Multi-objective Multi-perspective Numerical Optimization of Collision Avoidance Maneuvers Using Differential Evolution

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ABSTRACT

The design of collision avoidance maneuvers in real case scenarios involves intricate decision-making processes, demanding varying fidelity of data and processes at different stages. Mission constraints, propellant constraints, reliability of collision risk estimation, nature of secondary objects and even operator's schedules contribute to the process of decision making. Therefore, it is imperative to adopt a multi-perspective approach to the problem formulation involving many (if not all) of the above-mentioned aspects. In this context, the maneuver design for collision avoidance is formulated as a heuristic multi-objective multi-perspective optimization problem in this research and the solution is obtained using Differential Evolution (DE), an evolutionary optimization technique. The objective functions to minimize in the problem formulation are a) mass of fuel used b) the collision probability after maneuver(s) c) the deviation of the maneuvered trajectory from the non-maneuvered nominal trajectory and d) disruption time of routine payload operations (defined as the time span for which the spacecraft deviates from its nominal orbit). The solution search space consists of the number of maneuvers, start times and durations of each maneuver and the components of each maneuver. Altitude constraints are modelled as boundary conditions to the optimization problem. The optimal solutions found indicate that the primary objective of collision avoidance is accomplished, reducing the value of collision probability to commonly accepted limits. The penalties of conducting sub-optimal maneuvers are quantified by comparison with the optimal solution. When the objective function doesn't include the propellant consumed, it is found that there is a significant saving in the optimal solution equivalent to the fuel used for collision avoidance maneuver. Similarly, when the objective function doesn't consider the deviation of the maneuverd trajectory, unacceptably large deviations are incurred. The proposed methodology allows the satellite operators to explore a number of design scenarios and tune the solutions to the exact requirements.

1. INTRODUCTION AND BACKGROUND

Humanity's utilization of space has grown exponentially in the last two decades. There is a rapid increase in the number of Resident Space Objects (RSO) due to the launch of many new satellites and their associated debris. The rate at which close approaches of RSOs are being reported has alarmed the space community at large, calling for a robust infrastructure to monitor the RSOs, to replace legacy systems of sensors and for international cooperation towards sustainable and judicious use of space. Any event of an actual impact spells catastrophe, as a single collision not only leads to the loss of the objects, but also generates thousands of fragments of debris, dispersed practically randomly in new trajectories, further increasing space junk which poses a severe danger of future collision to other RSOs. Thus, a predicted conjunction with a quantified high level of risk warrants the need to consider and plan a Collision Avoidance Maneuver (CAM).

SpaceX alone performed more than 25,000 CAMs between Dec 1, 2022, and May 31, 2023 [1]. These numbers doubled to nearly 50,000 between Dec 1, 2023, and May 31, 2024, after SpaceX decreased the threshold for performing evasive maneuvers by an order of magnitude, while adding more satellites to its mega-constellation, increasing the count to 6,200 spacecraft [1]. The updates in collision risk metrics and increase in RSOs all point to the fact that the number of CAMs performed by operators are only going to increase moving forward.

Performing a CAM, however, is an undesirable event from a satellite operator's perspective, as it consumes available propellant from a satellite affecting the total mission lifetime and consequently, the revenue generated. Furthermore, performing a CAM requires an interruption of normal operations of the payload and deviation from the nominal trajectory. Studies have also shown that performing CAM for a predicted conjunction event increases the risk of collision downstream in the future, due to inadequate methods of calculating orbital uncertainties in the aftermath of a CAM, rendering subsequent collision predictions to be less than accurate for several days [2].

In practice, real case scenarios involve intricate decision-making processes, demanding varying fidelity of data and processes at different stages. Mission constraints, propellant constraints, reliability of collision risk estimation, nature of secondary objects involved and even operator’s schedules contribute to the process of decision making. Therefore, it is imperative to adopt a multi-perspective approach to the problem formulation involving many (if not all) of the above-mentioned aspects.

