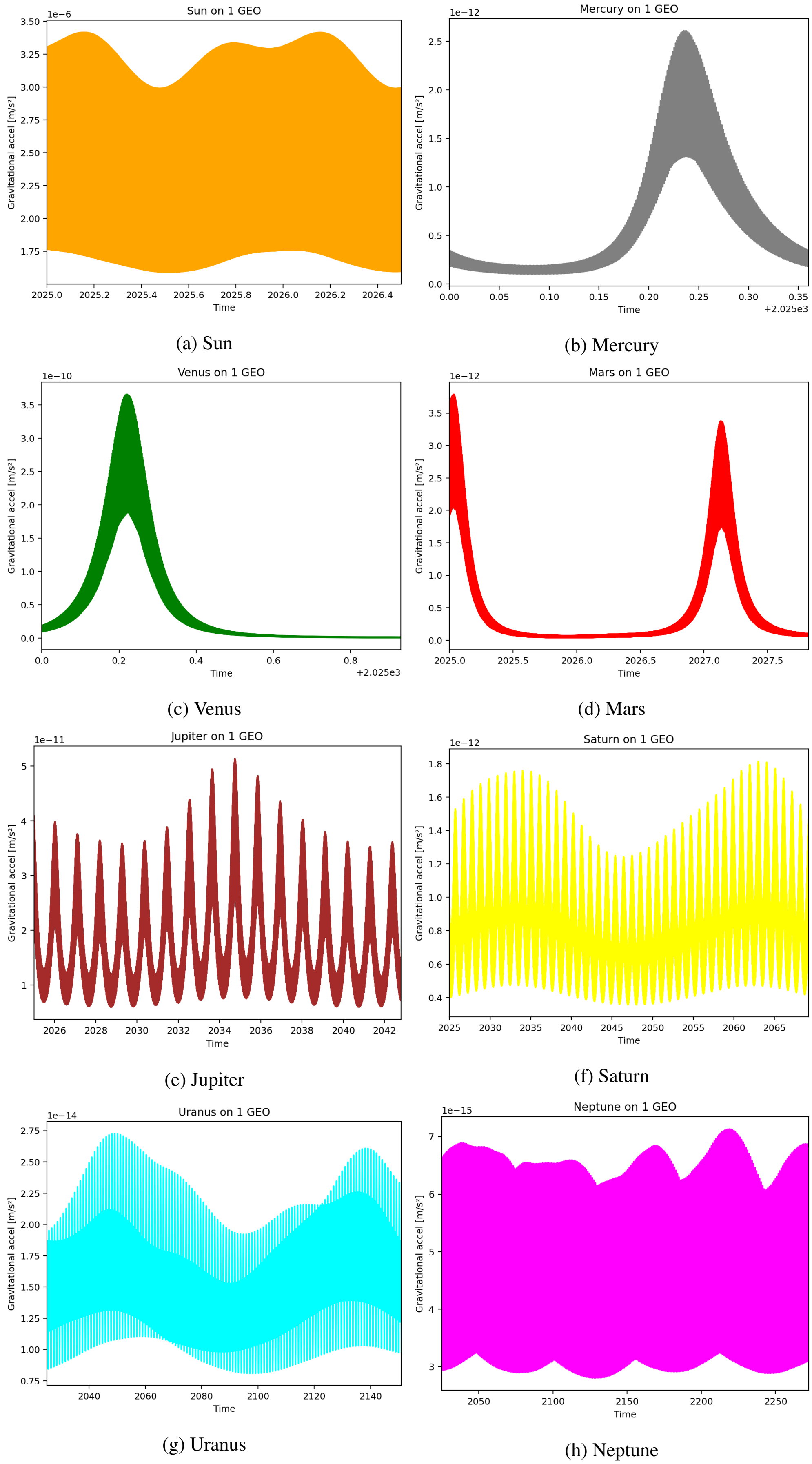


(b) Integrated by SSAPy for 90 days.



(c) Drift between GMAT and SSAPY over 90 days.

Fig. 6: The initial orbit is an orbit of semi-major axis 2 GEO, 0.3 eccentricity and no initial inclination. The model used for this comparison was Earth (EGM2008) and the Moon and Sun as point source and radiation pressure from both the Earth and Sun.