Taking into consideration the multitude of decisions and objectives to monitor for performing a CAM, tackling it as an optimization problem is beneficial to obtain maneuver solutions that minimize the amount of fuel used while also optimally reducing the Collision Probability (Pc), having a great significance on the mission lifetime. Formulation of an optimization problem allows for modelling of mission operations as well.

2. LITERATURE REVIEW

There is immense wealth of literature for the optimization of collision avoidance maneuvers and most of them adopt the reduction of collision probability and minimization of delta-V as the objective function. A gradient based optimization technique was employed in [3] to determine the optimal CAM solution and the determination of optimal maneuver direction and magnitude were decoupled. This allowed the optimal maneuvering direction to be determined using the gradient of the collision probability with respect to the maneuver direction and the optimal maneuver magnitude to be determined using simple root finding numerical techniques. The design of optimal CAM is posed as a convex optimization problem and solved using linearized dynamics in [4]. The maneuvers are modeled as a set of impulses allowing it to be used with propulsive systems across any thrust range and tackles the objective of propellant consumption with constraints either on collision risk or miss distance. An analytical solution to the problem of finding miss distance and collision probability is described in [5] wherein the optimal maneuver direction is determined as solution to the linearized eigenvalue problem. The solutions for maximizing the miss distance and minimizing the collision probability are presented and the differences are highlighted. The solved numerical examples indicate that when the maneuver is carried out less than several orbits ahead, the optimal direction is different from being purely tangential and involves a significant out-of-plane component [5].

As a contrast to treating collision probability and miss distance separately, [6] tackles these two aspects simultaneously as constraints to a non-linear optimization problem in addition to the maneuver time and time duration between the maneuver and TCA. A two-step process is described wherein the first step is to conduct a grid search for forming the initial guesses to the problem and the second step is the nonlinear optimization process. The optimality of the solutions is compared against a Monte-Carlo based simulation as the reference.

Apart from the above-mentioned studies, there have been several efforts to incorporate the actual mission constraints into the objective function formulation and increase the fidelity of the solution to better suit the requirements of the satellite owner/operators. Because no two orbital missions are essentially the same, the problem formulation approach is heuristic in nature and is typically solved using evolutionary optimization techniques. A multi-objective collision avoidance maneuver design strategy for satellites in both GEO and LEO orbital regimes is proposed in [7]. The different aspects considered are: 1) the fuel to be minimized, 2) the deviations of the maneuvered trajectory from the nominal trajectory and 3) reduction in the collision probability post maneuver. The objective function includes a scalarized approach where these three aspects are assigned different weights. Specific mission related constraints such as allowable deviations from the GEO orbital slot are modeled as constraints to the optimization problem. A genetic algorithm-based heuristic optimization process is used to derive optimal solutions. The flexibility of the evolutionary algorithms to model the intricacies of the CAM design is demonstrated. Another alternate approach to forming a multi-objective optimization problem is to consider the conflicting objective functions as separate, independent formulations. The solution space will then consist of pareto-optimal fronts, with the possibility of obtaining a number of optimal solutions rather than a single optimal solution in the case of the scalarization approach. A CAM design strategy based on this approach was proposed in [8] and the optimization problem is solved using a swarm particle optimization approach. The possibility of secondary and tertiary conjunctions was considered for the design of CAM for objects in GEO and solutions were ensured to avoid these.

A multi-objective optimization using the NSGA-II algorithm was performed in [9]. In this study multiple objects are considered for a collision threat with a satellite. The optimizing factors modelled were the delta-v and maneuver cycle. Maneuver cycle here is defined as the time interval within which ground track error is maintained. Three different burn strategies were employed for each simulation and their results compared in terms of different objective function contributions.

Optimization of collision avoidance maneuvers was performed using a formulation based on self-adaptive differential evolution in [10]. The single best minimal solution also includes re-insertion maneuvers to return to the nominal orbit using Lambert's problem, by exploring fuel-optimum transfer trajectories for re-insertion numerically. Miss distance and collision probability are tackled as constraints in the study. The solutions are presented for both the two and three impulse approaches for several cases of constraint profiles. The discussion of the solution also highlights the trade-off between ΔV and the maneuver cycle. Maneuver cycle here is defined as the duration the satellite spends away from it's nominal orbit.