(a) Sun

(b) Mercury

(c) Venus

(d) Mars

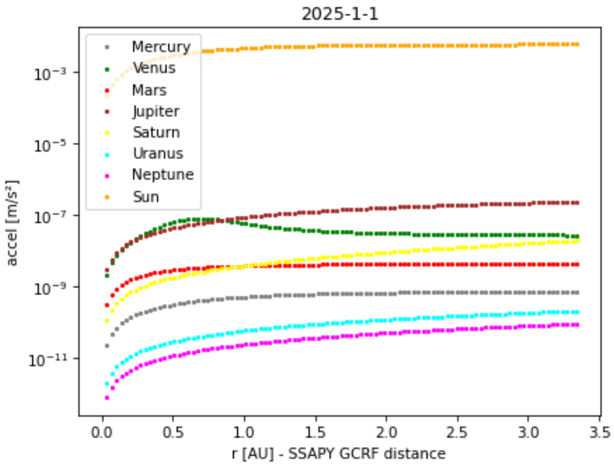
(e) Jupiter

(f) Saturn

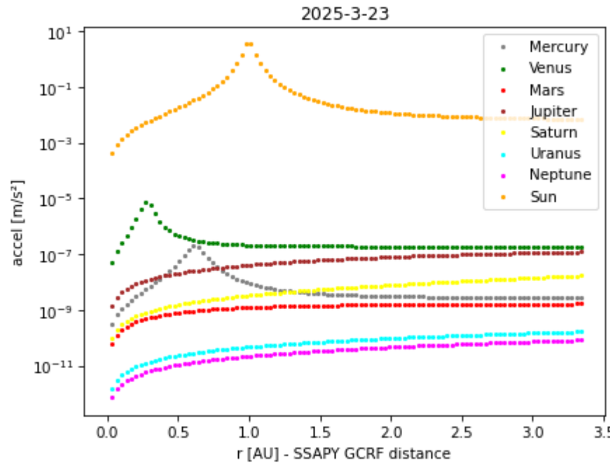
(g) Uranus

(h) Neptune

Fig. 7: The gravitational force due to the planets on an object in a GEO orbit over the course of one and a half periods of the planets orbit.

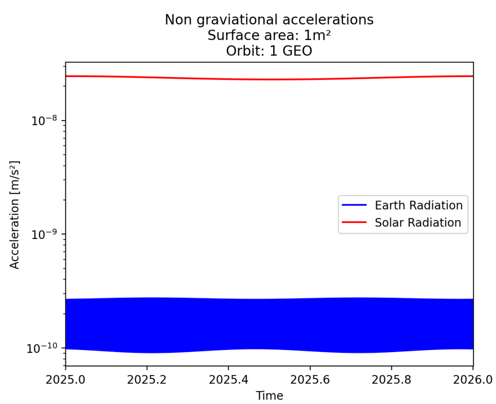


(a) Sun

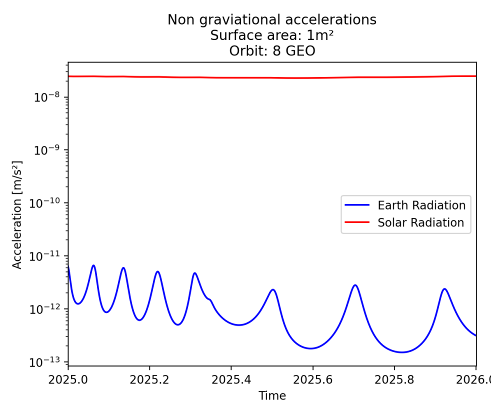


(b) Mercury

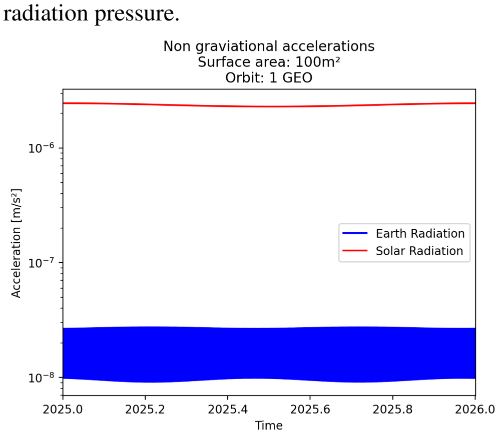
Fig. 8: The gravitational force due to the planets on an object in a GEO orbit over the course of one and a half periods of the planets orbit.



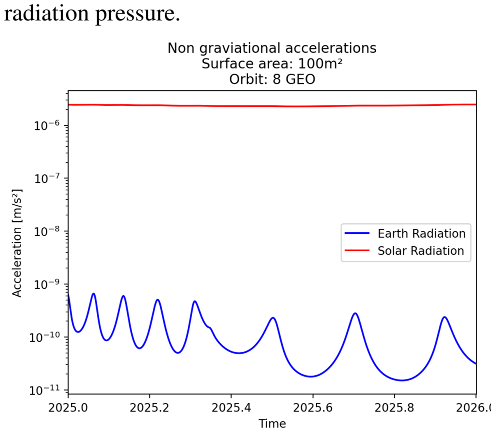
(a) Accelerations on a satellite with 1m^2 cross-section in an 1 GEO orbit due to Earth and solar radiation pressure.



(b) Accelerations on a satellite with 1m^2 cross-section in an 8 GEO orbit due to Earth and solar radiation pressure.

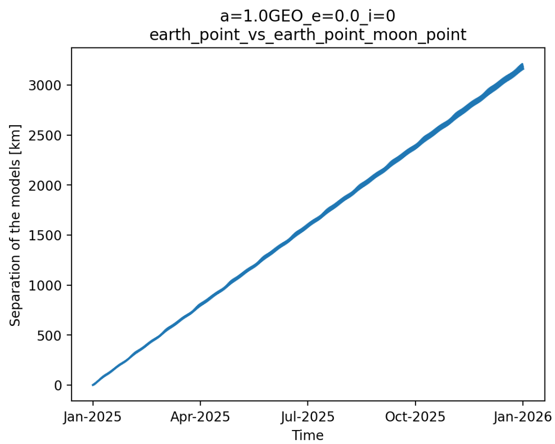


(c) Accelerations on a satellite with 100m^2 cross-section in an 1 GEO orbit due to Earth and solar radiation pressure.

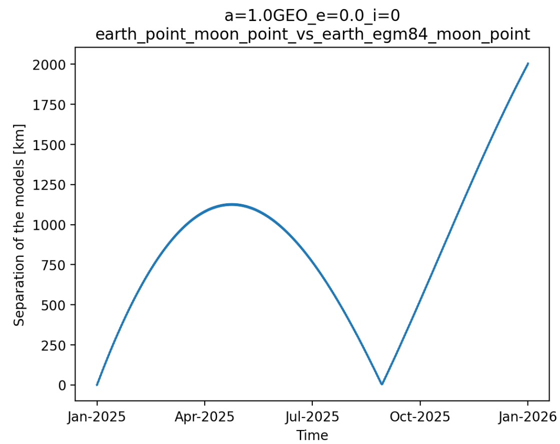


(d) Accelerations on a satellite with 100m^2 cross-section in an 8 GEO orbit due to Earth and solar radiation pressure.

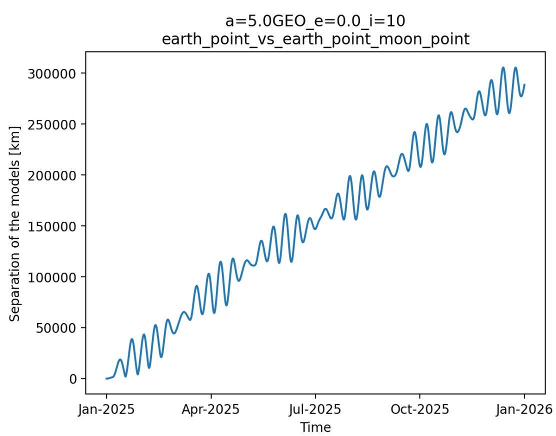
Fig. 9: Strength of the accelerations due to radiation forces on satellites at GEO and 8 GEO.



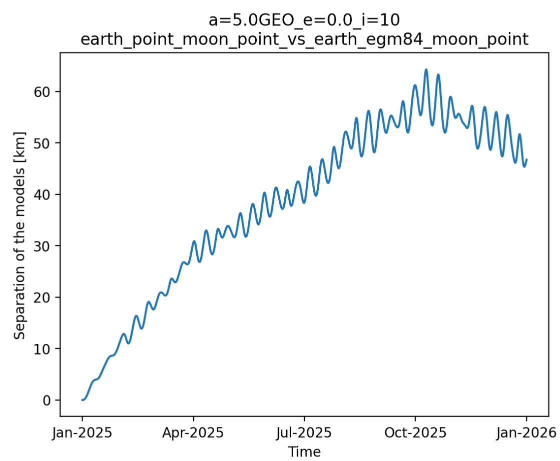
(a) 1 GEO drift due to including a point source moon vs no moon.