Based on the above-mentioned literature, the current work proposes a multi-perspective multi-objective formulation of the design of collision avoidance maneuvers. Similar to the work described in [7], a single objective function is formulated combining different aspects to be optimized using a scalarization approach. The solution approach employs an evolutionary algorithm known as differential evolution. The aim is to determine a single-best optimal solution which can be valuable to the satellite operator. The next section describes the methodology for the problem formulation.

3. METHODOLOGY

3.1 Differential Evolution

Differential Evolution (DE) is a stochastic direct search method which mimics the evolution of living species [11]. For a problem of D unknown parameters, the respective search domains are to be defined (for a vector space, the corresponding components are to be defined as well). From these search bounds, an initial population of size NP is built randomly, following uniform distribution and the objective function is evaluated for each member of the population. The members of this population are tested for violation of path constraints, if any. A new member is formed using three operations of mutation, crossover and selection. The new member will replace the existing member if the corresponding objective function value is lesser than that of the existing member. The process is repeated till a predefined convergence criterion is met. This basic variant of DE is denoted as DE/rand/1/bin.

3.2 Objective function

The motive of the work is to design optimal collision avoidance maneuvers considering the operational constraints of owner/operators of the satellites. The different factors considered are:

1. Mass of the propellant used for the maneuver(s).
2. The probability of collision before and after the maneuver(s)
3. Payload interruption time: defined as the time from the start of the first maneuver to the end of the last maneuver. This assumes that the satellite cannot perform its payload operations when it is maneuvering.
4. Deviation of the maneuvered trajectory from the nominal, non-maneuvered trajectory: quantified as the difference of the states between the non-maneuvered and maneuvered trajectory in the final half orbit at the end of the analysis duration.

Towards incorporating the above-mentioned aspects, the objective function to be minimized is formulated using a scalarization approach as follows:

$$F = F_1 + F_2 + F_3 + F_4 + F_5 \tag{1}$$

where

$$F_1 = W_1 * \left(log_{10}(P_c + 1)\right) \tag{2}$$

$$F_2 = W_2 * m_{prop} \tag{3}$$

$$F_3 = W_3 * t_{interruption} \tag{4}$$

$$F_4 = W_4 * \sum_{i=1}^{N} \left|\vec{P}_{Nominal\_Trajectory} - \vec{P}_{Maneuver\_Trajectory}\right| \tag{5}$$

$$F_5 = W_5 * \sum_{i=1}^{N} \left|\vec{V}_{Nominal\_Trajectory} - \vec{V}_{Maneuver\_Trajectory}\right| \tag{6}$$

In these equations, F is the composite objective function. W_1 to W_5 represent the weights assigned to the individual components of the objective function because the values of physical quantities have different orders of magnitudes. P_c is the value of the collision probability, as read from the Conjunction Data Message (CDM). The formulation of the component F_1 is inspired by the objective function formulated in [8]. m_{prop} is the mass of the propellant in the satellite and $t_{interruption}$ is the payload operations interruption time caused due to the maneuver(s). Terms F_4 and F_5 correspond to the deviation of the maneuvered trajectory from the nominal, non-maneuvered trajectory.

3.3 Design variables

The search space for the optimization problem consists of the following design variables: 1) the number of maneuvers (N_{man}) 2) the start times of each of the maneuvers 3) burn duration for each of the maneuvers and 4) the three components of each maneuver accelerations in radial (R), along-track (S) and cross-track (W) directions [13].

3.4 Search bounds and boundary conditions

In order to derive meaningful solutions from the design of collision avoidance maneuvers, the number of maneuvers (N_{man}) is restricted to lie between 2 and 5. It is ensured that at least one of the maneuvers is always conducted before the time of closest approach (TCA) to avoid illogical solutions where the conjunction is not avoided, but other objectives are met. Further, the analysis duration is restricted to span for two days; one day prior to TCA and one day post the TCA. The duration of each burn is restricted to lie between 1 and 30 seconds. The thrust for each burn is maintained constant at 5 N (with a specific impulse of 250s), resulting in search bounds for the acceleration components. The mass of the primary (maneuvering satellite) is constantly updated during the numerical integration of equations of motion. An assumed operational altitude constraint of 10km is also introduced, where the position deviation of the maneuvered trajectory from the nominal trajectory is checked at every time-step of the numerical integration process.

3.5 Computational algorithm

A step-by-step computational algorithm is mentioned below:

1. ***Re-creating the collision scenario:*** the purposes of this step are two-fold: a) it checks the conformance of the close approach to the (different) data source and (different) processes used in-house compared to those used for generating the CDM and b) generation of the non-maneuvered trajectory for comparison with the maneuvered trajectory. First, using the data ingested from the CDM, initial states one day prior to TCA are obtained through back propagation of states and covariances for both the objects involved in the conjunction. These initial states are then propagated numerically for two days, thereby obtaining the nominal, non-maneuvered trajectories for the entire analysis duration for both the objects.
2. ***Generation of initial population:*** An initial population of size NP is built. Each member of the population consists of $5 * N_{man}$ scalar components (N_{man} for start times, N_{man} for burn durations and $N_{man} * 3$ for acceleration components) and the corresponding objective function values. The values for these unknowns

are chosen randomly from their respective bounds. Evaluation of the objective function involves propagation from the initial state with the simulation of maneuvers and the subsequent re-evaluation of terms in the objective function (c.f. Eq. 1). For this, the collision probability is evaluated using the DCA of the maneuvered trajectory at TCA and the interruption time is evaluated as the difference between the epoch at the end of the last maneuver and the epoch the start of the first maneuver. The propellant used is calculated based on the burn duration and the assumed mass flow rate of the propulsion system. Finally, the error in the end states is obtained by the summation of the differences of the maneuvered trajectory from the nominal trajectory.

3. ***Generation of trial population:*** A trial member from the search bounds is generated for each member of the current population through the process of mutation, crossover and selection.
    a. Mutation: A mutant vector is generated using some randomly selected members from the current population such that they are not the same as the member under testing. A scaling factor denoted by F is used for the mutation process, and the mutant member V is generated according to the relation $V_i = U_{R1} + F * (U_{R2} - U_{R3})$. Here R_1, R_2 and R_3 are three distinct random integers chosen from $[1, NP]$ and the variable i varies between 1 and NP. These members are chosen such that they are different from the element under testing (i member), that is R_1, R_2 and R_3 must not be equal to i.
    b. Cross over: The member of the current population under testing and the mutant member together generates the trial member. A parameter 'crossover frequency' (CR) is used to generate a trial member [11]. A random number $rand(j)$ is generated between 0 and 1, for each component of the i th member U for which trial member is to be generated. For each of the component (j), $if\ rand(j) > CR$, the jth component of the ith member of the current population is retained for the trial vector and if $rand(j) \leq CR$, the component in the trial vector is replaced with the jth component of the mutant vector.
    c. The objective function for the trial member is evaluated and the member under testing is replaced by this trial member if the objective function value is lower.
4. Generation of trial members and subjecting them to the above three operations is carried out for all the members in the current population and thus a new population is generated.
5. The above-mentioned steps are repeated till a pre-defined convergence criterion is met. For the current problem, the convergence criterion chosen is that the difference between the maximum and minimum values of objective functions between two successive generations should be less than 1.E-3.

A high-level depiction of the above-mentioned computational algorithm is presented in Fig. 1.