(b) 1 GEO drift when including EGM 84 harmonics to a point source Earth and Moon model.

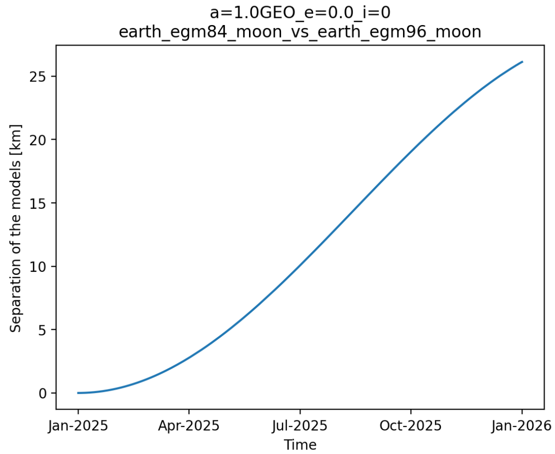


(c) 5 GEO drift due to including a point source moon vs no moon.

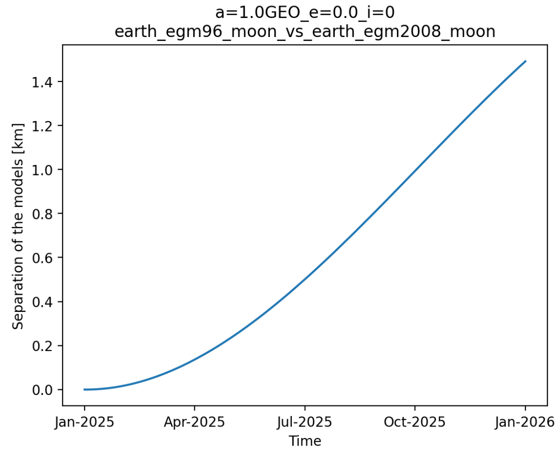


(d) 5 GEO drift when including EGM 84 harmonics to a point source Earth and Moon model.

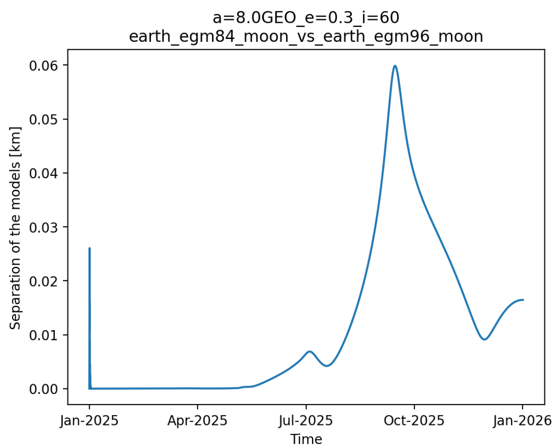
Fig. 10: Earth and Moon as point sources, vs Earth with harmonics.