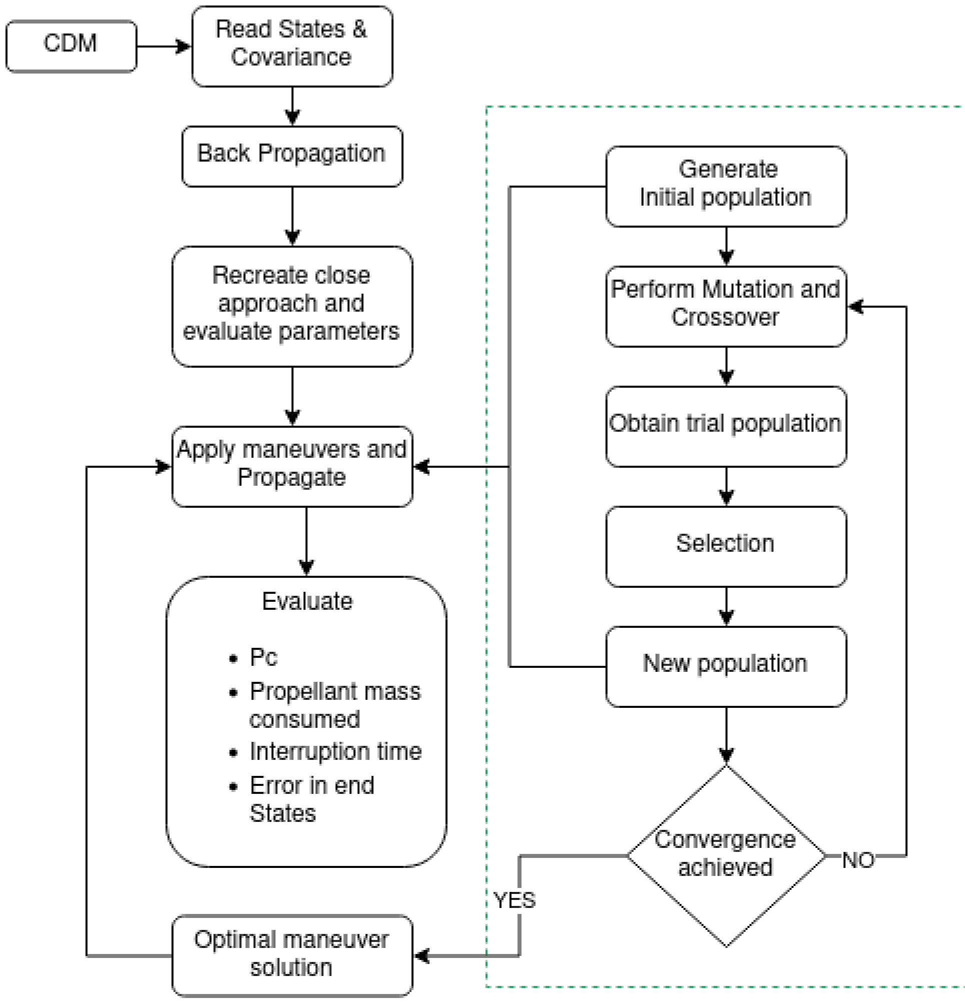

Fig. 1. A high level flowchart of the design of collision avoidance maneuvers. The aspects involving differential evolution are marked in green.

4. RESULTS

A real-world conjunction scenario is chosen for the analysis and is generated using proprietary data ingested into the in-house developed conjunction analysis tool. A close approach was identified between two active satellites 558xx and 394xx in LEO, predicted at 08:41:00 UTC on 07/08/2024. The Distance at Close Approach (DCA) was 1.28808 km, with radial, along-track, and cross-track separations of -1.11 km, 156.58m and 631.78 m respectively. The relative velocity at TCA was 14.09 km/s with the maximum collision probability value of 2.99598591E-04. The collision probability technique employed is Alfano Max Probability [12]. The parameters other than the DCA used are Aspect Ratio (AR) of 55 and a hard body radius of 5m. The value of aspect ratio, i.e. the ratio of standard deviations of major and minor axis of the combined covariance ellipse is derived from the computed covariances. The design variable number of maneuver (N_{man}) is fixed at three for illustration purposes. The DE-related parameters are set as Mutation factor F = 0.5, cross over ratio CR = 0.7 and NP = 75. The weights used in the objective function are: $W_1 = 5E5$, $W_2 = 50$, $W_3 = 1$, $W_4 = 10$ and $W_5 = 100$.

The optimal solution is presented in Table 1.

Table 1: Optimal maneuver design for the close approach

Burn no:	Maneuver start (UTC)	Maneuver duration (s)	Radial Acceleration (m/s^2)	Along track Acceleration (m/s^2)	Cross track Acceleration (m/s^2)
1	24/08/06 16:11:45.746	1.543	-0.025618	-0.010933	-0.019118
2	24/08/07 12:02:05.953	1.031	-0.006495	0.032982	-0.003367
3	24/08/08 08:38:01.549	2.130	-0.015870	-0.029657	-0.003162

The first maneuver avoids the close approach leading to a Pc reduction to a value of 5.2325e-5 with the new DCA being 3.1061 km. The second and the third maneuver performed restituting burns to minimize the error between the maneuvered primary satellite and its nominal orbit. The propellant mass used is 9.5944 grams. The ΔV for each burn is 5.21 cm/s, 3.48 cm/s and 7.19 cm/s bringing the total consumed ΔV to 15.89 cm/s. The evolution of the maneuverd trajectory in RSW coordinates is depicted in the Fig. 2.

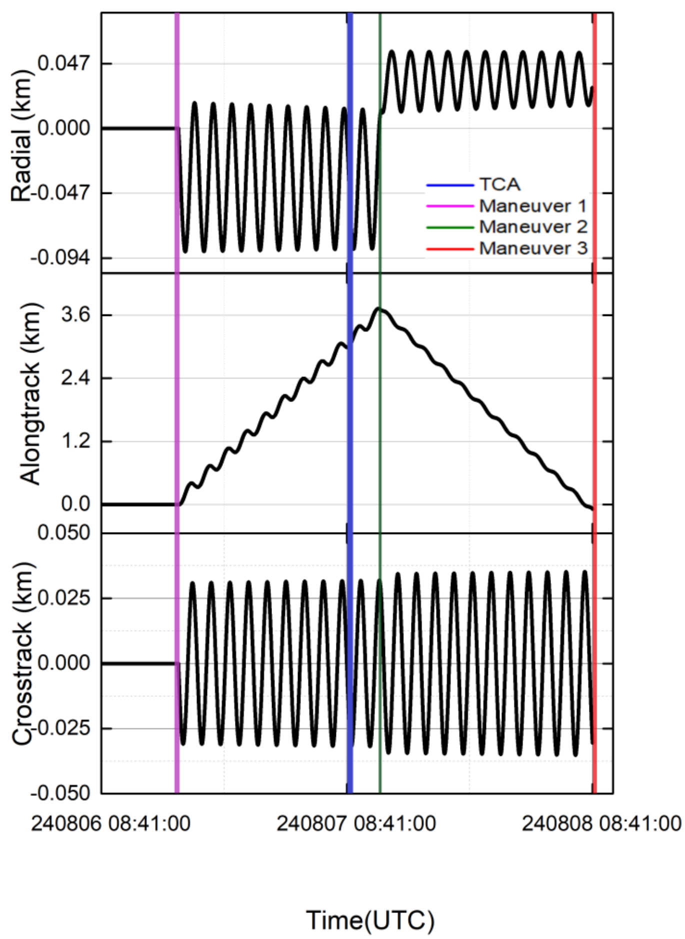

Fig. 2. Evolution of radial, along-track and cross-track differences between maneuvered and non-maneuvered trajectories

In order to compare the optimal solution obtained and validate its optimality, a comparative analysis is carried out. The comparative analysis excludes one objective from the objective function at a time by setting its associated weight to 0. The simulation is then carried out, keeping every other parameter unchanged. The solution consequently obtained is termed as a 'sub-optimal' solution. The penalty incurred in the objective excluded is then quantified against the optimal maneuver design solution obtained. This process is repeated for every objective and the results are presented in Table 2.