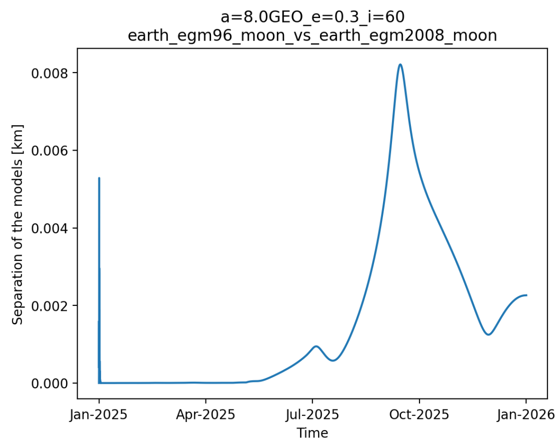
(a) EGM 84 vs EGM 96



(b) EGM 84 vs EGM 2008

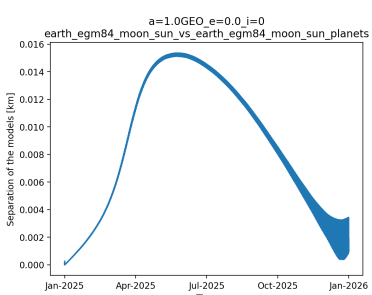


(c) EGM 84 vs EGM 96

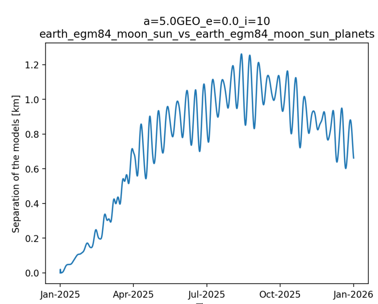


(d) EGM 84 vs EGM 2008

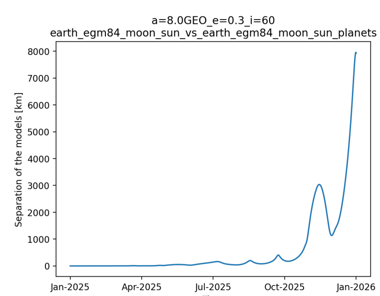
Fig. 11: Comparing Earth harmonic models



(a) Effect of planets on a 1 GEO orbit.

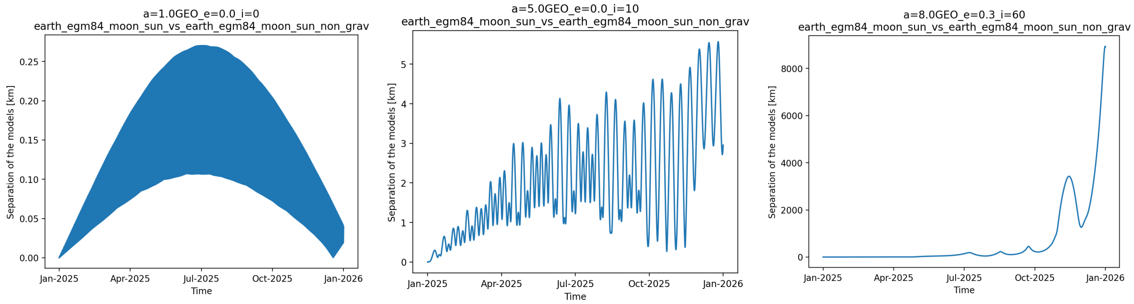


(b) Effect of planets on an 5 GEO cis-lunar orbit.



(c) Effect of planets on an 8 GEO cis-lunar orbit.

Fig. 12: Comparing models with and without the Solar System's planets. Each model has a base of the Earth and Moon with harmonic gravity and the Sun as a point source.

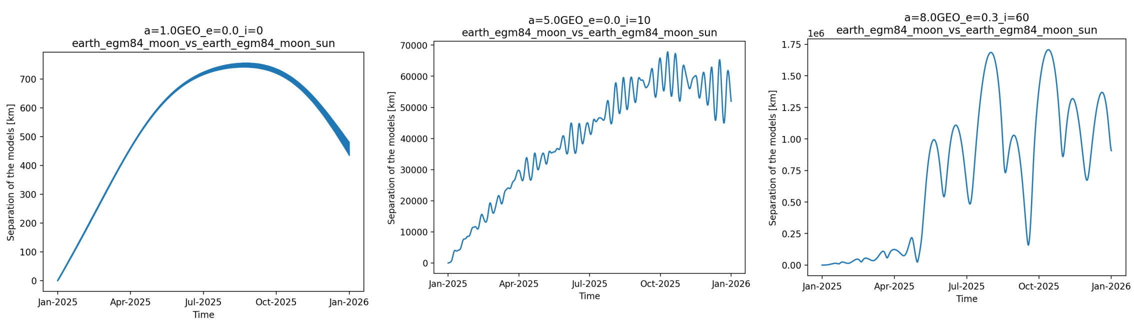


(a) Effect of radiation pressure on a 1 GEO orbit.

(b) Effect of radiation pressure on an 5 GEO cislunar orbit.

(c) Effect of radiation pressure on an 8 GEO cislunar orbit.

Fig. 13: Comparing models with and without the non gravitational forces, e.g. radiation pressure from the Sun and Earth. Each model has a base of the Earth and Moon with harmonic gravity and the Sun and all planets as a point sources.



(a) Effect of the Sun on a 1 GEO orbit.

(b) Effect of the Sun on 5 GEO cislunar orbit.

(c) Effect of the Sun on an 8 GEO cislunar orbit.

Fig. 14: Comparing models with and without the gravitational force from the Sun. Each model has a base of the Earth and Moon with harmonic gravity.