Table 2 Penalty incurred by exclusion of different components of objective function

Objective excluded	**Penalty incurred compared to optimal solution**
Propellant mass (F_2) / ΔV	11.9 grams of propellant lost (corresponds to ΔV of 19.72 cm/s)
Interruption time (F_3)	- (6 hr 44 min)
End states (F_4 & F_5)	53.16 km deviation at the end of analysis

As can be observed from Table 2, a position error of about 53 kms is incurred by excluding the end states. The high position error in the sub-optimal solution is due to the three maneuvers introduced in the optimization, where all the maneuvers have converged to a solution of a purely along-track burn, leading to a large displacement in the along-track direction.

By excluding the interruption time, the sub-optimal solution performed better by about 6 hours as compared to the optimal solution. This is identified as a trade-off solution where a higher interruption time is incurred, due to the conflicting nature of the objectives in the optimal result. Further exploration into the dynamics of the evolution of individual objective functions can be conducted to pinpoint the reason for this result. Finally, exclusion of the propellant mass leads to a penalty of 11.9 grams of propellant or a ΔV of 19.72 cm/s as compared to the optimal solution.

A normalized convergence is presented based on the difference between the maxima and minima in each generation. The Y-axis represents the objective function normalized with respect to the highest value in the difference of the objective function. The convergence is as depicted in Fig. 3.:

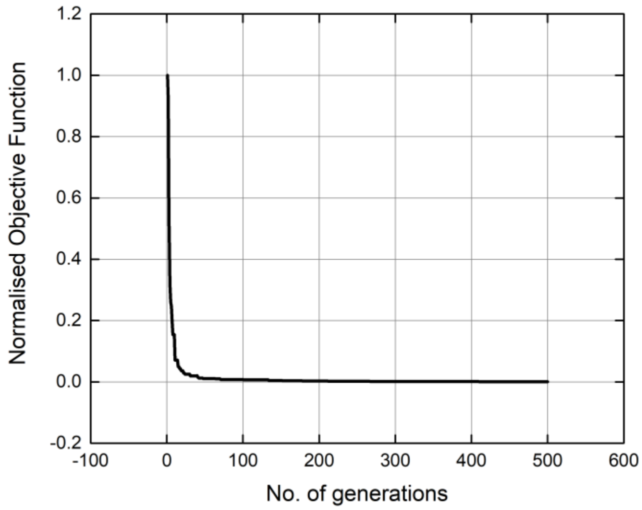

Fig. 3. Convergence pattern in the differential evolution algorithm

5. CONCLUSIONS

The heuristic optimization of collision avoidance maneuvers using differential evolution is presented. A number of objective functions including the mass of propellant, collision probability after maneuver, deviation of the maneuvered trajectory from the non-maneuvered trajectory and reduction of payload interruption time are modelled. The solution search space consists of the number, start time and duration of each maneuver and its components. Altitude constraints are modelled as boundary conditions to the optimization problem. The optimization process is implemented on a real-case close approach in LEO, and the optimal three burn solution is presented. The optimal propellant used for the collision avoidance in the investigated case is found to be 15.89 cm/s. Interestingly, when the simulation is carried out without considering the propellant used as an objective function, a penalty of 19.72 cm/s is found to be incurred. This shows that inclusion of multiple objectives into the problem formulation is essential to derive optimal solutions. Similarly, when the objective function doesn’t consider the deviation of the maneuvered trajectory, unacceptably large position deviations are found to be incurred. The applications of the proposed methodology will benefit the satellite owner/operators with optimal maneuver recommendations and informed decision-making contexts. The modular nature of the methodology allows for a multitude of operational constraints to be modelled, offering flexibility to the operator to tune the optimal it’s to their precise requirements. Extended mission life, confidence and reliability in collision avoidance maneuvers and overall financial gain are the broad outcomes of the proposed research.

Further efforts can made to extend the study towards developing pareto-optimal solutions to present a set of solutions due to the conflicting nature of the objectives. Use of more sophisticated collision probability techniques can be adopted, while also accounting for position and maneuver uncertainties into the problem to present a more complete picture of the performance of the suggested evasive maneuvers.

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